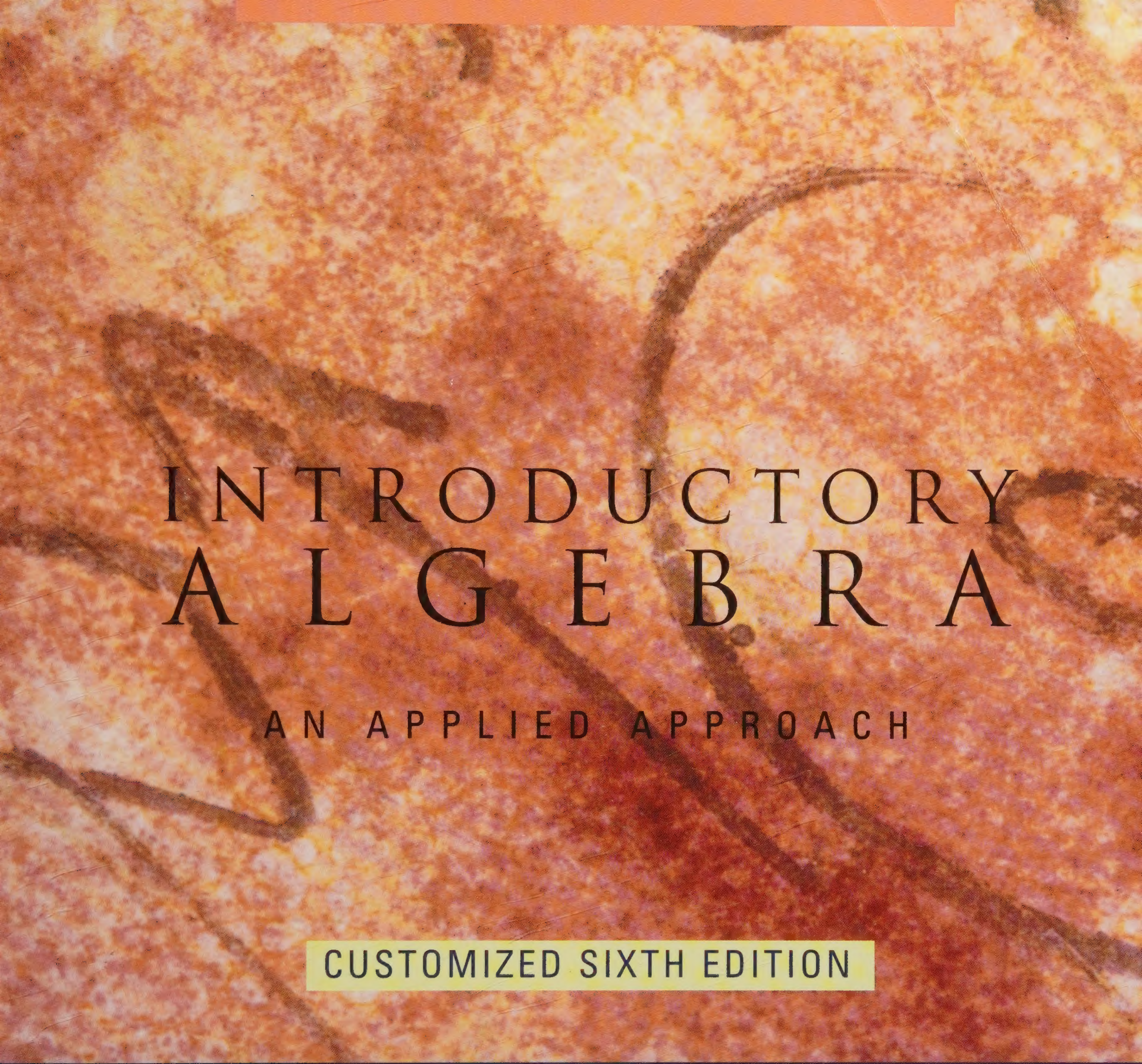

INTRODUCTORY ALGEBRA

AN APPLIED APPROACH

CUSTOMIZED SIXTH EDITION

AUFMANN | BARKER | LOCKWOOD

Table of Properties

Properties of Real Numbers

The Associative Property of Addition

If a, b, and c are real numbers, then
$(a + b) + c = a + (b + c)$.

The Commutative Property of Addition

If a and b are real numbers, then
$a + b = b + a$.

The Addition Property of Zero

If a is a real number, then
$a + 0 = 0 + a = a$.

The Multiplication Property of Zero

If a is a real number, then
$a \cdot 0 = 0 \cdot a = 0$.

The Inverse Property of Addition

If a is a real number, then
$a + (-a) = (-a) + a = 0$.

The Associative Property of Multiplication

If a, b, and c are real numbers, then
$(ab)c = a(bc)$.

The Commutative Property of Multiplication

If a and b are real numbers, then
$ab = ba$.

The Multiplication Property of One

If a is a real number, then
$a \cdot 1 = 1 \cdot a = a$.

The Inverse Property of Multiplication

If a is a real number and $a \neq 0$, then
$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$.

Distributive Property

If a, b, and c are real numbers, then
$a(b + c) = ab + ac$ or $(b + c)a = ba + ca$.

Properties of Equations

Addition Property of Equations

If $a = b$, then $a + c = b + c$.

Multiplication Property of Equations

If $a = b$ and $c \neq 0$, then $ac = bc$.

Properties of Exponents

If m and n are integers, then $x^m \cdot x^n = x^{m+n}$.

If m and n are integers, then $(x^m)^n = x^{mn}$.

If $x \neq 0$, then $x^0 = 1$.

If m and n are integers and $x \neq 0$, then $\frac{x^m}{x^n} = x^{m-n}$.

If m, n, and p are integers, then $(x^m y^n)^p = x^{mp} y^{np}$.

If n is a positive integer and $x \neq 0$, then
$x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$.

If m, n, and p are integers and $y \neq 0$, then $\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}$.

Properties of Inequalities

Addition Property of Inequalities

If $a > b$, then $a + c > b + c$.
If $a < b$, then $a + c < b + c$.

Multiplication Property of Inequalities

If $a > b$ and $c > 0$, then $ac > bc$.
If $a < b$ and $c > 0$, then $ac < bc$.
If $a > b$ and $c < 0$, then $ac < bc$.
If $a < b$ and $c < 0$, then $ac > bc$.

Principle of Zero Products

If $a \cdot b = 0$, then $a = 0$ or $b = 0$.

Properties of Radical Expressions

If a and b are positive real numbers, then $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

If a and b are positive real numbers, then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Property of Squaring Both Sides of an Equation

If a and b are real numbers and $a = b$, then $a^2 = b^2$.

INTRODUCTORY ALGEBRA

AN APPLIED APPROACH

CUSTOMIZED SIXTH EDITION

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Preface

The sixth edition of *Introductory Algebra: An Applied Approach* provides mathematically sound and comprehensive coverage of the topics considered essential in an introductory algebra course. The text has been designed not only to meet the needs of the traditional college student, but also to serve the needs of returning students whose mathematical proficiency may have declined during years away from formal education.

In this new edition of *Introductory Algebra: An Applied Approach,* we have continued to integrate some of the approaches suggested by AMATYC. Each chapter opens by illustrating and referencing a mathematical application within the chapter. At the end of each section there are "Applying the Concepts" exercises that include writing, synthesis, critical thinking, and challenge problems. At the end of each chapter there is a "Focus on Problem Solving" that introduces students to various problem-solving strategies. This is followed by "Projects and Group Activities" that can be used for cooperative learning activities.

NEW! Changes to this Edition

In response to user requests, Chapter 7, *Linear Equations in Two Variables,* now includes a discussion of parallel lines.

In Chapter 8, *Systems of Equations,* the discussion of solutions of systems of equations has been expanded and improved. This development should lead to greater student understanding of dependent, inconsistent, and independent systems of equations.

In Chapter 10, *Radical Expressions,* square roots are simplified by using perfect squares rather than by using prime factorization. Therefore, rather than writing a radicand of 20 as 2^2 times 5, 20 is written as the product of 4 and 5, an approach that better mirrors what the student is thinking mentally when simplifying radical expressions.

Concept-based writing exercises have been added at the beginning of many of the objective-specific exercise sets. These exercises require a student to verbalize basic concepts presented in the lesson. These are the concepts that the student must have an understanding of before attempting any exercises. For example, at the beginning of the exercises for Objective 7.3B, on finding the slope of a straight line, is the question, "What is the difference between a line that has zero slope and one whose slope is undefined?"

Many of the exercise sets now include developmental exercises, the intent of which is to reinforce the concepts underlying the skills presented in the lesson. For example, in Chapter 10, *Radical Expressions,* the student is asked, "Which of the numbers 2, 9, 20, 25, 50, 81, and 100 are *not* perfect squares?"

Throughout the text, data problems have been updated to reflect current data and trends. These application problems will demonstrate to students the variety of problems in real life that require mathematical analysis. Instructors will find that many of these problems may lead to interesting class discussions.

Another new feature of this edition is *AIM for Success,* which explains what is required of a student to be successful and how this text has been designed to foster student success. *AIM for Success* can be used as a lesson on the first day of class or as a project for students to complete to strengthen their study skills. There are suggestions for teaching this lesson in the *Instructor's Resource Manual.*

Related to *AIM for Success* are *Prep Tests* which occur at the beginning of each chapter. These tests focus on the particular prerequisite skills that will be used in the upcoming chapter. The answers to these questions can be found in the Answer Appendix along with a reference (except for Chapter 1) to the objective from which the question was taken. Students who miss a question are encouraged to review the objective from which the question was taken.

The *Go Figure* problem that follows the *Prep Test* is a puzzle problem designed to engage students in problem solving.

Chapter Opening Features

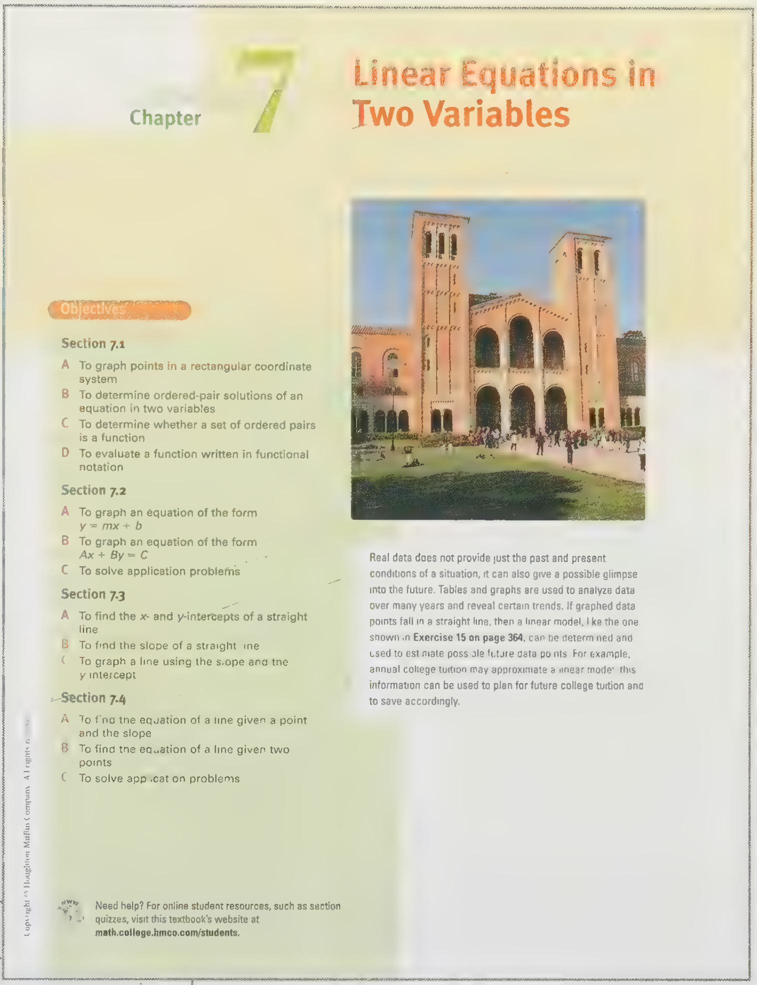
NEW! Chapter Opener

New, motivating chapter opener photos and captions have been added, illustrating and referencing a specific application from the chapter.

The WWW at the bottom of the page lets students know of additional on-line resources at math.college.hmco.com/students.

Objective-Specific Approach

Each chapter begins with a list of learning objectives which form the framework for a complete learning system. The objectives are woven throughout the text (i.e. Exercises, Prep Tests, Chapter Reviews, Chapter Tests, Cumulative Reviews) as well as through the print and multimedia ancillaries. This results in a seamless learning system delivered in one consistent voice.



Chapter 7

Linear Equations in Two Variables

Objectives

Section 7.1

- A To graph points in a rectangular coordinate system
- B To determine ordered-pair solutions of an equation in two variables
- C To determine whether a set of ordered pairs is a function
- D To evaluate a function written in functional notation

Section 7.2

- A To graph an equation of the form $y = mx + b$
- B To graph an equation of the form $Ax + By = C$
- C To solve application problems

Section 7.3

- A To find the x- and y-intercepts of a straight line
- B To find the slope of a straight line
- C To graph a line using the slope and the y-intercept

Section 7.4

- A To find the equation of a line given a point and the slope
- B To find the equation of a line given two points
- C To solve application problems

Real data does not provide just the past and present conditions of a situation, it can also give a possible glimpse into the future. Tables and graphs are used to analyze data over many years and reveal certain trends. If graphed data points fall in a straight line, then a linear model, like the one shown in **Exercise 15 on page 364**, can be determined and used to estimate possible future data points. For example, annual college tuition may approximate a linear model; this information can be used to plan for future college tuition and to save accordingly.

Need help? For online student resources, such as section quizzes, visit this textbook's website at **math.college.hmco.com/students.**

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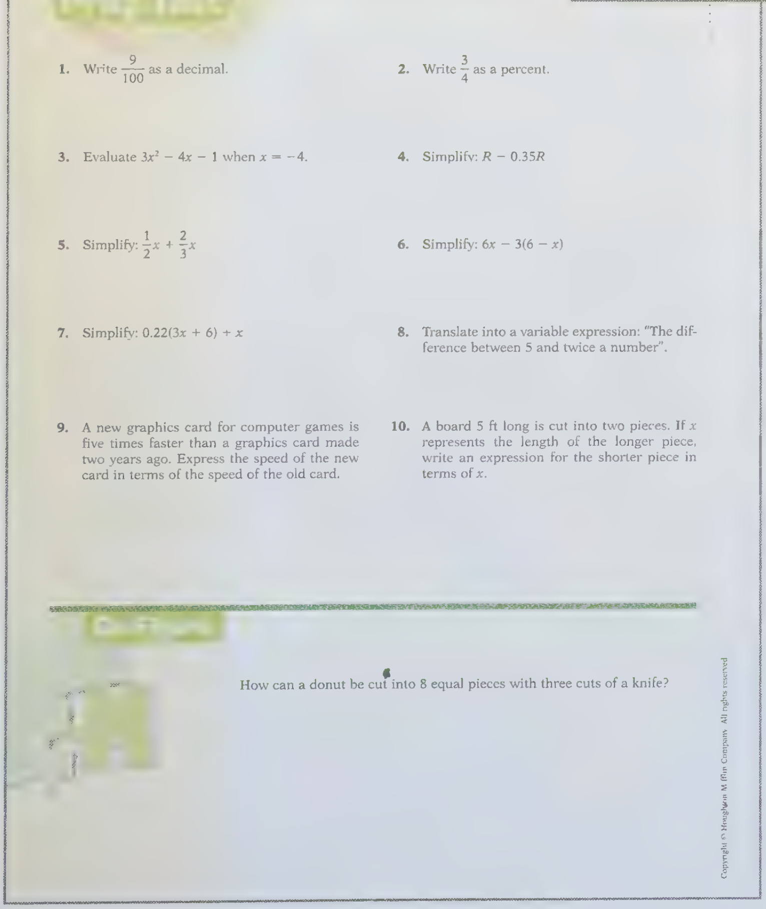


Prep Test

1. Write $\frac{9}{100}$ as a decimal.
2. Write $\frac{3}{4}$ as a percent.
3. Evaluate $3x^2 - 4x - 1$ when $x = -4$.
4. Simplify: $R - 0.35R$
5. Simplify: $\frac{1}{2}x + \frac{2}{3}x$
6. Simplify: $6x - 3(6 - x)$
7. Simplify: $0.22(3x + 6) + x$
8. Translate into a variable expression: "The difference between 5 and twice a number".
9. A new graphics card for computer games is five times faster than a graphics card made two years ago. Express the speed of the new card in terms of the speed of the old card.
10. A board 5 ft long is cut into two pieces. If x represents the length of the longer piece, write an expression for the shorter piece in terms of x.

How can a donut be cut into 8 equal pieces with three cuts of a knife?

NEW! Prep Test and Go Figure

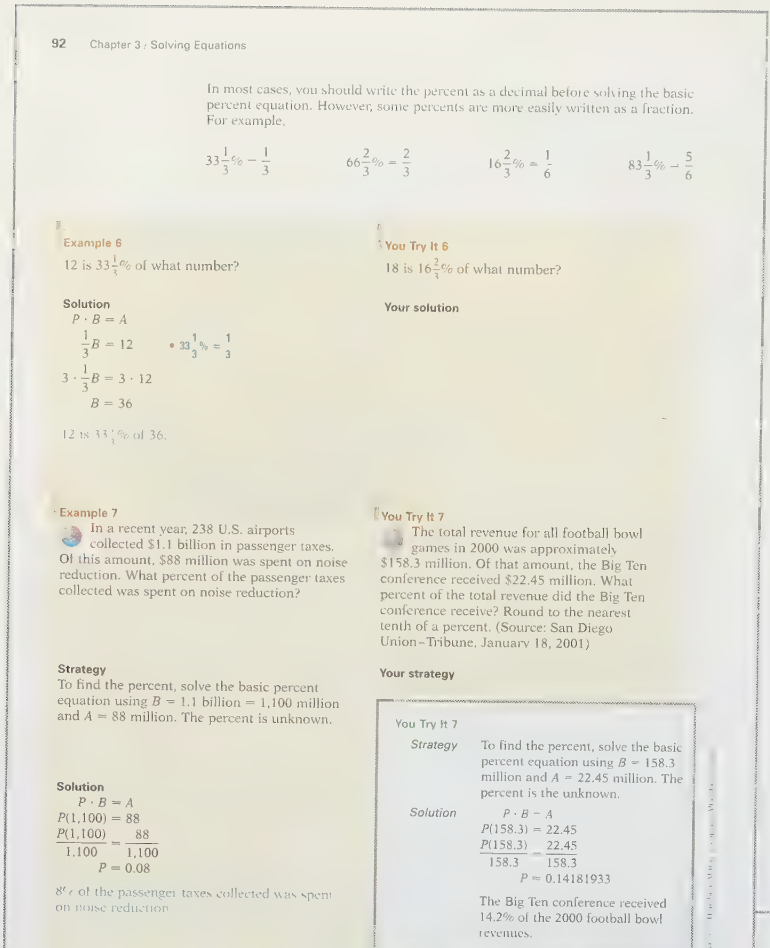
Prep Tests occur at the beginning of each chapter and test students on previously covered concepts that are required in the coming chapter. Answers are provided in the Answer Appendix. Objective references are also provided if a student needs to review specific concepts.

The Go Figure problem that follows the Prep Test is a playful puzzle problem designed to engage students in problem solving.

Aufmann Interactive Method (AIM)

An Interactive Approach

Introductory Algebra: An Applied Approach uses an interactive style that provides a student with an opportunity to try a skill as it is presented. Each section is divided into objectives, and every objective contains one or more sets of matched-pair examples. The first example in each set is worked out; the second example, called "You Try It," is for the student to work. By solving this problem, the student actively practices concepts as they are presented in the text.



92 Chapter 3 / Solving Equations

In most cases, you should write the percent as a decimal before solving the basic percent equation. However, some percents are more easily written as a fraction. For example,

$33\frac{1}{3}\% = \frac{1}{3}$ $66\frac{2}{3}\% = \frac{2}{3}$ $16\frac{2}{3}\% = \frac{1}{6}$ $83\frac{1}{3}\% = \frac{5}{6}$

Example 6

12 is $33\frac{1}{3}\%$ of what number?

Solution

$$P \cdot B = A$$
$$\frac{1}{3}B = 12 \qquad \bullet\ 33\frac{1}{3}\% = \frac{1}{3}$$
$$3 \cdot \frac{1}{3}B = 3 \cdot 12$$
$$B = 36$$

12 is $33\frac{1}{3}\%$ of 36.

You Try It 6

18 is $16\frac{2}{3}\%$ of what number?

Your solution

Example 7

In a recent year, 238 U.S. airports collected $1.1 billion in passenger taxes. Of this amount, $88 million was spent on noise reduction. What percent of the passenger taxes collected was spent on noise reduction?

Strategy

To find the percent, solve the basic percent equation using $B = 1.1$ billion $= 1{,}100$ million and $A = 88$ million. The percent is unknown.

Solution

$$P \cdot B = A$$
$$P(1{,}100) = 88$$
$$\frac{P(1{,}100)}{1{,}100} = \frac{88}{1{,}100}$$
$$P = 0.08$$

8% of the passenger taxes collected was spent on noise reduction.

You Try It 7

The total revenue for all football bowl games in 2000 was approximately $158.3 million. Of that amount, the Big Ten conference received $22.45 million. What percent of the total revenue did the Big Ten conference receive? Round to the nearest tenth of a percent. (Source: San Diego Union–Tribune, January 18, 2001)

Your strategy

You Try It 7

Strategy To find the percent, solve the basic percent equation using $B = 158.3$ million and $A = 22.45$ million. The percent is the unknown.

Solution

$$P \cdot B = A$$
$$P(158.3) = 22.45$$
$$\frac{P(158.3)}{158.3} = \frac{22.45}{158.3}$$
$$P \approx 0.14181933$$

The Big Ten conference received 14.2% of the 2000 football bowl revenues.

Pages 92 and S4

There are complete worked-out solutions to these examples in an appendix. By comparing their solution to the solution in the appendix, students obtain immediate feedback on, and reinforcement of, the concept.

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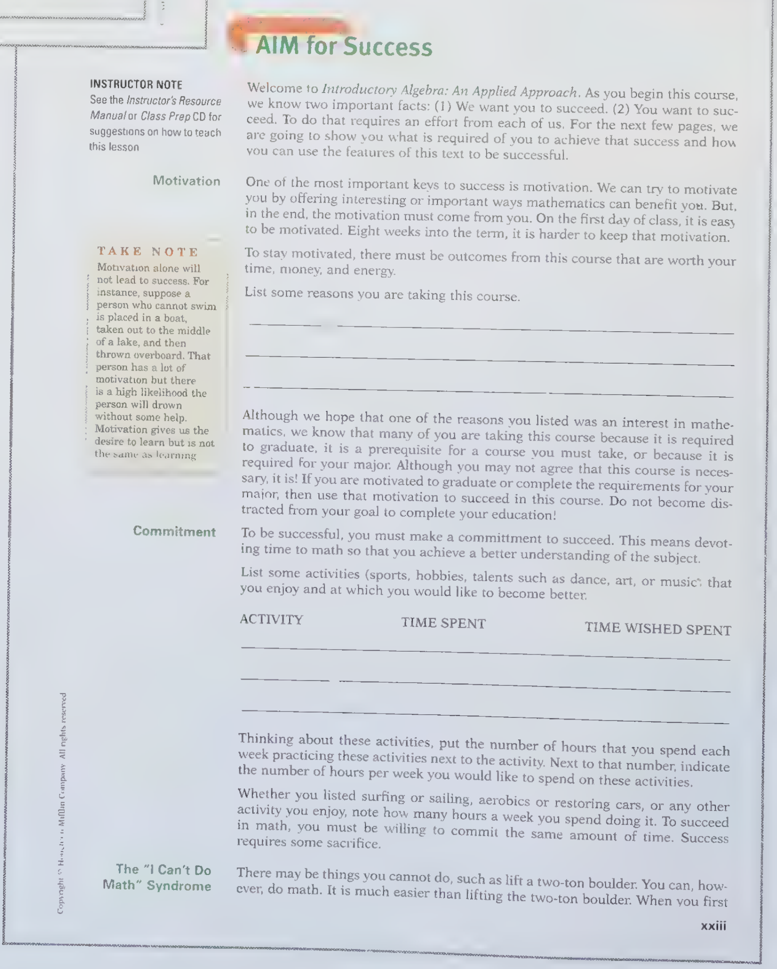

AIM for Success

INSTRUCTOR NOTE
See the *Instructor's Resource Manual* or *Class Prep* CD for suggestions on how to teach this lesson

Welcome to *Introductory Algebra: An Applied Approach*. As you begin this course, we know two important facts: (1) We want you to succeed. (2) You want to succeed. To do that requires an effort from each of us. For the next few pages, we are going to show you what is required of you to achieve that success and how you can use the features of this text to be successful.

Motivation

One of the most important keys to success is motivation. We can try to motivate you by offering interesting or important ways mathematics can benefit you. But, in the end, the motivation must come from you. On the first day of class, it is easy to be motivated. Eight weeks into the term, it is harder to keep that motivation.

To stay motivated, there must be outcomes from this course that are worth your time, money, and energy.

List some reasons you are taking this course.

____

____

____

TAKE NOTE
Motivation alone will not lead to success. For instance, suppose a person who cannot swim is placed in a boat, taken out to the middle of a lake, and then thrown overboard. That person has a lot of motivation but there is a high likelihood the person will drown without some help. Motivation gives us the desire to learn but is not the same as learning

Although we hope that one of the reasons you listed was an interest in mathematics, we know that many of you are taking this course because it is required to graduate, it is a prerequisite for a course you must take, or because it is required for your major. Although you may not agree that this course is necessary, it is! If you are motivated to graduate or complete the requirements for your major, then use that motivation to succeed in this course. Do not become distracted from your goal to complete your education!

Commitment

To be successful, you must make a committment to succeed. This means devoting time to math so that you achieve a better understanding of the subject.

List some activities (sports, hobbies, talents such as dance, art, or music) that you enjoy and at which you would like to become better.

ACTIVITY	TIME SPENT	TIME WISHED SPENT

Thinking about these activities, put the number of hours that you spend each week practicing these activities next to the activity. Next to that number, indicate the number of hours per week you would like to spend on these activities.

Whether you listed surfing or sailing, aerobics or restoring cars, or any other activity you enjoy, note how many hours a week you spend doing it. To succeed in math, you must be willing to commit the same amount of time. Success requires some sacrifice.

The "I Can't Do Math" Syndrome

There may be things you cannot do, such as lift a two-ton boulder. You can, however, do math. It is much easier than lifting the two-ton boulder. When you first

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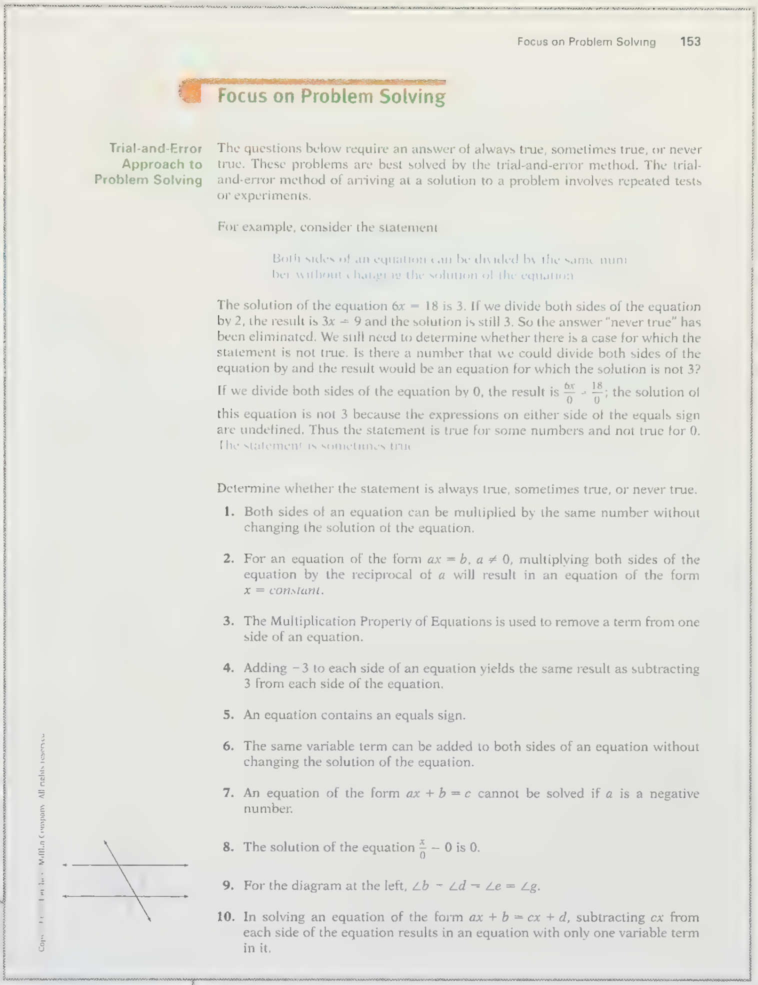
NEW! AIM for Success Student Preface

This new student 'how to use this book' preface explains what is required of a student to be successful and how this text has been designed to foster student success, including the Aufmann Interactive Method (AIM). *AIM for Success* can be used as a lesson on the first day of class or as a project for students to complete to strengthen their study skills. There are suggestions for teaching this lesson in the *Instructor's Resource Manual*.

Problem Solving

Focus on Problem Solving

At the end of each chapter is a Focus on Problem Solving feature which introduces the student to various successful problem-solving strategies. Strategies such as drawing a diagram, applying solutions to other problems, working backwards, inductive reasoning, and trial and error are some of the techniques that are demonstrated.



Focus on Problem Solving 153

Focus on Problem Solving

Trial-and-Error Approach to Problem Solving

The questions below require an answer of always true, sometimes true, or never true. These problems are best solved by the trial-and-error method. The trial-and-error method of arriving at a solution to a problem involves repeated tests or experiments.

For example, consider the statement

> Both sides of an equation can be divided by the same number without changing the solution of the equation.

The solution of the equation $6x = 18$ is 3. If we divide both sides of the equation by 2, the result is $3x = 9$ and the solution is still 3. So the answer "never true" has been eliminated. We still need to determine whether there is a case for which the statement is not true. Is there a number that we could divide both sides of the equation by and the result would be an equation for which the solution is not 3? If we divide both sides of the equation by 0, the result is $\frac{6x}{0} = \frac{18}{0}$; the solution of this equation is not 3 because the expressions on either side of the equals sign are undefined. Thus the statement is true for some numbers and not true for 0. The statement is sometimes true.

Determine whether the statement is always true, sometimes true, or never true.

1. Both sides of an equation can be multiplied by the same number without changing the solution of the equation.
2. For an equation of the form $ax = b$, $a \neq 0$, multiplying both sides of the equation by the reciprocal of a will result in an equation of the form $x = constant$.
3. The Multiplication Property of Equations is used to remove a term from one side of an equation.
4. Adding -3 to each side of an equation yields the same result as subtracting 3 from each side of the equation.
5. An equation contains an equals sign.
6. The same variable term can be added to both sides of an equation without changing the solution of the equation.
7. An equation of the form $ax + b = c$ cannot be solved if a is a negative number.
8. The solution of the equation $\frac{x}{0} = 0$ is 0.
9. For the diagram at the left, $\angle b = \angle d = \angle e = \angle g$.
10. In solving an equation of the form $ax + b = cx + d$, subtracting cx from each side of the equation results in an equation with only one variable term in it.

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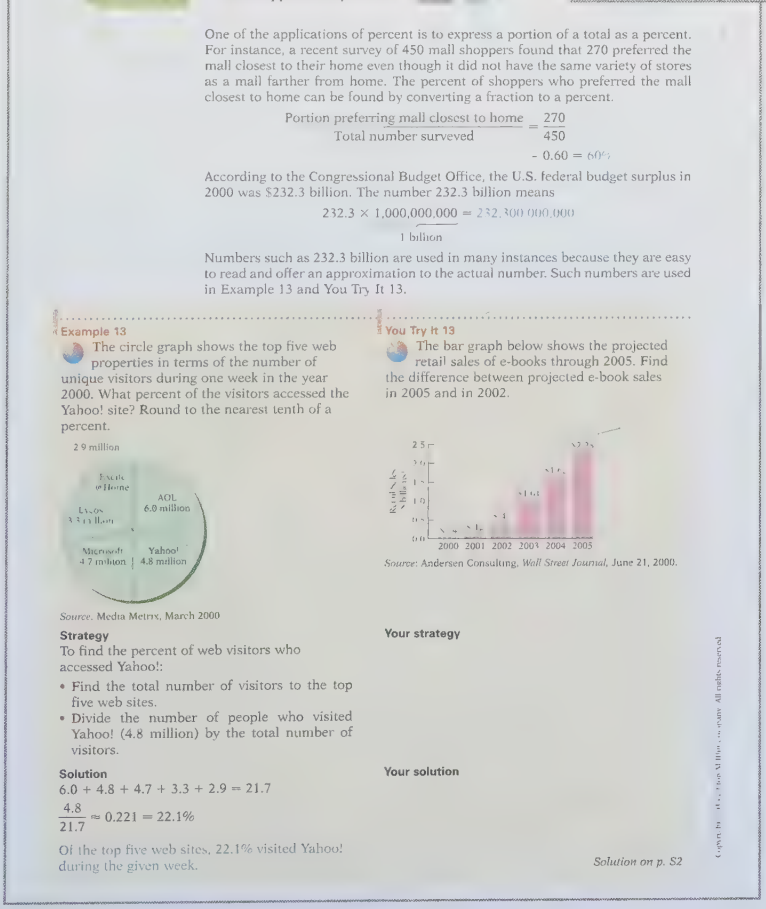


30 Chapter 1 / Real Numbers

To solve application problems

One of the applications of percent is to express a portion of a total as a percent. For instance, a recent survey of 450 mall shoppers found that 270 preferred the mall closest to their home even though it did not have the same variety of stores as a mall farther from home. The percent of shoppers who preferred the mall closest to home can be found by converting a fraction to a percent.

$$\frac{\text{Portion preferring mall closest to home}}{\text{Total number surveyed}} = \frac{270}{450}$$
$$= 0.60 = 60\%$$

According to the Congressional Budget Office, the U.S. federal budget surplus in 2000 was \$232.3 billion. The number 232.3 billion means

$$232.3 \times \underbrace{1{,}000{,}000{,}000}_{\text{1 billion}} = 232{,}300{,}000{,}000$$

Numbers such as 232.3 billion are used in many instances because they are easy to read and offer an approximation to the actual number. Such numbers are used in Example 13 and You Try It 13.

Example 13

The circle graph shows the top five web properties in terms of the number of unique visitors during one week in the year 2000. What percent of the visitors accessed the Yahoo! site? Round to the nearest tenth of a percent.

Source: Media Metrix, March 2000

Strategy

To find the percent of web visitors who accessed Yahoo!:

- Find the total number of visitors to the top five web sites.
- Divide the number of people who visited Yahoo! (4.8 million) by the total number of visitors.

Solution

$6.0 + 4.8 + 4.7 + 3.3 + 2.9 = 21.7$

$\frac{4.8}{21.7} \approx 0.221 = 22.1\%$

Of the top five web sites, 22.1% visited Yahoo! during the given week.

You Try It 13

The bar graph below shows the projected retail sales of e-books through 2005. Find the difference between projected e-book sales in 2005 and in 2002.

Source: Andersen Consulting, *Wall Street Journal*, June 21, 2000.

Your strategy

Your solution

Solution on p. S2

Problem-Solving Strategies

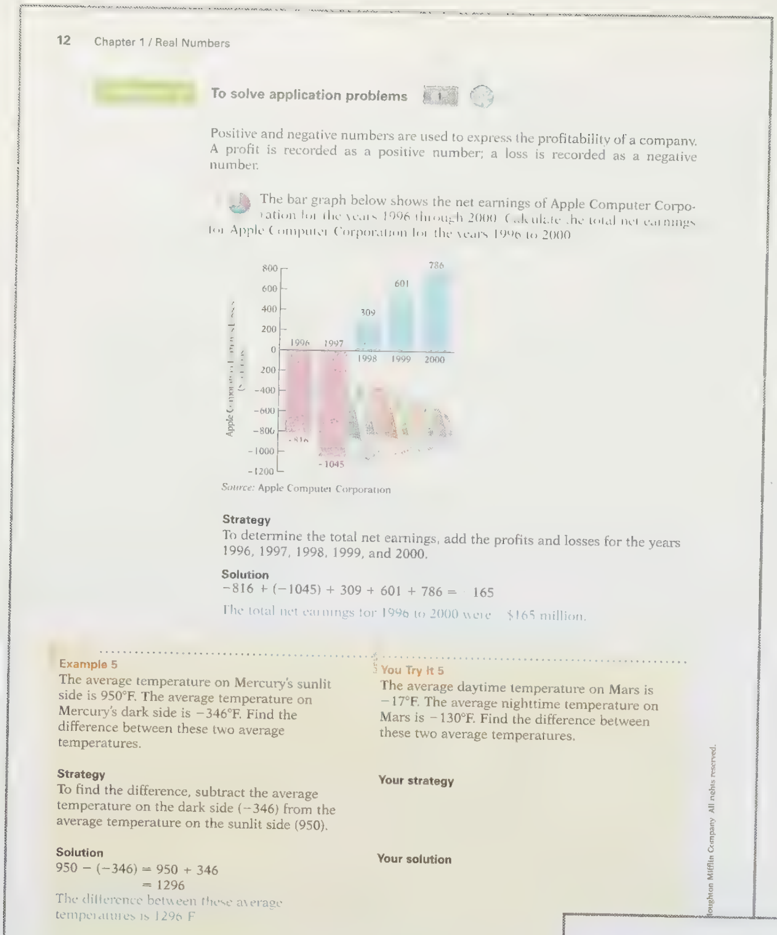
The text features a carefully developed approach to problem-solving that emphasizes the importance of *strategy* when solving problems. Students are encouraged to develop their own strategies—to draw diagrams, to write out the solution steps in words—as part of their solution to a problem. In each case, model strategies are presented as guides for students to follow as they attempt the "You Try It" problem. Having students provide strategies is a natural way to incorporate writing into the math curriculum.

Real Data and Applications

Applications

One way to motivate an interest in mathematics is through applications. Wherever appropriate, the last objective of a section presents applications that require the student to use problem-solving strategies, along with the skills covered in that section, to solve practical problems. This carefully integrated applied approach generates student awareness of the value of algebra as a real-life tool.

Applications are taken from many disciplines including agriculture, business, carpentry, chemistry, construction, Earth science, education, manufacturing, nutrition, real estate, and sociology.



12 Chapter 1 / Real Numbers

To solve application problems

Positive and negative numbers are used to express the profitability of a company. A profit is recorded as a positive number; a loss is recorded as a negative number.

The bar graph below shows the net earnings of Apple Computer Corporation for the years 1996 through 2000. Calculate the total net earnings for Apple Computer Corporation for the years 1996 to 2000.

Source: Apple Computer Corporation

Strategy
To determine the total net earnings, add the profits and losses for the years 1996, 1997, 1998, 1999, and 2000.

Solution
$-816 + (-1045) + 309 + 601 + 786 = -165$
The total net earnings for 1996 to 2000 were -$165 million.

Example 5
The average temperature on Mercury's sunlit side is 950°F. The average temperature on Mercury's dark side is −346°F. Find the difference between these two average temperatures.

Strategy
To find the difference, subtract the average temperature on the dark side (−346) from the average temperature on the sunlit side (950).

Solution
$950 - (-346) = 950 + 346$
$= 1296$
The difference between these average temperatures is 1296°F.

You Try It 5
The average daytime temperature on Mars is −17°F. The average nighttime temperature on Mars is −130°F. Find the difference between these two average temperatures.

Your strategy

Your solution

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Real Data

Real data examples and exercises, identified by ◔, ask students to analyze and solve problems taken from actual situations. Students are often required to work with tables, graphs, and charts drawn from a variety of disciplines.

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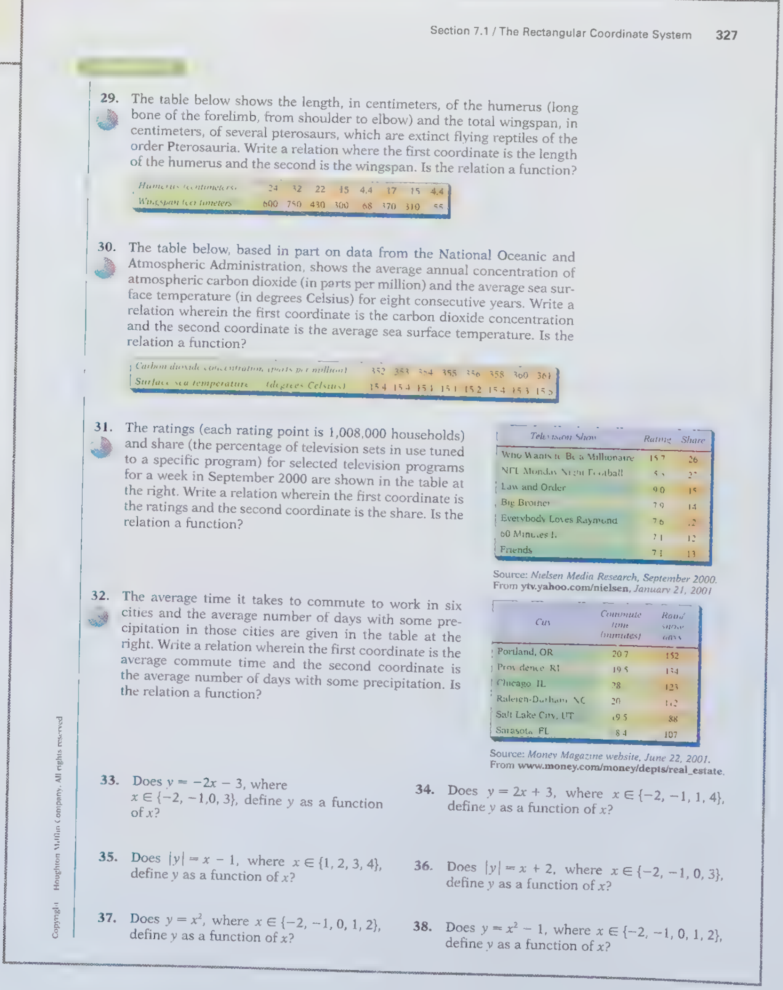


Section 7.1 / The Rectangular Coordinate System 327

29. The table below shows the length, in centimeters, of the humerus (long bone of the forelimb, from shoulder to elbow) and the total wingspan, in centimeters, of several pterosaurs, which are extinct flying reptiles of the order Pterosauria. Write a relation where the first coordinate is the length of the humerus and the second is the wingspan. Is the relation a function?

Humerus (centimeters)	24	32	22	15	4.4	17	15	4.4
Wingspan (centimeters)	600	750	430	300	68	370	310	55

30. The table below, based in part on data from the National Oceanic and Atmospheric Administration, shows the average annual concentration of atmospheric carbon dioxide (in parts per million) and the average sea surface temperature (in degrees Celsius) for eight consecutive years. Write a relation wherein the first coordinate is the carbon dioxide concentration and the second coordinate is the average sea surface temperature. Is the relation a function?

Carbon dioxide concentration (parts per million)	352	353	354	355	356	358	360	361
Surface sea temperature (degrees Celsius)	15.4	15.4	15.1	15.1	15.2	15.4	15.3	15.5

31. The ratings (each rating point is 1,008,000 households) and share (the percentage of television sets in use tuned to a specific program) for selected television programs for a week in September 2000 are shown in the table at the right. Write a relation wherein the first coordinate is the ratings and the second coordinate is the share. Is the relation a function?

Television Show	*Rating*	*Share*
Who Wants to Be a Millionaire	15.7	26
NFL Monday Night Football	5.5	2[illegible]
Law and Order	9.0	15
Big Brother	7.9	14
Everybody Loves Raymond	7.6	12
60 Minutes II	7.1	12
Friends	7.1	13

Source: *Nielsen Media Research, September 2000.* From **ytv.yahoo.com/nielsen**, *January 21, 2001*

32. The average time it takes to commute to work in six cities and the average number of days with some precipitation in those cities are given in the table at the right. Write a relation wherein the first coordinate is the average commute time and the second coordinate is the average number of days with some precipitation. Is the relation a function?

City	*Commute time (minutes)*	*Rain/snow days*
Portland, OR	20.7	152
Providence, RI	19.5	134
Chicago, IL	28	123
Raleigh-Durham, NC	20	112
Salt Lake City, UT	19.5	88
Sarasota, FL	8.4	107

Source: *Money Magazine website, June 22, 2001.* From **www.money.com/money/depts/real_estate**.

33. Does $y = -2x - 3$, where $x \in \{-2, -1, 0, 3\}$, define y as a function of x?

34. Does $y = 2x + 3$, where $x \in \{-2, -1, 1, 4\}$, define y as a function of x?

35. Does $|y| = x - 1$, where $x \in \{1, 2, 3, 4\}$, define y as a function of x?

36. Does $|y| = x + 2$, where $x \in \{-2, -1, 0, 3\}$, define y as a function of x?

37. Does $y = x^2$, where $x \in \{-2, -1, 0, 1, 2\}$, define y as a function of x?

38. Does $y = x^2 - 1$, where $x \in \{-2, -1, 0, 1, 2\}$, define y as a function of x?

Student Pedagogy

Icons

The and at each objective head remind students that both a video and a tutorial lesson are available for that objective.

Key Terms and Concepts

Key terms, in bold, emphasize important terms. The key terms are also provided in a **Glossary** at the back of the text.

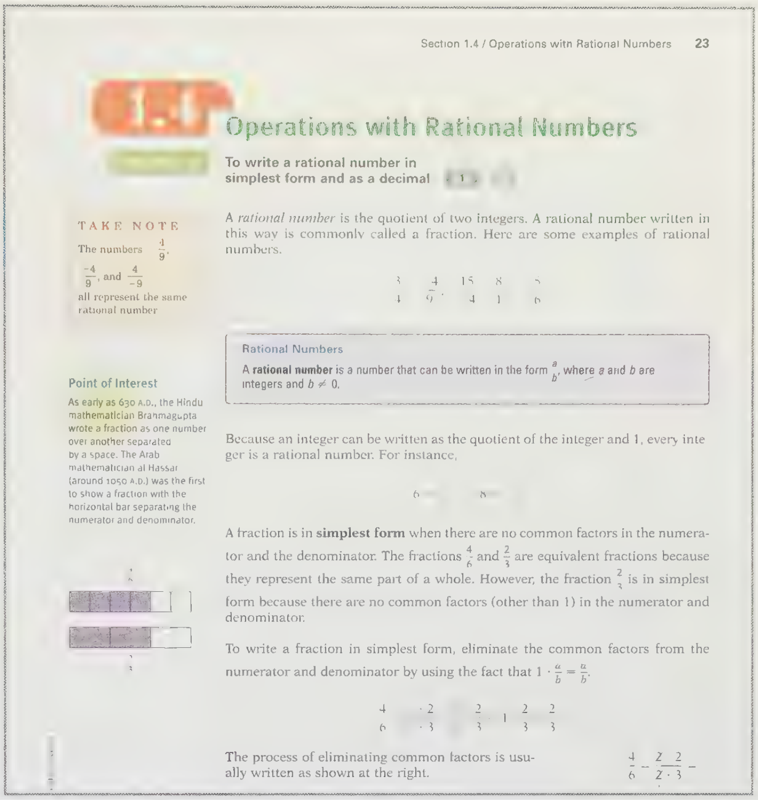
Key concepts are presented in purple boxes in order to highlight these important concepts and to provide for easy reference.

Take Note

These margin notes alert students to a point requiring special attention or are used to amplify the concept under discussion.

Point of Interest

These margin notes contain interesting sidelights about mathematics, its history, or its application.



Section 1.4 / Operations with Rational Numbers 23

Operations with Rational Numbers

To write a rational number in simplest form and as a decimal

TAKE NOTE

The numbers $\frac{4}{9}$, $\frac{-4}{9}$, and $\frac{4}{-9}$ all represent the same rational number

A *rational number* is the quotient of two integers. A rational number written in this way is commonly called a fraction. Here are some examples of rational numbers.

$$\frac{3}{4} \quad \frac{4}{9} \quad \frac{15}{4} \quad \frac{8}{1} \quad \frac{5}{6}$$

Rational Numbers

A **rational number** is a number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Point of Interest

As early as 630 A.D., the Hindu mathematician Brahmagupta wrote a fraction as one number over another separated by a space. The Arab mathematician al Hassar (around 1050 A.D.) was the first to show a fraction with the horizontal bar separating the numerator and denominator.

Because an integer can be written as the quotient of the integer and 1, every integer is a rational number. For instance,

$$6 \quad [illegible] \qquad 8 \quad [illegible]$$

A fraction is in **simplest form** when there are no common factors in the numerator and the denominator. The fractions $\frac{4}{6}$ and $\frac{2}{3}$ are equivalent fractions because they represent the same part of a whole. However, the fraction $\frac{2}{3}$ is in simplest form because there are no common factors (other than 1) in the numerator and denominator.

To write a fraction in simplest form, eliminate the common factors from the numerator and denominator by using the fact that $1 \cdot \frac{a}{b} = \frac{a}{b}$.

$$\frac{4}{6} \quad \frac{\cdot 2}{\cdot 3} \quad \frac{2}{3} \quad 1 \quad \frac{2}{3} \quad \frac{2}{3}$$

The process of eliminating common factors is usually written as shown at the right. $\frac{4}{6} = \frac{2 \cdot 2}{2 \cdot 3} =$

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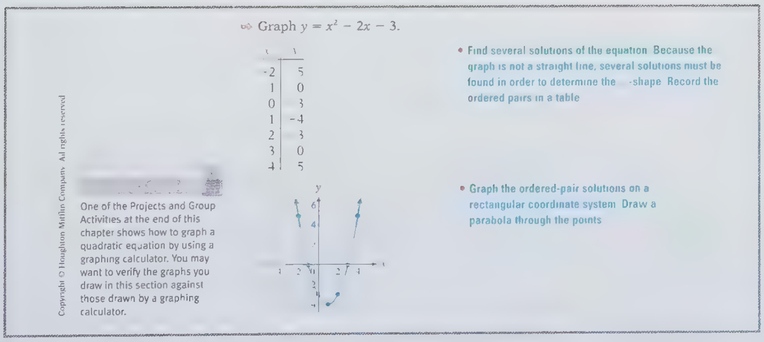


➾ Graph $y = x^2 - 2x - 3$.

x	y
-2	5
1	0
0	3
1	-4
2	3
3	0
4	5

- Find several solutions of the equation. Because the graph is not a straight line, several solutions must be found in order to determine the -shape. Record the ordered pairs in a table.
- Graph the ordered-pair solutions on a rectangular coordinate system. Draw a parabola through the points.

One of the Projects and Group Activities at the end of this chapter shows how to graph a quadratic equation by using a graphing calculator. You may want to verify the graphs you draw in this section against those drawn by a graphing calculator.

Calculator Note

These margin notes provide suggestions for using a calculator in certain situations.

Annotated Examples

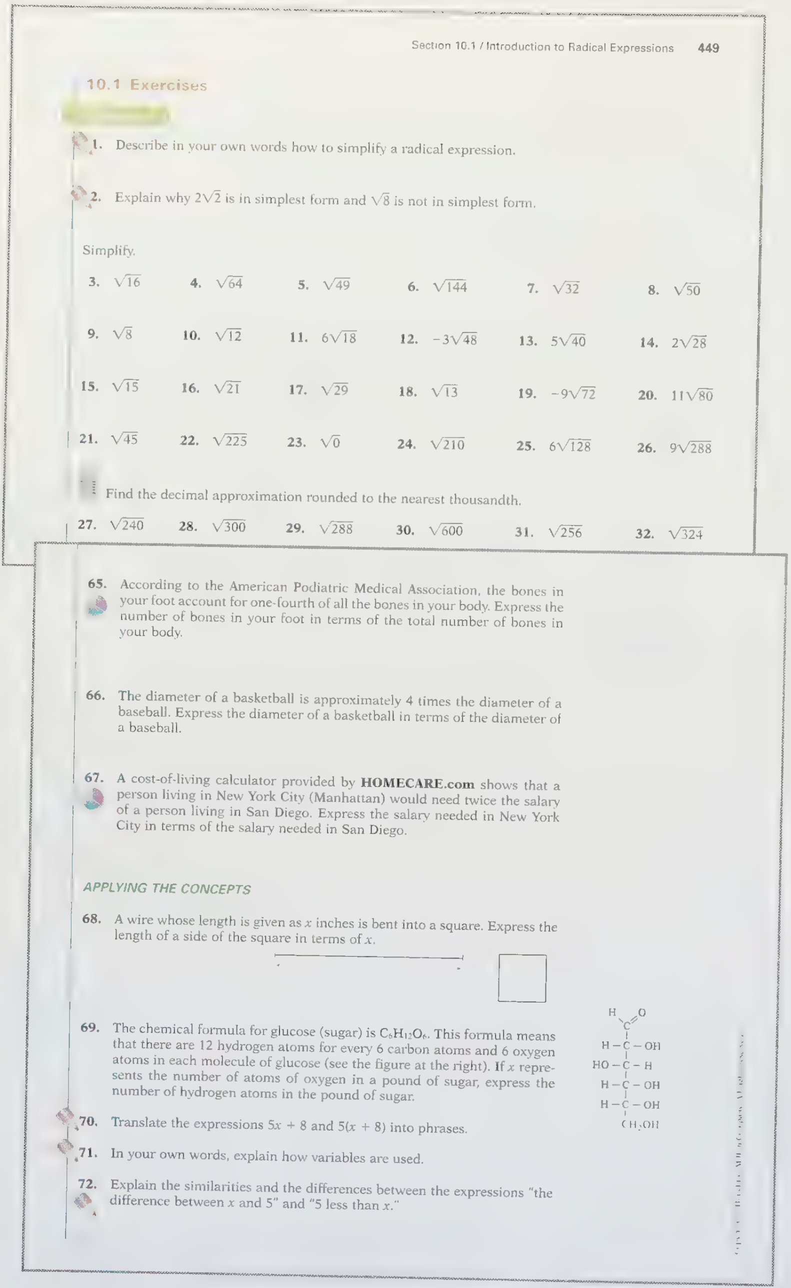
Examples indicated by ➾ use annotations in blue to explain what is happening in key steps of the complete, worked-out solutions.

Exercises and Projects

Exercises

The exercise sets of *Introductory Algebra: An Applied Approach* emphasize skill building, skill maintenance, and applications. Concept-based writing or developmental exercises have been integrated with the exercise sets. Icons identify appropriate writing , data analysis , and calculator exercises.

Included in each exercise set are **Applying the Concepts** that present extensions of topics, require analysis, or offer challenge problems. The writing exercises ask students to explain answers, write about a topic in the section, or research and report on a related topic.


Section 10.1 / Introduction to Radical Expressions 449

10.1 Exercises

1. Describe in your own words how to simplify a radical expression.

2. Explain why $2\sqrt{2}$ is in simplest form and $\sqrt{8}$ is not in simplest form.

Simplify.

3. $\sqrt{16}$ 4. $\sqrt{64}$ 5. $\sqrt{49}$ 6. $\sqrt{144}$ 7. $\sqrt{32}$ 8. $\sqrt{50}$
9. $\sqrt{8}$ 10. $\sqrt{12}$ 11. $6\sqrt{18}$ 12. $-3\sqrt{48}$ 13. $5\sqrt{40}$ 14. $2\sqrt{28}$
15. $\sqrt{15}$ 16. $\sqrt{21}$ 17. $\sqrt{29}$ 18. $\sqrt{13}$ 19. $-9\sqrt{72}$ 20. $11\sqrt{80}$
21. $\sqrt{45}$ 22. $\sqrt{225}$ 23. $\sqrt{0}$ 24. $\sqrt{210}$ 25. $6\sqrt{128}$ 26. $9\sqrt{288}$

Find the decimal approximation rounded to the nearest thousandth.

27. $\sqrt{240}$ 28. $\sqrt{300}$ 29. $\sqrt{288}$ 30. $\sqrt{600}$ 31. $\sqrt{256}$ 32. $\sqrt{324}$

65. According to the American Podiatric Medical Association, the bones in your foot account for one-fourth of all the bones in your body. Express the number of bones in your foot in terms of the total number of bones in your body.

66. The diameter of a basketball is approximately 4 times the diameter of a baseball. Express the diameter of a basketball in terms of the diameter of a baseball.

67. A cost-of-living calculator provided by **HOMECARE.com** shows that a person living in New York City (Manhattan) would need twice the salary of a person living in San Diego. Express the salary needed in New York City in terms of the salary needed in San Diego.

APPLYING THE CONCEPTS

68. A wire whose length is given as x inches is bent into a square. Express the length of a side of the square in terms of x.

69. The chemical formula for glucose (sugar) is $C_6H_{12}O_6$. This formula means that there are 12 hydrogen atoms for every 6 carbon atoms and 6 oxygen atoms in each molecule of glucose (see the figure at the right). If x represents the number of atoms of oxygen in a pound of sugar, express the number of hydrogen atoms in the pound of sugar.

70. Translate the expressions $5x + 8$ and $5(x + 8)$ into phrases.

71. In your own words, explain how variables are used.

72. Explain the similarities and the differences between the expressions "the difference between x and 5" and "5 less than x."

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Projects and Group Activities

The Projects and Group Activities feature at the end of each chapter can be used as extra credit or for cooperative learning activities. The projects cover various aspects of mathematics, including the use of calculators, collecting data from the internet, data analysis, and extended applications.

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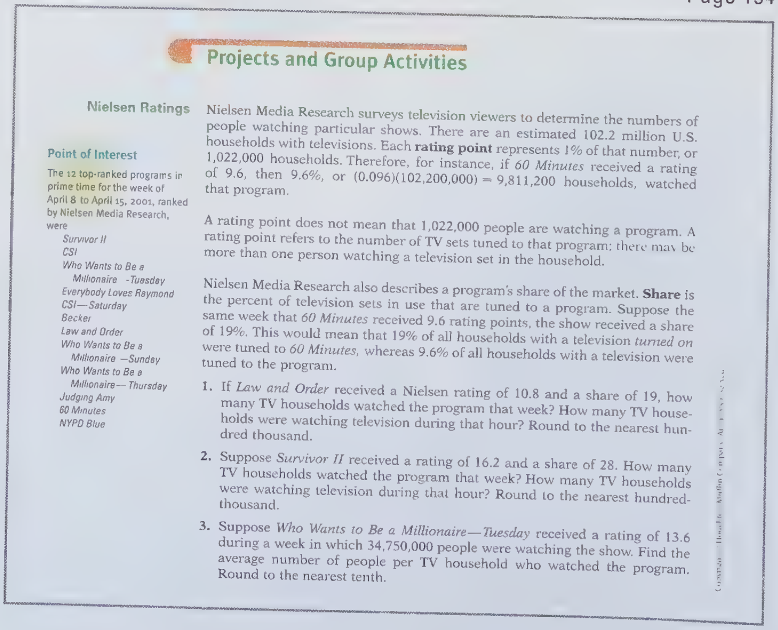

Projects and Group Activities

Nielsen Ratings

Nielsen Media Research surveys television viewers to determine the numbers of people watching particular shows. There are an estimated 102.2 million U.S. households with televisions. Each **rating point** represents 1% of that number, or 1,022,000 households. Therefore, for instance, if *60 Minutes* received a rating of 9.6, then 9.6%, or (0.096)(102,200,000) = 9,811,200 households, watched that program.

A rating point does not mean that 1,022,000 people are watching a program. A rating point refers to the number of TV sets tuned to that program; there may be more than one person watching a television set in the household.

Nielsen Media Research also describes a program's share of the market. **Share** is the percent of television sets in use that are tuned to a program. Suppose the same week that *60 Minutes* received 9.6 rating points, the show received a share of 19%. This would mean that 19% of all households with a television *turned on* were tuned to *60 Minutes*, whereas 9.6% of all households with a television were tuned to the program.

1. If *Law and Order* received a Nielsen rating of 10.8 and a share of 19, how many TV households watched the program that week? How many TV households were watching television during that hour? Round to the nearest hundred thousand.
2. Suppose *Survivor II* received a rating of 16.2 and a share of 28. How many TV households watched the program that week? How many TV households were watching television during that hour? Round to the nearest hundred-thousand.
3. Suppose *Who Wants to Be a Millionaire—Tuesday* received a rating of 13.6 during a week in which 34,750,000 people were watching the show. Find the average number of people per TV household who watched the program. Round to the nearest tenth.

Point of Interest

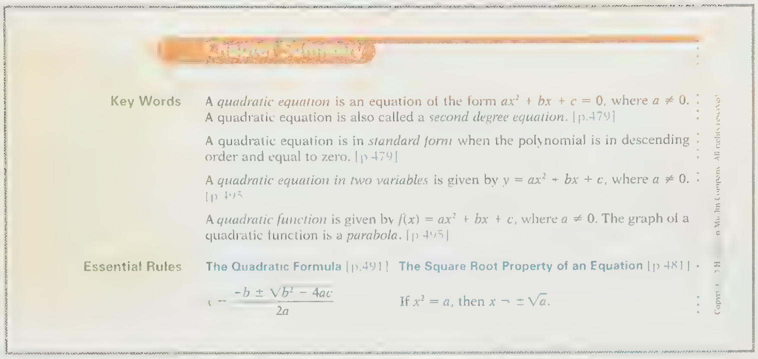
The 12 top-ranked programs in prime time for the week of April 8 to April 15, 2001, ranked by Nielsen Media Research, were

- *Survivor II*
- *CSI*
- *Who Wants to Be a Millionaire -Tuesday*
- *Everybody Loves Raymond*
- *CSI—Saturday*
- *Becker*
- *Law and Order*
- *Who Wants to Be a Millionaire —Sunday*
- *Who Wants to Be a Millionaire— Thursday*
- *Judging Amy*
- *60 Minutes*
- *NYPD Blue*

End of Chapter

Chapter Summary

At the end of each chapter there is a Chapter Summary that includes Key Words and Essential Rules that were covered in the chapter. These chapter summaries provide a single point of reference as the student prepares for a test. Each concept references the page number from the lesson where the concept is introduced.



Chapter Summary

Key Words A *quadratic equation* is an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$. A quadratic equation is also called a *second degree equation*. [p.479]

A quadratic equation is in *standard form* when the polynomial is in descending order and equal to zero. [p.479]

A *quadratic equation in two variables* is given by $y = ax^2 + bx + c$, where $a \neq 0$. [p.495]

A *quadratic function* is given by $f(x) = ax^2 + bx + c$, where $a \neq 0$. The graph of a quadratic function is a *parabola*. [p.495]

Essential Rules The Quadratic Formula [p.491] The Square Root Property of an Equation [p.481]

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ If $x^2 = a$, then $x = \pm\sqrt{a}$.

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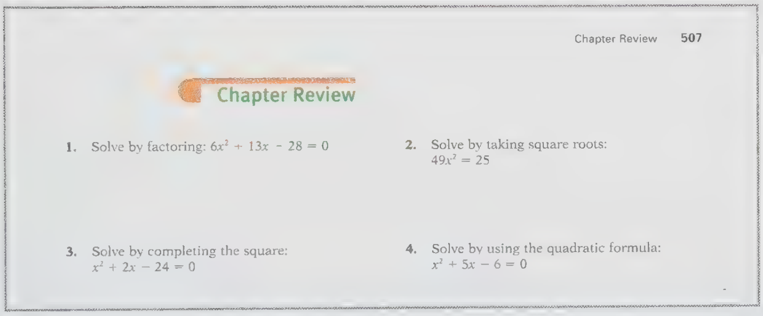

Chapter Review 507

Chapter Review

1. Solve by factoring: $6x^2 + 13x - 28 = 0$
2. Solve by taking square roots: $49x^2 = 25$
3. Solve by completing the square: $x^2 + 2x - 24 = 0$
4. Solve by using the quadratic formula: $x^2 + 5x - 6 = 0$

Chapter Review

Review exercises are found at the end of each chapter. These exercises are selected to help the student integrate all of the topics presented in the chapter.

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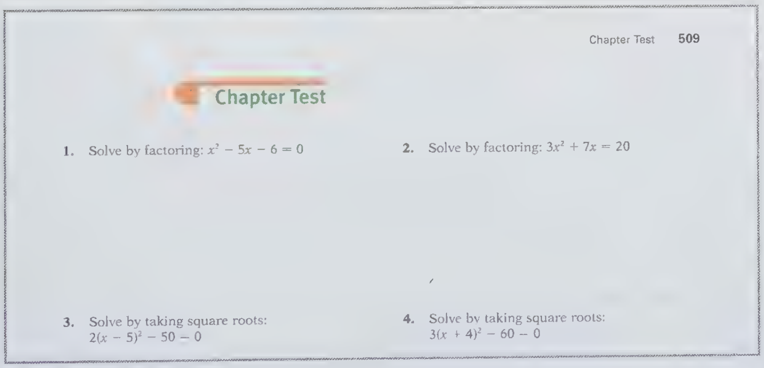

Chapter Test 509

Chapter Test

1. Solve by factoring: $x^2 - 5x - 6 = 0$
2. Solve by factoring: $3x^2 + 7x = 20$
3. Solve by taking square roots: $2(x - 5)^2 - 50 = 0$
4. Solve by taking square roots: $3(x + 4)^2 - 60 = 0$

Chapter Test

The Chapter Test exercises are designed to simulate a possible test of the material in the chapter.

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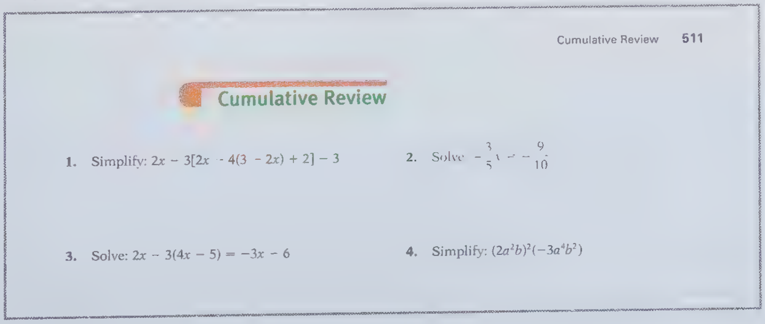

Cumulative Review 511

Cumulative Review

1. Simplify: $2x - 3[2x - 4(3 - 2x) + 2] - 3$
2. Solve: $-\frac{3}{5}x = -\frac{9}{10}$
3. Solve: $2x - 3(4x - 5) = -3x - 6$
4. Simplify: $(2a^2b)^2(-3a^4b^2)$

Cumulative Review

Cumulative Review exercises, which appear at the end of each chapter (beginning with Chapter 2), help students maintain skills learned in previous chapters.

The answers to all Chapter Review exercises, all Chapter Test exercises, and all Cumulative Review exercises are given in the Answer Section. Along with the answer, there is a reference to the objective that pertains to each exercise.

Supplements for the Instructor

Introductory Algebra: An Applied Approach has a complete set of teaching aids for the instructor.

Instructor's Annotated Edition This edition contains a replica of the student text and additional items just for the instructor. These include: *Instructor Notes, Transparency Master icons, In-Class Examples, Concept Checks, Discuss the Concepts, New Vocabulary/Symbols, etc.,Vocabulary/Symbols, etc. to Review, Optional Student Activities, Quick Quizzes, Answers to Writing Exercises,* and *Suggested Assignments.* Answers to all exercises are also provided.

Instructor's Solutions Manual The *Instructor's Solutions Manual* contains worked-out solutions for all exercises in the text.

Instructor's Resource Manual with Chapter Tests The *Instructor's Resource Manual* contains ready-to-use printed Chapter Tests, which is the first of three sources of testing material. Eight printed tests (in two formats - free response and multiple choice) are provided for each chapter. Cumulative tests and final exams are also provided. These tests are available on the *Class Prep* CD or can be downloaded from our web site at math.college.hmco.com/instructors. The tests are in Microsoft Word® format and can be edited to suit the needs of the instructor. The *Instructor's Resource Manual* also includes a lesson plan for the *AIM for Success*.

Printed Test Bank The *Printed Test Bank,* the second component of the testing material, offers a printout of the items in the *HM Testing* computerized testing software, including an example of each algorithmic item. All multiple-choice items are carefully worded to make them appropriate to use as free-response items also.

NEW! ***HM Testing*** *HM Testing,* our computerized test generator, is our third source of testing material. The database contains test items and algorithms and is designed to produce an unlimited number of tests for each chapter of the text, including cumulative tests and final exams. These questions are unique to *HM Testing* and do not repeat items provided in the Chapter Tests of the *Instructor's Resource Manual*. It is available for Microsft Windows® and the Macintosh. Both versions provide **on-line testing** and **gradebook** functions.

HM³ Tutorial (Instructor version) This state-of-the-art tutorial CD software package for Microsoft Windows® was written and developed by the authors specifically for use with this text. Every objective is supported by the HM³ Tutor, exercises and quizzes are algorithmically generated, solution steps are animated, lessons and exercises are presented in a colorful, lively manner, and an integrated classroom management system tracks student performance.

NEW! ***WebCT ePacks*** *WebCT ePacks* provide instructors with a flexible, Internet-based education platform providing multiple ways to present learning materials. The *WebCT ePacks* come with a full array of features to enrich the online learning experience.

NEW! ***Blackboard Cartridges*** The *Houghton Mifflin Blackboard cartridge* allows flexible, efficient, and creative ways to present learning materials and opportunities. In addition to course management benefits, instructors may make use of an electronic grade book, receive papers from students enrolled in the course via the Internet, and track student use of the communication and collaboration functions.

NEW! ***HMClassPrep*** These CD-ROMs contain a multitude of text-specific resources for instructors to use to enhance the classroom experience. The resources, or assets, are available as pdf and/or customizable Microsoft Word®

files and include transparency masters and Chapter Tests from the IRM, to name only a few. These resources can be easily accessed from the CD-ROM by chapter or resource type. The CD can also link you to the text's web site.

NEW! ***Instructor Text-specific website*** The resources available on the *Class Prep CD* are also available on the instructor web site at math.college.hmco.com/instructors. Appropriate items will be password protected. Instructors also have access to the student part of the text's web site.

Supplements for the Student

Student Solutions Manual The *Student Solutions Manual* contains complete solutions to all odd-numbered exercises in the text.

Math Study Skills Workbook *by Paul D. Nolting* This workbook is designed to reinforce skills and minimize frustration for students in any math class, lab, or study skills course. It offers a wealth of study tips and sound advice on note taking, time management, and reducing math anxiety. In addition, numerous opportunities for self assessment enable students to track their own progress.

NEW! ***eduSpace*** *eduSpace* is a new content delivery system, combining an algorithmic tutorial program, online delivery of course materials, and classroom management functions. The interactive online content correlates directly to this text and can be accessed 24 hours a day.

HM^3 Tutorial (Student version) This state-of-the-art tutorial CD software package for Microsoft Windows® was written and developed by the authors specifically for use with this text. HM^3 is an interactive tutorial containing lessons, exercises, and quizzes for every section of the text—lessons are presented in a colorful, lively manner and solution steps are animated; exercises and quizzes are algorithmically generated. The lessons provide additional instruction and practice and can be used in several ways: (1) to cover material that was missed because of absence from class; (2) to reinforce instruction on a concept that has not yet been mastered; (3) to review material in preparation for an examination. Following each lesson there are practice exercises for the student to try. These exercises are created by carefully constructed algorithms that enable a student to practice a variety of exercise types. Because the exercises are created algorithmically, each time the student uses the tutorial, the student is presented with different problems. Following the practice exercises there is an algorithmically-generated quiz. Next to every objective head in the text, the 💿 serves as a reminder that there is an HM^3 tutorial lesson corresponding to that objective.

NEW! ***SMARTHINKING™ live, on-line tutoring*** Houghton Mifflin has partnered with SMARTHINKING to provide an easy-to-use and effective on-line tutorial service. **Whiteboard Simulations** and **Practice Area** promote real-time visual interaction.

Three levels of service are offered.

- **Text-specific Tutoring** provides real-time, one-on-one instruction with a specially qualified 'e-structor.'
- **Questions Any Time** allows students to submit questions to the tutor outside the scheduled hours and receive a reply within 24 hours.
- **Independent Study Resources** connect students with around-the-clock access to additional educational services, including interactive web sites, diagnostic tests and Frequently Asked Questions posed to SMARTHINKING e-structors.

NEW! Videos This edition offers brand new text-specific videos, hosted by Dana Mosely, covering all sections of the text. These videos, professionally produced specifically for this text, offer a valuable resource for further instruction and review. Next to every objective head in the text, the 🎞 serves as a reminder that the objective is covered in a video lesson.

NEW! Student Text-Specific Web Site On-line student resources, such as section quizzes, can be found at this text's website at math.college.hmco.com/students.

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AIM for Success

Welcome to *Introductory Algebra: An Applied Approach*. As you begin this course, we know two important facts: (1) We want you to succeed. (2) You want to succeed. To do that requires an effort from each of us. For the next few pages, we are going to show you what is required of you to achieve that success and how you can use the features of this text to be successful.

Motivation

One of the most important keys to success is motivation. We can try to motivate you by offering interesting or important ways mathematics can benefit you. But, in the end, the motivation must come from you. On the first day of class, it is easy to be motivated. Eight weeks into the term, it is harder to keep that motivation.

TAKE NOTE
Motivation alone will not lead to success. For instance, suppose a person who cannot swim is placed in a boat, taken out to the middle of a lake, and then thrown overboard. That person has a lot of motivation but there is a high likelihood the person will drown without some help. Motivation gives us the desire to learn but is not the same as learning.

To stay motivated, there must be outcomes from this course that are worth your time, money, and energy.

List some reasons you are taking this course.

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Although we hope that one of the reasons you listed was an interest in mathematics, we know that many of you are taking this course because it is required to graduate, it is a prerequisite for a course you must take, or because it is required for your major. Although you may not agree that this course is necessary, it is! If you are motivated to graduate or complete the requirements for your major, then use that motivation to succeed in this course. Do not become distracted from your goal to complete your education!

Commitment

To be successful, you must make a committment to succeed. This means devoting time to math so that you achieve a better understanding of the subject.

List some activities (sports, hobbies, talents such as dance, art, or music) that you enjoy and at which you would like to become better.

ACTIVITY	TIME SPENT	TIME WISHED SPENT

Thinking about these activities, put the number of hours that you spend each week practicing these activities next to the activity. Next to that number, indicate the number of hours per week you would like to spend on these activities.

Whether you listed surfing or sailing, aerobics or restoring cars, or any other activity you enjoy, note how many hours a week you spend doing it. To succeed in math, you must be willing to commit the same amount of time. Success requires some sacrifice.

The "I Can't Do Math" Syndrome

There may be things you cannot do, such as lift a two-ton boulder. You can, however, do math. It is much easier than lifting the two-ton boulder. When you first learned the activities you listed above, you probably could not do them well. With practice, you got better. With practice, you will be better at math. Stay focused, motivated, and committed to success.

It is difficult for us to emphasize how important it is to overcome the "I Can't Do Math Syndrome." If you listen to interviews of very successful atheletes after a particularly bad performance, you will note that they focus on the positive aspect of what they did, not the negative. Sports psychologists encourage athletes to always be positive—to have a "Can Do" attitude. Develop this attitude toward math.

Strategies for Success

Textbook Reconnaissance Right now, do a 15-minute "textbook reconnaissance" of this book. Here's how:

First, read the table of contents. Do it in three minutes or less. Next, look through the entire book, page by page. Move quickly. Scan titles, look at pictures, notice diagrams.

A textbook reconnaissance shows you where a course is going. It gives you the big picture. That's useful because brains work best when going from the general to the specific. Getting the big picture before you start makes details easier to recall and understand later on.

Your textbook reconnaissance will work even better if, as you scan, you look for ideas or topics that are interesting to you. List three facts, topics, or problems that you found interesting during your textbook reconnaissance.

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The idea behind this technique is simple: It's easier to work at learning material if you know it's going to be useful to you.

Not all the topics in this book will be "interesting" to you. But that is true of any subject. Surfers find that on some days the waves are better than others, musicians find some music more appealing than other music, computer gamers find some computer games more interesting than others, car enthusiasts find some cars more exciting than others. Some car enthusiasts would rather have a completely restored 1957 Chevrolet than a new Ferrari.

Know the Course Requirements To do your best in this course, you must know exactly what your instructor requires. Course requirements may be stated in a *syllabus*, which is a printed outline of the main topics of the course, or they may be presented orally. When they are listed in a syllabus or on other printed pages, keep them in a safe place. When they are presented orally, make sure to take complete notes. In either case, it is important that you understand them completely and follow them exactly. Be sure you know the answer to each of the following questions.

1. What is your instructor's name?
2. Where is your instructor's office?
3. At what times does your instructor hold office hours?
4. Besides the textbook, what other materials does your instructor require?
5. What is your instructor's attendance policy?
6. If you must be absent from a class meeting, what should you do before returning to class? What should you do when you return to class?

7. What is the instructor's policy regarding collection or grading of homework assignments?
8. What options are available if you are having difficulty with an assignment? Is there a math tutoring center?
9. If there is a math lab at your school, where is it located? What hours is it open?
10. What is the instructor's policy if you miss a quiz?
11. What is the instructor's policy if you miss an exam?
12. Where can you get help when studying for an exam?

Remember: Your instructor wants to see you succeed. If you need help, ask! Do not fall behind. If you are running a race and fall behind by 100 yards, you may be able to catch up but it will require more effort than had you not fallen behind.

TAKE NOTE

Besides time management, there must be realistic ideas of how much time is available. There are very few people who can *successfully* work full-time and go to school full-time. If you work 40 hours a week, take 15 units, spend the recommended study time given at the right, and sleep 8 hours a day, you will use over 80% of the available hours in a week. That leaves less than 20% of the hours in a week for family, friends, eating, recreation, and other activities.

Time Management We know that there are demands on your time. Family, work, friends, and entertainment all compete for your time. We do not want to see you receive poor job evaluations because you are studying math. However, it is also true that we do not want to see you receive poor math test scores because you devoted too much time to work. When several competing and important tasks require your time and energy, the only way to manage the stress of being successful at both is to manage your time efficiently.

Instructors often advise students to spend twice the amount of time outside of class studying as they spend in the classroom. Time management is important if you are to accomplish this goal and succeed in school. The following activity is intended to help you structure your time more efficiently.

List the name of each course you are taking this term, the number of class hours each course meets, and the number of hours you should spend studying each subject outside of class. Then fill in a weekly schedule like the one printed below. Begin by writing in the hours spent in your classes, the hours spent at work (if you have a job), and any other commitments that are not flexible with respect to the time that you do them. Then begin to write down commitments that are more flexible, including hours spent studying. Remember to reserve time for activities such as meals and exercise. You should also schedule free time.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
7–8 a.m.							
8–9 a.m.							
9–10 a.m.							
10–11 a.m.							
11–12 p.m.							
12–1 p.m.							
1–2 p.m.							
2–3 p.m.							
3–4 p.m.							
4–5 p.m.							
5–6 p.m.							
6–7 p.m.							
7–8 p.m.							
8–9 p.m.							
9–10 p.m.							
10–11 p.m.							
11–12 a.m.							

We know that many of you must work. If that is the case, realize that working 10 hours a week at a part-time job is equivalent to taking a three-unit class. If you must work, consider letting your education progress at a slower rate to allow you to be successful at both work and school. There is no rule that says you must finish school in a certain time frame.

Schedule Study Time As we encouraged you to do by filling out the time management form above, schedule a certain time to study. You should think of this time the way you would the time for work or class—that is, reasons for missing study time should be as compelling as reasons for missing work or class. "I just didn't feel like it" is not a good reason to miss your scheduled study time.

Although this may seem like an obvious exercise, list a few reasons you might want to study.

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Of course we have no way of knowing the reasons you listed, but from our experience one reason given quite frequently is "To pass the course." There is nothing wrong with that reason. If that is the most important reason for you to study, then use it to stay focused.

One method of keeping to a study schedule is to form a ***study group***. Look for people who are committed to learning, who pay attention in class, and who are punctual. Ask them to join your group. Choose people with similar educational goals but different methods of learning. You can gain insight from seeing the material from a new perspective. Limit groups to four or five people; larger groups are unwieldy.

There are many ways to conduct a study group. Begin with the following suggestions and see what works best for your group.

1. Test each other by asking questions. Each group member might bring two or three sample test questions to each meeting.
2. Practice teaching each other. Many of us who are teachers learned a lot about our subject when we had to explain it to someone else.
3. Compare class notes. You might ask other students about material in your notes that is difficult for you to understand.
4. Brainstorm test questions.
5. Set an agenda for each meeting. Set approximate time limits for each agenda item and determine a quitting time.

And finally, probably the most important aspect of studying is that it should be done in relatively small chunks. If you can only study three hours a week for this course (probably not enough for most people), do it in blocks of one hour on three separate days, preferably after class. Three hours of studying on a Sunday is not as productive as three hours of paced study.

Text Features That Promote Success

There are 11 chapters in this text. Each chapter is divided into sections, and each section is subdivided into learning objectives. Each learning objective is labeled with a letter from A to E.

Preparing for a Chapter Before you begin a new chapter, you should take some time to review previously learned skills. There are two ways to do this. The first is to complete the ***Cumulative Review***, which occurs after every chapter (except Chapter 1). For instance, turn to page 253. The questions in this review are taken from the previous chapters. The answers for all these exercises can be found on page A11. Turn to that page now and locate the answers for the Chapter 5 Cumulative Review. After the answer to the first exercise, which is 7 you will see the objective reference [1.2B]. This means that this question was taken from Chapter 1, Section 2, Objective B. If you missed this question, you should return to that objective and restudy the material.

A second way of preparing for a new chapter is to complete the ***Prep Test***. This test focuses on the particular skills that will be required for the new chapter. Turn to page 206 to see a Prep Test. The answers for the Prep Test are the first set of answers in the answer section for a chapter. Turn to page A9 to see the answers for the Chapter 5 Prep Test. Note that an objective reference is given for each question. If you answer a question incorrectly, restudy the objective from which the question was taken.

Before the class meeting in which your professor begins a new section, you should read each objective statement for that section. Next, browse through the objective material, being sure to note each word in bold type. These words indicate important concepts that you must know in order to learn the material. Do not worry about trying to understand all the material. Your professor is there to assist you with that endeavor. The purpose of browsing through the material is so that your brain will be prepared to accept and organize the new information when it is presented to you.

Turn to page 3. Write down the title of the first objective in Section 1.1. Under the title of the objective, write down the words in the objective that are in bold print. It is not necessary for you to understand the meaning of these words. You are in this class to learn their meaning.

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________________ ________________ ________________ ________________

Math is Not a Spectator Sport To learn mathematics you must be an active participant. Listening and watching your professor do mathematics is not enough. Mathematics requires that you interact with the lesson you are studying. If you filled in the blanks above, you were being interactive. There are other ways this textbook has been designed to help you be an active learner.

Annotated Examples An orange arrow indicates an example with explanatory remarks to the right of the work. Using paper and pencil, you should work along as you go through the example.

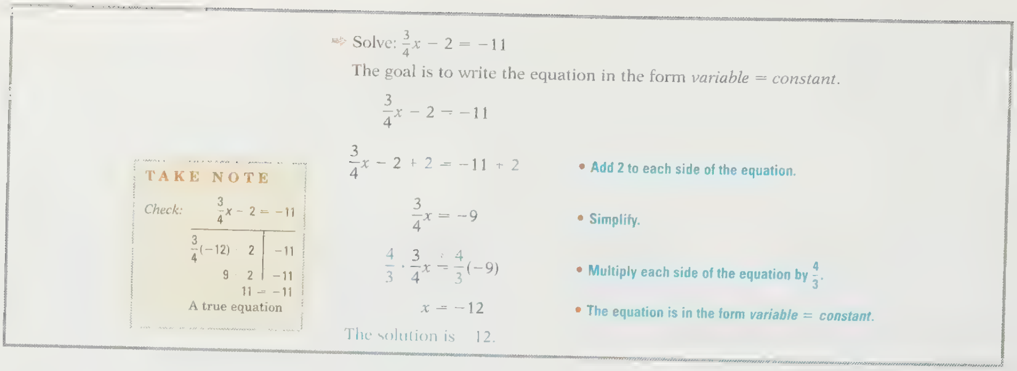
$$\frac{3}{4}x - 2 = -11$$

$$\frac{3}{4}x - 2 + 2 = -11 + 2$$

$$\frac{3}{4}x = -9$$

$$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3}(-9)$$

$$x = -12$$



Solve: $\frac{3}{4}x - 2 = -11$

The goal is to write the equation in the form *variable = constant*.

$$\frac{3}{4}x - 2 = -11$$

$$\frac{3}{4}x - 2 + 2 = -11 + 2$$ • Add 2 to each side of the equation.

$$\frac{3}{4}x = -9$$ • Simplify.

$$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3}(-9)$$ • Multiply each side of the equation by $\frac{4}{3}$.

$$x = -12$$ • The equation is in the form *variable = constant*.

The solution is −12.

TAKE NOTE

Check: $\frac{3}{4}x - 2 = -11$

$$\frac{3}{4}(-12) - 2 \;\Big|\; -11$$

$$-9 - 2 \;\Big|\; -11$$

$$-11 = -11$$

A true equation

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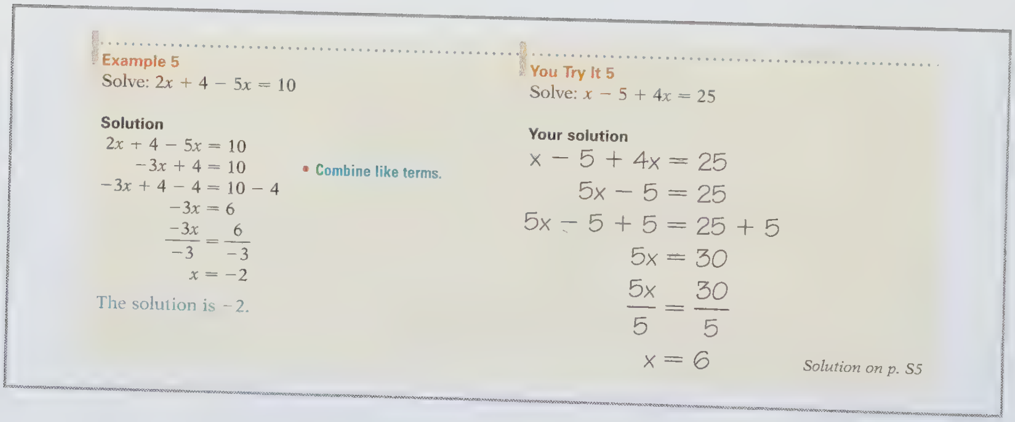
When you complete the example, get a clean sheet of paper. Write down the problem and then try to complete the solution without referring to your notes or the book. When you can do that, move on to the next part of the objective.

Leaf through the book now and write down the page numbers of two other occurrences of an arrowed example.

You Try Its One of the key instructional features of this text is the paired examples. Notice that in each example box, the example on the left is completely worked out and the "You Try It" example on the right is not. Study the worked-out example carefully by working through each step. Then work the You Try It. If you get stuck, refer to the page number at the end of the example, which directs you to the place where the You Try It is solved—a complete worked-out solution is provided. Try to use the given solution to get a hint for the step you are stuck on. Then try to complete your solution.



Example 5

Solve: $2x + 4 - 5x = 10$

Solution

$$2x + 4 - 5x = 10$$

$$-3x + 4 = 10$$ • Combine like terms.

$$-3x + 4 - 4 = 10 - 4$$

$$-3x = 6$$

$$\frac{-3x}{-3} = \frac{6}{-3}$$

$$x = -2$$

The solution is −2.

You Try It 5

Solve: $x - 5 + 4x = 25$

Your solution

$$x - 5 + 4x = 25$$

$$5x - 5 = 25$$

$$5x - 5 + 5 = 25 + 5$$

$$5x = 30$$

$$\frac{5x}{5} = \frac{30}{5}$$

$$x = 6$$

Solution on p. S5

Page 102

When you have completed your solution, check your work against the solution we provided. (Turn to page S5 to see the solution of You Try It 2.) Be aware that frequently there is more than one way to solve a problem. Your answer, however, should be the same as the given answer. If you have any question as to whether your method will "always work," check with your instructor or with someone in the math center.

Browse through the textbook and write down the page numbers where two other paired example features occur.

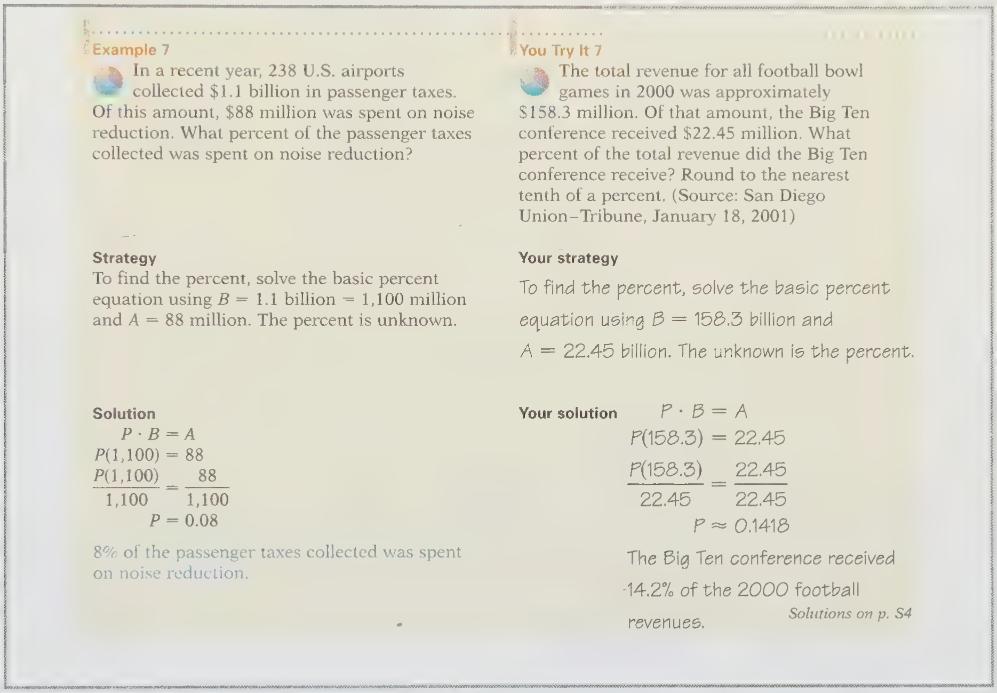
Remember: Be an active participant in your learning process. When you are sitting in class watching and listening to an explanation, you may think that you understand. However, until you actually try to do it, you will have no confirmation of the new knowledge or skill. Most of us have had the experience of sitting in class thinking we knew how to do something only to get home and realize that we didn't.

TAKE NOTE

There is a strong connection between reading and being a successful student in math or any other subject. If you have difficulty reading, consider taking a reading course. Reading is much like other skills. There are certain things you can learn that will make you a better reader.

Word Problems Word problems are difficult because we must read the problem, determine the quantity we must find, think of a method to do that, and then actually solve the problem. In short, we must formulate a *strategy* to solve the problem and then devise a *solution*.

Note in the paired example below that part of every word problem is a strategy and part is a solution. The strategy is a written description of how we will solve the problem. In the corresponding You Try It, you are asked to formulate a strategy. Do not skip this step, and be sure to write it out.



Example 7

In a recent year, 238 U.S. airports collected \$1.1 billion in passenger taxes. Of this amount, \$88 million was spent on noise reduction. What percent of the passenger taxes collected was spent on noise reduction?

Strategy

To find the percent, solve the basic percent equation using $B = 1.1$ billion $= 1{,}100$ million and $A = 88$ million. The percent is unknown.

Solution

$$P \cdot B = A$$
$$P(1{,}100) = 88$$
$$\frac{P(1{,}100)}{1{,}100} = \frac{88}{1{,}100}$$
$$P = 0.08$$

8% of the passenger taxes collected was spent on noise reduction.

You Try It 7

The total revenue for all football bowl games in 2000 was approximately \$158.3 million. Of that amount, the Big Ten conference received \$22.45 million. What percent of the total revenue did the Big Ten conference receive? Round to the nearest tenth of a percent. (Source: San Diego Union-Tribune, January 18, 2001)

Your strategy

To find the percent, solve the basic percent equation using $B = 158.3$ billion and $A = 22.45$ billion. The unknown is the percent.

Your solution

$$P \cdot B = A$$
$$P(158.3) = 22.45$$
$$\frac{P(158.3)}{22.45} = \frac{22.45}{22.45}$$
$$P \approx 0.1418$$

The Big Ten conference received 14.2% of the 2000 football revenues.

Solutions on p. S4

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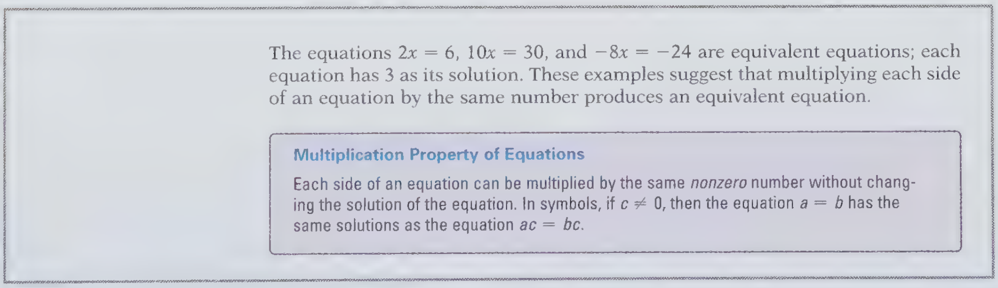
TAKE NOTE

If a rule has more than one part, be sure to make a notation to that effect.

Rule Boxes Pay special attention to rules placed in boxes. These rules give you the reasons certain types of problems are solved the way they are. When you see a rule, try to rewrite the rule in your own words.

Multiply each side of an equation by the same number. Do not use zero.



The equations $2x = 6$, $10x = 30$, and $-8x = -24$ are equivalent equations; each equation has 3 as its solution. These examples suggest that multiplying each side of an equation by the same number produces an equivalent equation.

Multiplication Property of Equations

Each side of an equation can be multiplied by the same *nonzero* number without changing the solution of the equation. In symbols, if $c \neq 0$, then the equation $a = b$ has the same solutions as the equation $ac = bc$.

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TAKE NOTE

If you are working at home and need assistance, there is on-line help available at math.college.hmco.com/students, at this text's web site.

Chapter Exercises When you have completed studying an objective, do the exercises in the exercise set that correspond with that objective. The exercises are labeled with the same letter as the objective. Math is a subject that needs to be learned in small sections and practiced continually in order to be mastered. Doing all of the exercises in each exercise set will help you master the problem-solving techniques necessary for success. As you work through the exercises for an objective, check your answers to the odd-numbered exercises with those in the back of the book.

Preparing for a Test There are important features of this text that can be used to prepare for a test.

- Chapter Summary
- Chapter Review
- Chapter Test

After completing a chapter, read the Chapter Summary. (See page 195 for the Chapter 4 Summary.) This summary highlights the important topics covered in the chapter. The page number following each topic refers you to the page in the text on which you can find more information about the concept.

Following the Chapter Summary are a Chapter Review (see page 196) and a Chapter Test (see page 199). Doing the review exercises is an important way of testing your understanding of the chapter. The answer to each review exercise is given at the back of the book, along with its objective reference. After checking your answers, restudy any objective from which a question you missed was taken. It may be helpful to retry some of the exercises for that objective to reinforce your problem-solving techniques.

The Chapter Test should be used to prepare for an exam. We suggest that you try the Chapter Test a few days before your actual exam. Take the test in a quiet place and try to complete the test in the same amount of time you will be allowed for your exam. When taking the Chapter Test, practice the strategies of successful test takers: (1) scan the entire test to get a feel for the questions; (2) read the directions carefully; (3) work the problems that are easiest for you first; and perhaps most importantly, (4) try to stay calm.

When you have completed the Chapter Test, check your answers. If you missed a question, review the material in that objective and rework some of the exercises from that objective. This will strengthen your ability to perform the skills in that objective.

Write your major here.

Is it difficult to be successful? YES! Successful music groups, artists, professional athletes, chefs, and ______________ have to work very hard to achieve their goals. They focus on their goals and ignore distractions. The things we ask you to do to achieve success take time and commitment. We are confident that if you follow our suggestions, you will succeed.

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Chapter 1 Real Numbers

Signed numbers are used to express the amount of change in the price of a share of stock. Positive numbers indicate an increase in the price, and negative numbers indicate a decrease. Stock market analysts calculate the moving average of a stock, which is the average of the changes in a stock's price over several days. The moving average is used to discover trends in the movement of the price and to help advise investors. The **Exercises on pages 20 and 22** and the **Project on page 44** involve calculating moving averages for different stocks.

Objectives

Section 1.1

- A To use inequality symbols with integers
- B To use opposites and absolute value

Section 1.2

- A To add integers
- B To subtract integers
- C To solve application problems

Section 1.3

- A To multiply integers
- B To divide integers
- C To solve application problems

Section 1.4

- A To write a rational number in simplest form and as a decimal
- B To convert between percents, fractions, and decimals
- C To add or subtract rational numbers
- D To multiply or divide rational numbers
- E To solve application problems

Section 1.5

- A To evaluate exponential expressions
- B To use the Order of Operations Agreement to simplify expressions

Need help? For online student resources, such as section quizzes, visit this textbook's website at **math.college.hmco.com/students.**

Prep Test

1. What is 127.1649 rounded to the nearest hundredth?

2. Add: 49.147 + 5.96

3. Subtract: 5004 − 487

4. Multiply: 407 × 28

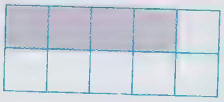
5. Divide: 456 ÷ 19

6. What is the smallest number that both 8 and 12 divide evenly?

7. What is the greatest number that divides both 16 and 20 evenly?

8. Without using 1, write 21 as a product of two whole numbers.

9. Represent the shaded portion of the figure as a fraction.



10. Which of the following, if any, is not possible?
a. 6 + 0 **b.** 6 − 0 **c.** 6 × 0 **d.** 6 ÷ 0

Go Figure

If you multiply the first 20 natural numbers

$$(1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot \ldots \cdot 17 \cdot 18 \cdot 19 \cdot 20),$$

how many zeros will be at the end of the number?

1.1 Introduction to Whole Numbers

Objective A

To use inequality symbols with integers

It seems to be a human characteristic to group similar items. For instance, a biologist places similar animals in groups called *species*. Nutritionists classify foods according to *food groups*; for example, pasta, crackers, and rice are among the foods in the bread group.

Mathematicians place objects with similar properties in groups called *sets*. A **set** is a collection of objects. The objects in a set are called the **elements** of the set.

The **roster method** of writing sets encloses a list of the elements in braces. Thus the set of sections within an orchestra is written {brass, percussion, string, woodwind}. When the elements of a set are listed, each element is listed only once. For instance, if the list of numbers 1, 2, 3, 2, 3 were placed in a set, the set would be $\{1, 2, 3\}$.

The numbers that we use to count objects, such as the students in a classroom or the horses on a ranch, are the *natural numbers*.

$$\textbf{Natural numbers} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \ldots\}$$

The three dots mean that the list of natural numbers continues on and on and that there is no largest natural number.

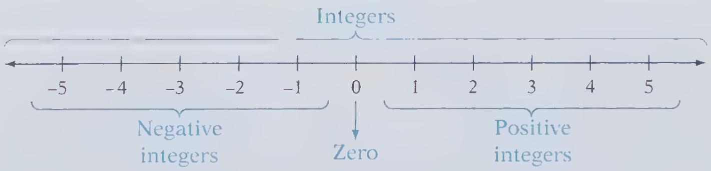
Point of Interest

The Alexandrian astronomer Ptolemy began using *omicron*, *O*, the first letter of the Greek word that means "nothing," as the symbol for zero in 150 A.D. It was not until the 13th century, however, that Fibonacci introduced 0 to the Western world as a placeholder so that we could distinguish, for example, 45 from 405.

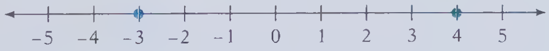
The natural numbers alone do not provide all the numbers that are useful in applications. For instance, a meteorologist also needs the number zero and numbers below zero.

$$\textbf{Integers} = \{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$$

Each integer can be shown on a number line. The integers to the left of zero on the number line are called **negative integers.** The integers to the right of zero are called **positive integers,** or natural numbers. Zero is neither a positive nor a negative integer.



The **graph** of an integer is shown by placing a heavy dot on the number line directly above the number. The graphs of −3 and 4 are shown on the number line below.



Consider the following sentences.

> The quarterback threw the football and the receiver caught *it*.
> A student purchased a computer and used *it* to write history papers.

In the first sentence, *it* is used to mean the football; in the second sentence, *it* means the computer. In language, the word *it* can stand for many different objects. Similarly, in mathematics, a letter of the alphabet can be used to stand for a number. Such a letter is called a **variable.** Variables are used in the following definition of inequality symbols.

Point of Interest

The symbols for "is less than" and "is greater than" were introduced by Thomas Harriot around 1630. Before that, ⊏ and ⊐ were used for $>$ and $<$, respectively.

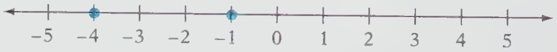
Inequality Symbols

If a and b are two numbers and a is to the left of b on the number line, then a **is less than** b. This is written $a < b$.

If a and b are two numbers and a is to the right of b on the number line, then a **is greater than** b. This is written $a > b$.

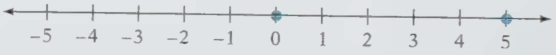
Negative 4 is less than negative 1.

$-4 < -1$



5 is greater than 0.

$5 > 0$



There are also inequality symbols for **is less than or equal to** ($\leq$) and **is greater than or equal to** ($\geq$).

$7 \leq 15$	7 is less than or equal to 15. This is true because $7 < 15$.
$6 \leq 6$	6 is less than or equal to 6. This is true because $6 = 6$.

Point of Interest

The Greek influence is again felt with the symbol $\in$ for "is an element of." Giuseppe Peano used this symbol in 1889 as an abbreviation for the Greek word for "is."

The symbol $\in$ means "is an element of." $2 \in B$ is read "2 is an element of set B."

Given $C = \{3, 5, 9\}$, then $3 \in C$, $5 \in C$, and $9 \in C$. $7 \notin C$ is read "7 is not an element of set C."

Example 1

Use the roster method to write the set of negative integers greater than or equal to -4.

Solution

$A = \{-4, -3, -2, -1\}$

- **A set is designated by a capital letter.**

You Try It 1

Use the roster method to write the set of positive integers less than 7.

Your solution

Solution on p. S1

Example 2

Given $A = \{-6, -2, 0\}$, which elements of set A are less than or equal to -2?

Solution

Find the order relation between each element of set A and -2.

$-6 < -2$
$-2 = -2$
$0 > -2$

The elements -6 and -2 are less than or equal to -2.

You Try It 2

Given $B = \{-5, -1, 5\}$, which elements of set B are greater than -1?

Your solution

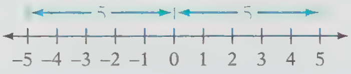
Solution on p. S1

[illegible] To use opposites and absolute value

Two numbers that are the same distance from zero on the number line but are on opposite sides of zero are **opposite numbers,** or **opposites.** The opposite of a number is also called its **additive inverse.**

The opposite of 5 is -5.

The opposite of -5 is 5.

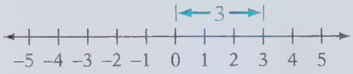


The negative sign can be read "the opposite of."

$-(2) = -2$ The opposite of 2 is -2.
$-(-2) = 2$ The opposite of -2 is 2.

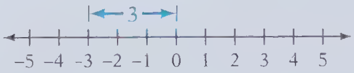
The **absolute value** of a number is its distance from zero on the number line. Therefore, the absolute value of a number is a positive number or zero. The symbol for absolute value is two vertical bars, | |.

The distance from 0 to 3 is 3. Therefore, the absolute value of 3 is 3.



$|3| = 3$

The distance from 0 to -3 is 3. Therefore, the absolute value of -3 is 3.



$|-3| = 3$

Point of Interest

The definition of *absolute value* given in the box is written in what is called rhetorical style. That is, it is written without the use of variables. This is how *all* mathematics was written prior to the Renaissance. During that period from the 14th to the 16th century, the idea of expressing a variable symbolically was developed. In terms of that symbolism, the definition of absolute value is

$$|x| = \begin{cases} x, & x > 0 \\ 0, & x = 0 \\ -x, & x < 0 \end{cases}$$

Absolute Value

The absolute value of a positive number is the number itself. For example, $|9| = 9$.

The absolute value of zero is zero. $|0| = 0$

The absolute value of a negative number is the opposite of the negative number. For example, $|-7| = 7$.

➡ Evaluate: $-|-12|$

$-|-12| = -12$

- The absolute value sign does not affect the negative sign in front of the absolute value sign.

Example 3

Evaluate $|-4|$ and $-|-10|$.

Solution

$|-4| = 4$

$-|-10| = -10$

You Try It 3

Evaluate $|-5|$ and $-|-23|$.

Your solution

Example 4

Given $A = \{-12, 0, 4\}$, find the additive inverse of each element of set A.

Solution

$-(-12) = 12$

$-0 = 0$

- Zero is neither positive nor negative.

$-(4) = -4$

You Try It 4

Given $B = \{-11, 0, 8\}$, find the additive inverse of each element of set B.

Your solution

Example 5

Given $C = \{-17, 0, 14\}$, find the absolute value of each element of set C.

Solution

$|-17| = 17$

$|0| = 0$

$|14| = 14$

You Try It 5

Given $D = \{-37, 0, 29\}$, find the absolute value of each element of set D.

Your solution

Solutions on p. S1

1.1 Exercises

[illegible]

1. Explain the difference between the natural numbers and the integers.

2. Name the smallest integer that is larger than any negative integer.

Place the correct symbol, < or >, between the two numbers.

3. $8 \quad -6$

4. $-14 \quad 16$

5. $-12 \quad 1$

6. $35 \quad 28$

7. $42 \quad 19$

8. $-42 \quad 27$

9. $0 \quad -31$

10. $-17 \quad 0$

11. $53 \quad -46$

12. $-27 \quad -38$

Answer True or False.

13. $-13 > 0$

14. $-20 > 3$

15. $12 > -31$

16. $9 > 7$

17. $-5 > -2$

18. $-44 > -21$

19. $-4 > -120$

20. $0 > -8$

21. $-1 \geq -1$

22. $-10 \leq -10$

For Exercises 23–28, use the roster method to write the set.

23. The natural numbers less than 9

24. The natural numbers less than or equal to 6

25. The positive integers less than or equal to 8

26. The positive integers less than 4

27. The negative integers greater than −7

28. The negative integers greater than or equal to −5

29. Given $A = \{-7, 0, 2, 5\}$, which elements of set A are greater than 2?

30. Given $B = \{-8, 0, 7, 15\}$, which elements of set B are greater than 7?

31. Given $D = \{-23, -18, -8, 0\}$, which elements of set D are less than -8?

32. Given $C = \{-33, -24, -10, 0\}$, which elements of set C are less than -10?

33. Given $E = \{-35, -13, 21, 37\}$, which elements of set E are greater than -10?

34. Given $F = \{-27, -14, 14, 27\}$, which elements of set F are greater than -15?

35. Given $B = \{-52, -46, 0, 39, 58\}$, which elements of set B are less than or equal to 0?

36. Given $A = \{-12, -9, 0, 12, 34\}$, which elements of set A are greater than or equal to 0?

37. Given $C = \{-23, -17, 0, 4, 29\}$, which elements of set C are greater than or equal to -17?

38. Given $D = \{-31, -12, 0, 11, 45\}$, which elements of set D are less than or equal to -12?

39. Given that set A is the positive integers less than 10, which elements of set A are greater than or equal to 5?

40. Given that set B is the positive integers less than or equal to 12, which elements of set B are greater than 6?

41. Given that set D is the negative integers greater than or equal to -10, which elements of set D are less than -4?

42. Given that set C is the negative integers greater than -8, which elements of set C are less than or equal to -3?

Objective B

Find the additive inverse.

43. 4 **44.** 8 **45.** -9 **46.** -28 **47.** -36

Evaluate.

48. $-(-14)$ **49.** $-(-40)$ **50.** $-(77)$ **51.** $-(39)$ **52.** $-(-13)$

53. $|-74|$ **54.** $|-96|$ **55.** $-|-82|$ **56.** $-|-53|$ **57.** $-|81|$

Place the correct symbol, < or >, between the values of the two numbers.

58. $|-83| \quad |58|$ **59.** $|22| \quad |-19|$ **60.** $|43| \quad |-52|$ **61.** $|-71| \quad |-92|$

62. $|-68| \quad |-42|$ **63.** $|12| \quad |-31|$ **64.** $|-45| \quad |-61|$ **65.** $|-28| \quad |43|$

66. Given $A = \{-8, -5, -2, 1, 3\}$, find
a. the opposite of each element of set A
b. the absolute value of each element of set A

67. Given $B = \{-11, -7, -3, 1, 5\}$, find
a. the opposite of each element of set B
b. the absolute value of each element of set B

APPLYING THE CONCEPTS

68. If x represents a negative integer, then $-x$ represents a ____________ integer.

69. If x is an integer, is the inequality $|x| < -3$ always true, sometimes true, or never true?

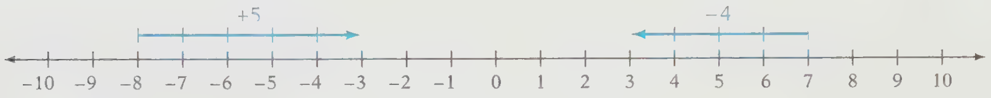
70. In your own words, explain the meaning of the absolute value of a number and the additive inverse of a number.

1.2 Addition and Subtraction of Integers

Objective A

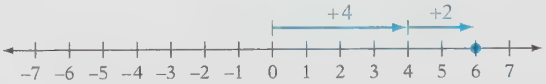
To add integers

A number can be represented anywhere along the number line by an arrow. A positive number is represented by an arrow pointing to the right, and a negative number is represented by an arrow pointing to the left. The size of the number is represented by the length of the arrow.

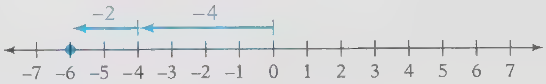


Addition is the process of finding the total of two numbers. The numbers being added are called **addends.** The total is called the **sum.** Addition of integers can be shown on the number line. To add integers, start at zero and draw, above the number line, an arrow representing the first number. At the tip of the first arrow, draw a second arrow representing the second number. The sum is below the tip of the second arrow.

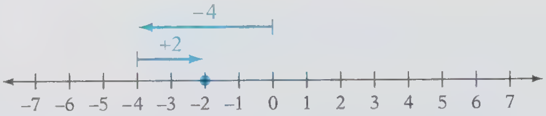
$4 + 2 = 6$



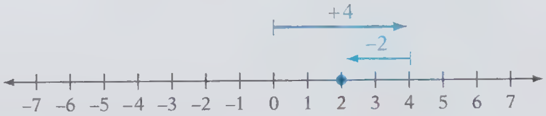
$-4 + (-2) = -6$



$-4 + 2 = -2$



$4 + (-2) = 2$



The pattern for addition shown on the number lines above is summarized in the following rules for adding integers.

Addition of Integers

To add two numbers with the same sign, add the absolute values of the numbers. Then attach the sign of the addends.

To add two numbers with different signs, find the absolute value of each number. Subtract the smaller of the two numbers from the larger. Then attach the sign of the number with the larger absolute value.

Add: $-12 + (-26)$

$-12 + (-26) = -38$

- The signs are the same. Add the absolute values of the numbers (12 + 26). Attach the sign of the addends.

Add: $-19 + 8$

$|-19| = 19$

$|8| = 8$

- The signs are different. Find the absolute value of each number.

$19 - 8 = 11$

- Subtract the smaller number from the larger.

$-19 + 8 = -11$

- Attach the sign of the number with the larger absolute value. Because $|-19| > |8|$, use the sign of -19.

Add: $-23 + 47 + (-18) + (-5)$

$-23 + 47 + (-18) + (-5)$
$= 24 + (-18) + (-5)$
$= 6 + (-5)$
$= 1$

- To add more than two numbers, add the first two numbers. Then add the sum to the third number. Continue until all the numbers are added.

Example 1
Add: $-52 + (-39)$

Solution
$-52 + (-39) = 91$

You Try It 1
Add: $100 + (-43)$

Your solution

Example 2
Add: $37 + (-52) + (-21) + (-7)$

Solution
$37 + (-52) + (-21) + (-7)$
$= -15 + (-21) + (-7)$
$= -36 + (-7)$
$= -43$

You Try It 2
Add: $-51 + 42 + 17 + (-102)$

Your solution

Solutions on p. S1

[illegible]

To subtract integers

1

Look at the two expressions below and note that each expression equals the same number.

$8 - 3 = 5$ 8 minus 3 is 5.
$8 + (-3) = 5$ 8 plus the opposite of 3 is 5.

This example suggests that to subtract two numbers, we add the opposite of the second number to the first number.

$$\boxed{\begin{matrix}\text{First}\\ \text{number}\end{matrix}} - \boxed{\begin{matrix}\text{second}\\ \text{number}\end{matrix}} = \boxed{\begin{matrix}\text{First}\\ \text{number}\end{matrix}} + \boxed{\begin{matrix}\text{the opposite of the}\\ \text{second number}\end{matrix}}$$

$$40 - 60 = 40 + (-60) = -20$$

$$-40 - 60 = -40 + (-60) = -100$$

$$-40 - (-60) = -40 + 60 = 20$$

$$40 - (-60) = 40 + 60 = 100$$

Subtract: $-21 - (-40)$

Change this sign to plus.

$$-21 - (-40) = -21 + 40 = 19$$

Change -40 to the opposite of -40.

- Rewrite each subtraction as addition of the opposite. Then add.

Subtract: $15 - 51$

Change this sign to plus.

$$15 - 51 = 15 + (-51) = -36$$

Change 51 to the opposite of 51.

- Rewrite each subtraction as addition of the opposite. Then add.

Subtract: $-12 - (-21) - 15$

$$\begin{aligned}-12 - (-21) - 15 &= -12 + 21 + (-15)\\ &= 9 + (-15)\\ &= -6\end{aligned}$$

- Rewrite each subtraction as addition of the opposite. Then add.

Example 3 Subtract: $-11 - 15$

Solution

$$\begin{aligned}-11 - 15 &= -11 + (-15)\\ &= -26\end{aligned}$$

You Try It 3 Subtract: $19 - (-32)$

Your solution

Example 4 Subtract: $-14 - 18 - (-21) - 4$

Solution

$$\begin{aligned}&-14 - 18 - (-21) - 4\\ &= -14 + (-18) + 21 + (-4)\\ &= -32 + 21 + (-4)\\ &= -11 + (-4)\\ &= -15\end{aligned}$$

You Try It 4 Subtract: $-9 - (-12) - 17 - 4$

Your solution

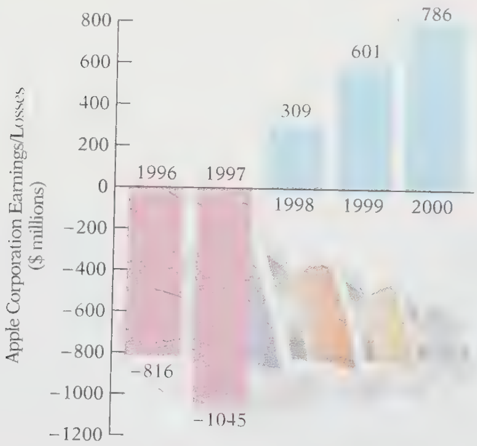
Solutions on p. S1

[illegible]

To solve application problems

Positive and negative numbers are used to express the profitability of a company. A profit is recorded as a positive number; a loss is recorded as a negative number.

The bar graph below shows the net earnings of Apple Computer Corporation for the years 1996 through 2000. Calculate the total net earnings for Apple Computer Corporation for the years 1996 to 2000.



Source: Apple Computer Corporation

Strategy
To determine the total net earnings, add the profits and losses for the years 1996, 1997, 1998, 1999, and 2000.

Solution
$-816 + (-1045) + 309 + 601 + 786 = -165$

The total net earnings for 1996 to 2000 were −$165 million.

Example 5
The average temperature on Mercury's sunlit side is 950°F. The average temperature on Mercury's dark side is −346°F. Find the difference between these two average temperatures.

Strategy
To find the difference, subtract the average temperature on the dark side (−346) from the average temperature on the sunlit side (950).

Solution

$$\begin{aligned} 950 - (-346) &= 950 + 346 \\ &= 1296 \end{aligned}$$

The difference between these average temperatures is 1296°F.

You Try It 5
The average daytime temperature on Mars is −17°F. The average nighttime temperature on Mars is −130°F. Find the difference between these two average temperatures.

Your strategy

Your solution

Solution on p. S1

1.2 Exercises

[illegible]

1. Explain how to add two integers with the same sign.

2. Explain how to add two integers with different signs.

Add.

3. $-3 + (-8)$
4. $-6 + (-9)$
5. $-8 + 3$
6. $-9 + 2$
7. $-3 + (-80)$
8. $-12 + (-1)$
9. $-23 + (-23)$
10. $-12 + (-12)$
11. $16 + (-16)$
12. $-17 + 17$
13. $48 + (-53)$
14. $19 + (-41)$
15. $-17 + (-3) + 29$
16. $13 + 62 + (-38)$
17. $-3 + (-8) + 12$
18. $-27 + (-42) + (-18)$
19. $13 + (-22) + 4 + (-5)$
20. $-14 + (-3) + 7 + (-21)$
21. $-22 + 20 + 2 + (-18)$
22. $-6 + (-8) + 14 + (-4)$
23. $-16 + (-17) + (-18) + 10$
24. $-25 + (-31) + 24 + 19$
25. $26 + (-15) + (-11) + (-12)$
26. $-32 + 40 + (-8) + (-19)$
27. $-17 + (-18) + 45 + (-10)$
28. $23 + (-15) + 9 + (-15)$
29. $46 + (-17) + (-13) + (-50)$
30. $-37 + (-17) + (-12) + (-15)$
31. $-14 + (-15) + (-11) + 40$
32. $28 + (-19) + (-8) + (-1)$
33. $-23 + (-22) + (-21) + 5$
34. $-31 + 9 + (-16) + (-15)$

[illegible]

35. What is the difference between the terms *minus* and *negative*?

36. Explain how to subtract two integers.

Subtract.

37. $16 - 8$

38. $12 - 3$

39. $7 - 14$

40. $6 - 9$

41. $-7 - 2$

42. $-9 - 4$

43. $7 - (-2)$

44. $3 - (-4)$

45. $-6 - (-3)$

46. $-4 - (-2)$

47. $6 - (-12)$

48. $-12 - 16$

49. $-4 - 3 - 2$

50. $4 - 5 - 12$

51. $12 - (-7) - 8$

52. $-12 - (-3) - (-15)$

53. $-19 - (-19) - 18$

54. $-8 - (-8) - 14$

55. $-17 - (-8) - (-9)$

56. $7 - 8 - (-1)$

57. $-30 - (-65) - 29 - 4$

58. $42 - (-82) - 65 - 7$

59. $-16 - 47 - 63 - 12$

60. $42 - (-30) - 65 - (-11)$

61. $-47 - (-67) - 13 - 15$

62. $-18 - 49 - (-84) - 27$

63. $-19 - 17 - (-36) - 12$

64. $48 - 19 - 29 - 51$

65. $21 - (-14) - 43 - 12$

66. $17 - (-17) - 14 - 21$

Application Problems

67. The temperature at which mercury boils is 360°C. Mercury freezes at −39°C. Find the difference between the temperature at which mercury boils and the temperature at which it freezes.

68. The temperature at which radon boils is −62°C. Radon freezes at −71°C. Find the difference between the temperature at which radon boils and the temperature at which it freezes.

The elevation, or height, of places on Earth is measured in relation to sea level, or the average level of the ocean's surface. The table below shows height above sea level as a positive number and depth below sea level as a negative number. Use the table for Exercises 69 to 72.

Continent	*Highest Elevation (meters)*		*Lowest Elevation (meters)*	
Africa	Mt. Kilimanjaro	5895	Qattara Depression	−133
Asia	Mt. Everest	8848	Dead Sea	400
Europe	Mt. Elbrus	5634	Caspian Sea	−28
America	Mt. Aconcagua	6960	Death Valley	−86

69. Find the difference in elevation between Mt. Aconcagua and Death Valley.

70. What is the difference in elevation between Mt. Kilimanjaro and the Qattara Depression?

71. For which continent shown is the difference between the highest and lowest elevations greatest?

72. For which continent shown is the difference between the highest and lowest elevations smallest?

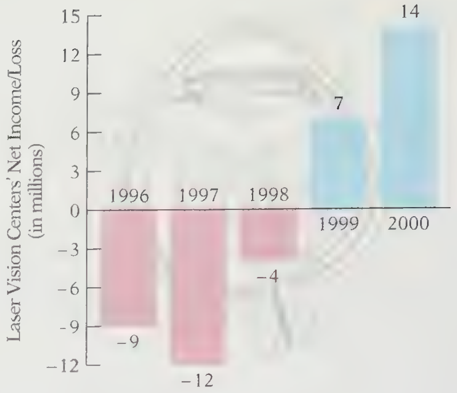
The table at the right shows the boiling point and the melting point in degrees Celsius of three chemical elements. Use this table for Exercises 73 and 74.

Chemical Element	*Boiling Point*	*Melting Point*
Carbon	4827	−355
Radon	−62	−71
Xenon	−107	−112

73. Find the difference between the boiling point and the melting point of carbon.

74. Find the difference between the boiling point and the melting point of xenon.

The graph at the right shows the net income or loss for the Laser Vision Centers for the years 1996 through 2000. Use this graph for Exercises 75 to 77.



Source: Laser Vision Centers, Inc.

75. Determine the total net loss for Laser Vision Centers for the years 1997 to 1999.

76. Find the difference between the company's net income for 1997 and for 1996.

77. For the years 1996 through 2000, did Laser Vision Centers have a net income gain or loss? Explain your answer.

A meteorologist may report a wind-chill temperature. This is the equivalent temperature, including the effects of wind and temperature, that a person would feel in calm air conditions. The table below gives the wind-chill temperature for various wind speeds and temperatures. For instance, when the temperature is 5°F and the wind is blowing at 15 mph, the wind-chill temperature is −25°F. Use this table for Exercises 78 and 79.

Wind-Chill Factors

Wind Speed (mph)	*Thermometer Reading (degrees Fahrenheit)*																
	35	**30**	**25**	**20**	**15**	**10**	**5**	**0**	**−5**	**−10**	**−15**	**−20**	**−25**	**−30**	**−35**	**−40**	**−45**
5	33	27	21	19	12	7	0	−5	−10	−15	−21	−26	−31	−36	−42	−47	−52
10	22	16	10	3	−3	−9	−15	−22	−27	−34	−40	−46	−52	−58	−64	−71	−77
15	16	9	2	−5	−11	−18	−25	−31	−38	−45	−51	−58	−65	−72	−78	−85	−92
20	12	4	−3	−10	−17	−24	−31	−39	−46	−53	−60	−67	−74	−81	−88	−95	−103
25	8	1	−7	−15	−22	−29	−36	−44	−51	−59	−66	−74	−81	−88	−96	−103	−110
30	6	−2	−10	−18	−25	−33	−41	−49	−56	−64	−71	−79	−86	−93	−101	−109	−116
35	4	−4	−12	−20	−27	−35	−43	−52	−58	−67	−74	−82	−89	−97	−105	−113	−120
40	3	−5	−13	−21	−29	−37	−45	−53	−60	−69	−76	−84	−92	−100	−107	−115	−123
45	2	−6	−14	−22	−30	−38	−46	−54	−62	−70	−78	−85	−93	−102	−109	−117	−125

78. When the thermometer reading is −5°F, what is the difference between the wind-chill factor when the wind is blowing 10 mph and when the wind is blowing 30 mph?

79. When the thermometer reading is −20°F, what is the difference between the wind-chill factor when the wind is blowing 15 mph and when the wind is blowing 25 mph?

APPLYING THE CONCEPTS

80. If a and b are integers, is the expression $|a + b| = |a| + |b|$ always true, sometimes true, or never true?

81. Is the difference between two integers always smaller than either one of the numbers in the difference? If not, give an example for which the difference between two integers is greater than either integer.

1.3 Multiplication and Division of Integers

To multiply integers

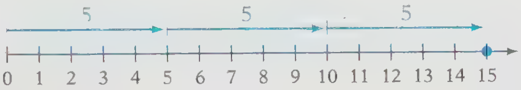
Point of Interest

The cross × was first used as a symbol for multiplication in 1631 in a book titled *The Key to Mathematics*. Also in that year, another book, *Practice of the Analytical Art*, advocated the use of a dot to indicate multiplication.

Several different symbols are used to indicate multiplication. The numbers being multiplied are called **factors;** for instance, 3 and 2 are factors in each of the examples at the right. The result is called the **product.** Note that when parentheses are used and there is no arithmetic symbol, the operation is multiplication.

$$3 \times 2 = 6$$
$$3 \cdot 2 = 6$$
$$(3)(2) = 6$$
$$3(2) = 6$$
$$(3)2 = 6$$

Multiplication is repeated addition of the same number. The product 3 × 5 is shown on the number line below.



$$3 \times 5 = \overbrace{5 + 5 + 5}^{\text{5 is added 3 times.}} = 15$$

Now consider the product of a positive and a negative number.

$$3(-5) = \overbrace{(-5) + (-5) + (-5)}^{-5 \text{ is added 3 times.}} = -15$$

This suggests that the product of a positive number and a negative number is negative. Here are a few more examples.

$$4(-7) = -28 \qquad -6 \cdot 7 = -42 \qquad (-8)7 = -56$$

To find the product of two negative numbers, look at the pattern at the right. As −5 multiplies a sequence of decreasing integers, the products increase by 5.

The pattern can be continued by requiring that the product of two negative numbers be positive.

These numbers decrease by 1.

These numbers increase by 5.

$$-5 \times 3 = -15$$
$$-5 \times 2 = -10$$
$$-5 \times 1 = -5$$
$$-5 \times 0 = 0$$
$$-5 \times (-1) = 5$$
$$-5 \times (-2) = 10$$
$$-5 \times (-3) = 15$$

Multiplication of Integers

To multiply two numbers with the same sign, multiply the absolute values of the numbers. The product is positive.

To multiply two numbers with different signs, multiply the absolute values of the numbers. The product is negative.

Multiply: $-2(5)(-7)(-4)$

$$-2(5)(-7)(-4) = -10(-7)(-4)$$
$$= 70(-4) = -280$$

- To multiply more than two numbers, multiply the first two. Then multiply the product by the third number. Continue until all the numbers are multiplied.

Consider the products shown at the right. Note that when there is an even number of negative factors, the product is positive. When there is an odd number of negative factors, the product is negative.

$$(-3)(-5) = 15$$
$$(-2)(-5)(-6) = -60$$
$$(-4)(-3)(-5)(-7) = 420$$
$$(-3)(-3)(-5)(-4)(-5) = -900$$
$$(-6)(-3)(-4)(-2)(-10)(-5) = 7200$$

This idea can be summarized by the following useful rule: **The product of an even number of negative factors is positive; the product of an odd number of negative factors is negative.**

Example 1 Multiply: $(-3)4(-5)$

Solution $(-3)4(-5) = (-12)(-5) = 60$

You Try It 1 Multiply: $8(-9)10$

Your solution

Example 2 Multiply: $12(-4)(-3)(-5)$

Solution

$$12(-4)(-3)(-5) = (-48)(-3)(-5)$$
$$= 144(-5)$$
$$= -720$$

You Try It 2 Multiply: $(-2)3(-8)7$

Your solution

Solutions on p. S1

Objective B To divide integers

TAKE NOTE
Think of the fraction bar as "divided by." Thus $\frac{8}{2}$ is 8 divided by 2.

For every division problem there is a related multiplication problem.

$$\underbrace{\frac{8}{2} = 4}_{\text{Division}} \quad \text{because} \quad \underbrace{4 \cdot 2 = 8.}_{\text{Related multiplication}}$$

This fact and the rules for multiplying integers can be used to illustrate the rules for dividing integers.

Note in the following examples that the quotient of two numbers with the same sign is positive.

$$\frac{12}{3} = 4 \text{ because } 4 \cdot 3 = 12. \qquad \frac{-12}{-3} = 4 \text{ because } 4(-3) = -12.$$

The next two examples illustrate that the quotient of two numbers with different signs is negative.

$$\frac{12}{-3} = -4 \text{ because } (-4)(-3) = 12. \qquad \frac{-12}{3} = -4 \text{ because } (-4)3 = -12.$$

Point of Interest

There was quite a controversy over the date the new millennium started because of the number zero. When our current calendar was created, numbering began with the year 1 because 0 had not yet been invented. Thus at the beginning of year 2, 1 year had elapsed; at the beginning of year 3, 2 years had elapsed; and so on. This means that at the beginning of year 2000, 1999 years had elapsed and it was not until the beginning of year 2001 that 2000 years had elapsed and a new millennium began.

Division of Integers

To divide two numbers with the same sign, divide the absolute values of the numbers. The quotient is positive.

To divide two numbers with different signs, divide the absolute values of the numbers. The quotient is negative.

Simplify: $-\frac{-56}{7}$

$$-\frac{-56}{7} = -\left(\frac{-56}{7}\right) = -(-8) = 8$$

The properties of division are stated below. In these statements, the symbol ≠ is read "is not equal to."

Properties of Zero and One in Division

If $a \neq 0$, $\frac{0}{a} = 0$.	Zero divided by any number other than zero is zero.
If $a \neq 0$, $\frac{a}{a} = 1$.	Any number other than zero divided by itself is one.
$\frac{a}{1} = a$.	A number divided by one is the number.
$\frac{a}{0}$ is undefined.	Division by zero is not defined.

The fact that $\frac{-12}{3} = -4$, $\frac{12}{-3} = -4$, and $-\frac{12}{3} = -4$ suggests the following rule.

If a and b are integers, and $b \neq 0$, then $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$.

Example 3 Divide: $(-120) \div (-8)$

Solution $(-120) \div (-8) = 15$

You Try It 3 Divide: $(-135) \div (-9)$

Your solution

Example 4

Divide: $\frac{95}{-5}$

Solution $\frac{95}{-5} = -19$

You Try It 4

Divide: $\frac{-72}{4}$

Your solution

Example 5

Simplify: $-\frac{-81}{3}$

Solution $-\frac{-81}{3} = -(-27) = 27$

You Try It 5

Simplify: $-\frac{36}{-12}$

Your solution

Solutions on p. S1

To solve application problems 1

In many courses, your course grade depends on the *average* of all your test scores. You compute the average by calculating the sum of all your test scores and then dividing that result by the number of tests. Statisticians call this average an **arithmetic mean.** Besides its application to finding the average of your test scores, the arithmetic mean is used in many other situations.

Stock market analysts calculate the **moving average** of a stock. This is the arithmetic mean of the changes in the value of a stock for a given number of days. To illustrate the procedure, we will calculate the 5-day moving average of a stock. In actual practice, a stock market analyst may use 15 days, 30 days, or some other number.

The table below shows the amount of increase or decrease, in cents, in the closing price of a stock for a 10-day period.

Day 1	*Day 2*	*Day 3*	*Day 4*	*Day 5*	*Day 6*	*Day 7*	*Day 8*	*Day 9*	*Day 10*
+50	−175	+225	0	−275	−75	−50	+50	−475	−50

To calculate the 5-day moving average of this stock, determine the average of the stock for days 1 through 5, days 2 through 6, days 3 through 7, and so on.

Days 1–5	*Days 2–6*	*Days 3–7*	*Days 4–8*	*Days 5–9*	*Days 6–10*
+50	−175	+225	0	−275	−75
−175	+225	0	−275	−75	−50
+225	0	−275	−75	−50	+50
0	−275	−75	−50	+50	−475
−275	−75	−50	+50	−475	−50
Sum = −175	Sum = −300	Sum = −175	Sum = −350	Sum = −825	Sum = −600
Av = $\frac{-175}{5} = -35$	Av = $\frac{-300}{5} = -60$	Av = $\frac{-175}{5} = -35$	Av = $\frac{-350}{5} = -70$	Av = $\frac{-825}{5} = -165$	Av = $\frac{-600}{5} = -120$

The 5-day moving average is the list of means: −35, −60, −35, −70, −165, and −120. If the list tends to increase, the price of the stock is showing an upward trend; if it decreases, the price of the stock is showing a downward trend. These trends help an analyst recommend stocks.

Example 6

The daily high temperatures (in degrees Celsius) for six days in Anchorage, Alaska, were −14°, 3°, 0°, −8°, 2°, and −1°. Find the average daily high temperature.

Strategy

To find the average daily high temperature:

- Add the six temperature readings.
- Divide the sum by 6.

Solution

$-14 + 3 + 0 + (-8) + 2 + (-1) = -18$

$-18 \div 6 = -3$

The average daily high temperature was −3°C.

You Try It 6

The daily low temperatures (in degrees Celsius) during one week were recorded as −6°, −7°, 0°, −5°, −8°, −1°, and −1°. Find the average daily low temperature.

Your strategy

Your solution

Solution on p. S1

1.3 Exercises

[illegible]

1. Explain how to multiply two integers with the same sign.

2. Explain how to multiply two integers with different signs.

Multiply.

3. $(14)3$
4. $17(6)$
5. $-7 \cdot 4$
6. $-8 \cdot 7$
7. $(-12)(-5)$
8. $(-13)(-9)$
9. $-11(23)$
10. $-8(21)$
11. $(-17)14$
12. $(-15)12$
13. $6(-19)$
14. $17(-13)$
15. $7(5)(-3)$
16. $(-3)(-2)8$
17. $9(-7)(-4)$
18. $(-2)(6)(-4)$
19. $-3(-8)(-9)$
20. $-7(-6)(-5)$
21. $(-9)7(5)$
22. $(-8)7(10)$
23. $7(-2)(5)(-6)$
24. $(-3)7(-2)8$
25. $-9(-4)(-8)(-10)$
26. $-11(-3)(-5)(-2)$
27. $7(9)(-11)4$
28. $-12(-4)7(-2)$
29. $(-14)9(-11)0$
30. $(-13)(15)(-19)0$

[illegible]

Divide.

31. $12 \div (-6)$
32. $18 \div (-3)$
33. $(-72) \div (-9)$
34. $(-64) \div (-8)$
35. $-42 \div 6$
36. $(-56) \div 8$
37. $(-144) \div 12$
38. $(-93) \div (-3)$
39. $48 \div (-8)$
40. $57 \div (-3)$
41. $\frac{-49}{7}$
42. $\frac{-45}{5}$
43. $\frac{-44}{-4}$
44. $\frac{-36}{-9}$
45. $\frac{98}{-7}$
46. $\frac{85}{-5}$
47. $-\frac{-120}{8}$
48. $-\frac{-72}{4}$
49. $-\frac{-80}{-5}$
50. $-\frac{-114}{-6}$
51. $0 \div (-9)$
52. $0 \div (-14)$
53. $\frac{-261}{9}$
54. $\frac{-128}{4}$
55. $9 \div 0$
56. $(-21) \div 0$
57. $\frac{132}{-12}$
58. $\frac{250}{-25}$
59. $\frac{0}{0}$
60. $\frac{-58}{0}$

Application Problems

61. The high temperatures for a 6-day period in Barrow, Alaska, were −23°F, −29°F, −21°F, −28°F, −28°F, and −27°F. Calculate the average daily high temperature.

62. The low temperatures for a 10-day period in a midwestern city were −4°F, −9°F, −5°F, −2°F, 4°F, −1°F, −1°F, −2°F, −2°F, and 2°F. Calculate the average daily low temperature for this city.

63. The value of a share of IBM's stock on November 3, 2000, was $100.13. The table below shows the amount of increase or decrease, to the nearest 10 cents, from the November 3 closing price of the stock for a 10-day period. Calculate the 5-day moving average for this stock.

Day 1	*Day 2*	*Day 3*	*Day 4*	*Day 5*	*Day 6*	*Day 7*	*Day 8*	*Day 9*	*Day 10*
20	200	−230	−60	−640	440	210	−10	−110	370

64. The value of a share of Dell Computer's stock on November 3, 2000, was $32.56. The table below shows the amount of increase or decrease, to the nearest 10 cents, from the November 3 closing price of the stock for a 10-day period. Calculate the 5-day moving average for this stock.

Day 1	*Day 2*	*Day 3*	*Day 4*	*Day 5*	*Day 6*	*Day 7*	*Day 8*	*Day 9*	*Day 10*
−110	110	−230	−190	−540	110	170	−60	−30	0

65. To discourage random guessing on a multiple-choice exam, a professor assigns 5 points for a correct answer, −2 points for an incorrect answer, and 0 points for leaving the question blank. What is the score for a student who had 20 correct answers, had 13 incorrect answers, and left 7 questions blank?

66. To discourage random guessing on a multiple-choice exam, a professor assigns 7 points for a correct answer, −3 points for an incorrect answer, and −1 point for leaving the question blank. What is the score for a student who had 17 correct answers, had 8 incorrect answers, and left 2 questions blank?

APPLYING THE CONCEPTS

67. If $x \in \{-6, -2, 7\}$, for which value of x does the expression $-3x$ have the greatest value?

68. Explain why $0 \div 0$ is not defined.

69. If $-4x$ equals a positive integer, is x a positive or a negative integer? Explain your answer.

1.4 Operations with Rational Numbers

[illegible]

To write a rational number in simplest form and as a decimal 1

TAKE NOTE

The numbers $-\frac{4}{9}$, $\frac{-4}{9}$, and $\frac{4}{-9}$ all represent the same rational number.

A *rational number* is the quotient of two integers. A rational number written in this way is commonly called a fraction. Here are some examples of rational numbers.

$$\frac{3}{4},\quad \frac{-4}{9},\quad \frac{15}{-4},\quad \frac{8}{1},\quad -\frac{5}{6}$$

Rational Numbers

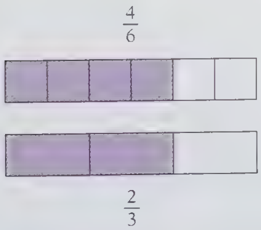
A **rational number** is a number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Point of Interest

As early as 630 A.D., the Hindu mathematician Brahmagupta wrote a fraction as one number over another separated by a space. The Arab mathematician al Hassar (around 1050 A.D.) was the first to show a fraction with the horizontal bar separating the numerator and denominator.

Because an integer can be written as the quotient of the integer and 1, every integer is a rational number. For instance,

$$6 = \frac{6}{1} \qquad -8 = \frac{-8}{1}$$



A fraction is in **simplest form** when there are no common factors in the numerator and the denominator. The fractions $\frac{4}{6}$ and $\frac{2}{3}$ are equivalent fractions because they represent the same part of a whole. However, the fraction $\frac{2}{3}$ is in simplest form because there are no common factors (other than 1) in the numerator and denominator.

To write a fraction in simplest form, eliminate the common factors from the numerator and denominator by using the fact that $1 \cdot \frac{a}{b} = \frac{a}{b}$.

$$\frac{4}{6} = \frac{\boxed{2}\cdot 2}{\boxed{2}\cdot 3} = \frac{\boxed{2}}{\boxed{2}}\cdot\frac{2}{3} = 1\cdot\frac{2}{3} = \frac{2}{3}$$

The process of eliminating common factors is usually written as shown at the right.

$$\frac{4}{6} = \frac{\overset{1}{\cancel{2}}\cdot 2}{\underset{1}{\cancel{2}}\cdot 3} = \frac{2}{3}$$

If you have difficulty determining the common factors, write the numerator and denominator in terms of prime factors. (Recall that a **prime number** is a number greater than 1 that is divisible only by itself and 1. The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.)

➡ Write $\frac{18}{30}$ in simplest form.

$$\frac{18}{30} = \frac{\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{3}}\cdot 3}{\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{3}}\cdot 5} = \frac{3}{5}$$

A rational number can also be written in **decimal notation.**

three tenths $0.3 = \frac{3}{10}$ forty-three thousandths $0.043 = \frac{43}{1000}$

A rational number written as a fraction can be written in decimal notation by dividing the numerator of the fraction by the denominator. Think of the fraction bar as meaning "divided by." Here are two examples of changing a fraction to a decimal.

Write $\frac{5}{8}$ as a decimal.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ -4\,8 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

- Divide the numerator, 5, by the denominator, 8.

When a resulting remainder is zero, the decimal is called a **terminating decimal.** The decimal 0.625 is a terminating decimal.

$\frac{5}{8} = 0.625$

Write $\frac{4}{11}$ as a decimal.

$$\begin{array}{r} 0.3636 \\ 11\overline{)4.0000} \\ -3\,3 \\ \hline 70 \\ -66 \\ \hline 40 \\ -33 \\ \hline 70 \\ -66 \\ \hline 4 \end{array}$$

- Divide the numerator, 4, by the denominator, 11.

No matter how long we continue to divide, the remainder is never zero. The decimal $0.\overline{36}$ is a **repeating decimal.** The bar over the 36 indicates that these digits repeat.

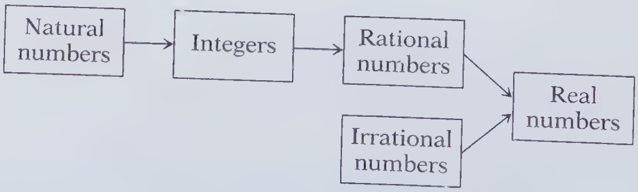
$\frac{4}{11} = 0.\overline{36}$

Every rational number can be written as a terminating or a repeating decimal. Some numbers—for example, $\sqrt{7}$ and π—have decimal representations that never terminate or repeat. These numbers are called **irrational numbers.**

$\sqrt{7} \approx 2.6457513\ldots$ $\pi \approx 3.1415926\ldots$

The rational numbers and the irrational numbers taken together are called the **real numbers.**

The diagram below shows the relationship between some of the sets of numbers we have discussed. The arrows indicate that one set is contained completely within the other set.



Note that there is no arrow between the rational numbers and the irrational numbers. Any given real number is either a rational number or an irrational number. It cannot be both. However, a natural number such as 7 can also be called an integer, a rational number, and a real number.

TAKE NOTE

Rational numbers are fractions, such as $-\frac{6}{7}$ or $\frac{10}{3}$, in which the numerator and denominator are integers. Rational numbers are also represented by repeating decimals such as 0.25767676... or terminating decimals such as 1.73. An irrational number is neither a terminating decimal nor a repeating decimal. For instance, 2.454454445444445... is an irrational number.

Example 1

Write $\frac{3}{20}$ as a decimal.

Solution $\frac{3}{20} = 3 \div 20 = 0.15$

You Try It 1

Write $\frac{4}{9}$ as a decimal. Place a bar over the repeating digits of the decimal.

Your solution

Solution on p. S1

To convert between percents, fractions, and decimals

"A population growth rate of 3%," "a manufacturer's discount of 25%," and "an 8% increase in pay" are typical examples of the many ways in which percent is used in applied problems. **Percent** means "parts of 100." Thus 27% means 27 parts of 100.

In applied problems involving a percent, it may be necessary to rewrite a percent as a fraction or as a decimal, or to rewrite a fraction or a decimal as a percent.

To write a percent as a fraction, remove the percent sign and multiply by $\frac{1}{100}$.

$$27\% = 27\left(\frac{1}{100}\right) = \frac{27}{100}$$

To write a percent as a decimal, remove the percent sign and multiply by 0.01.

$$33\% \quad = \quad 33(0.01) \quad = \quad 0.33$$

Move the decimal point two places to the left. Then remove the percent sign.

TAKE NOTE

The decimal equivalent of 100% is 1. Therefore, multiplying by 100% is the same as multiplying by 1 and does not change the value of the fraction.

$$\frac{5}{8} = \frac{5}{8}(1) = \frac{5}{8}(100\%)$$

A fraction or decimal can be written as a percent by multiplying by 100%. For example, $\frac{5}{8}$ is changed to a percent as follows:

$$\frac{5}{8} = \frac{5}{8}(100\%) = \frac{500}{8}\% = 62.5\%, \qquad \text{or} \qquad 62\frac{1}{2}\%$$

To write a decimal as a percent, multiply by 100%.

$$0.82 \quad = \quad 0.82(100\%) \quad = \quad 82\%$$

Move the decimal point two places to the right. Then write the percent sign.

Example 2

Write 130% as a fraction and as a decimal.

Solution

$$130\% = 130\left(\frac{1}{100}\right) = \frac{130}{100} = \frac{13}{10}$$

$$130\% = 130(0.01) = 1.30$$

You Try It 2

Write 125% as a fraction and as a decimal.

Your solution

Solution on p. S1

Example 3

Write $33\frac{1}{3}\%$ as a fraction.

Solution

$$33\frac{1}{3}\% = 33\frac{1}{3}\left(\frac{1}{100}\right) = \frac{100}{3}\left(\frac{1}{100}\right) = \frac{1}{3}$$

You Try It 3

Write $16\frac{2}{3}\%$ as a fraction.

Your solution

Example 4

Write $\frac{5}{6}$ as a percent.

Solution

$$\frac{5}{6} = \frac{5}{6}(100\%) = \frac{500}{6}\% = 83\frac{1}{3}\%$$

You Try It 4

Write $\frac{9}{16}$ as a percent.

Your solution

Example 5

Write 0.027 as a percent.

Solution

$$0.027 = 0.027(100\%) = 2.7\%$$

You Try It 5

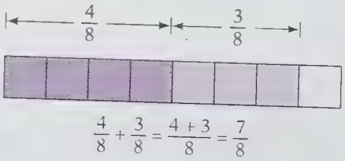
Write 0.043 as a percent.

Your solution

Solutions on p. S1

Objective B To add or subtract rational numbers 1

Four of the 8 squares have dark shading. This is $\frac{4}{8}$ of the entire rectangle. Three of the 8 squares have light shading. This is $\frac{3}{8}$ of the entire rectangle. Thus 7 of the 8 squares, or $\frac{7}{8}$, of the entire rectangle is shaded.



Addition of Fractions

To add two fractions with the same denominator, add the numerators and place the sum over the common denominator.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

To add fractions with different denominators, first rewrite the fractions as equivalent fractions with a common denominator. Then add the fractions. The common denominator is the **least common multiple** (LCM) of the denominators. This is the smallest number that is a multiple of each of the denominators. It can be found by first writing each denominator as a product of prime factors. The LCM must contain the factors of each denominator.

$$6 = 2 \cdot 3$$
$$10 = 2 \cdot 5$$

$$\text{LCM} = \overbrace{\underbrace{2 \cdot 3}_{\text{Factors of 6}} \cdot 5}^{\text{Factors of 10}} = 30$$

TAKE NOTE
The least common multiple of the denominators is frequently called the **least common denominator** (LCD).

Add: $-\frac{5}{6} + \frac{3}{10}$

The LCM of 6 and 10 is 30. Rewrite each fraction as an equivalent fraction with a denominator of 30. Then add the fractions.

$$-\frac{5}{6} + \frac{3}{10} = -\frac{25}{30} + \frac{9}{30} = \frac{-25+9}{30} = \frac{-16}{30} = -\frac{8}{15}$$

To subtract fractions with the same denominator, subtract the numerators and place the difference over the common denominator.

Subtract: $-\frac{4}{9} - \left(-\frac{7}{12}\right)$

The LCM of 9 and 12 is 36. Rewrite each fraction as an equivalent fraction with a denominator of 36. Then subtract the fractions.

$$-\frac{4}{9} - \left(-\frac{7}{12}\right) = -\frac{16}{36} - \left(-\frac{21}{36}\right) = \frac{-16-(-21)}{36} = \frac{-16+21}{36} = \frac{5}{36}$$

Point of Interest

One of the earliest written mathematical documents is called the Rhind Papyrus. It was discovered in Egypt in 1858 but it is estimated to date back to 1650 B.C. The Papyrus shows that the earlier Egyptian method of calculating with fractions was much different from methods used today. These early Egyptians used *unit fractions* which are fractions whose numerator is 1. All other fractions were represented in terms of those fractions. For instance, the fraction $\frac{2}{11}$ was represented as $\frac{1}{6} + \frac{1}{66}$.

To add or subtract decimals, write the numbers so that the decimal points are in a vertical line. Then proceed as in the addition or subtraction of integers. Write the decimal point in the answer directly below the decimal points in the problem.

Add: $-114.039 + 84.76$

$$\begin{array}{r} 114.039 \\ -\ 84.76 \\ \hline 29.279 \end{array}$$

- The signs are different. Find the difference between the absolute values of the numbers. $|-114.039| = 114.039$; $|84.76| = 84.76$

$$-114.039 + 84.76 = -29.279$$

- Attach the sign of the number with the larger absolute value. Because $|-114.039| > |84.76|$, use the sign of -114.039.

Example 6

Subtract: $\frac{5}{16} - \frac{7}{40}$

Solution
The LCM of 16 and 40 is 80.

$$\frac{5}{16} - \frac{7}{40} = \frac{25}{80} - \frac{14}{80} = \frac{25-14}{80} = \frac{11}{80}$$

You Try It 6

Subtract: $\frac{5}{9} - \frac{11}{12}$

Your solution

Example 7

Simplify: $-\frac{3}{4} + \frac{1}{6} - \frac{5}{8}$

Solution
The LCM of 4, 6, and 8 is 24.

$$-\frac{3}{4} + \frac{1}{6} - \frac{5}{8} = -\frac{18}{24} + \frac{4}{24} - \frac{15}{24}$$
$$= \frac{-18+4-15}{24} = \frac{-29}{24} = -\frac{29}{24}$$

You Try It 7

Simplify: $-\frac{7}{8} - \frac{5}{6} + \frac{3}{4}$

Your solution

Solutions on p. S2

Example 8 Subtract: 42.987 − 98.61

Solution 42.987 − 98.61

$= 42.987 + (-98.61)$

$= -55.623$

You Try It 8 Subtract: 16.127 − 67.91

Your solution

Solution on p. S2

Objective D To multiply or divide rational numbers 1

The product of two fractions is the product of the numerators divided by the product of the denominators.

Multiply: $\frac{3}{8} \times \frac{12}{17}$

$$\frac{3}{8} \times \frac{12}{17} = \frac{3 \cdot 12}{8 \cdot 17}$$

- Multiply the numerators. Multiply the denominators.

$$= \frac{3 \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3}{2 \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 17} = \frac{9}{34}$$

- Divide the numerator and denominator by their common factors.

The **reciprocal** of a fraction is the fraction with the numerator and denominator interchanged. For instance, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, and the reciprocal of $-\frac{5}{2}$ is $-\frac{2}{5}$. To divide fractions, multiply the dividend by the reciprocal of the divisor.

TAKE NOTE
The method for dividing fractions is sometimes stated, "To divide fractions, invert the divisor and then multiply." Inverting the divisor means writing its reciprocal.

Divide: $\frac{3}{10} \div \left(-\frac{18}{25}\right)$

$$\frac{3}{10} \div \left(-\frac{18}{25}\right) = -\left(\frac{3}{10} \div \frac{18}{25}\right) = -\left(\frac{3}{10} \cdot \frac{25}{18}\right) = -\left(\frac{3 \cdot 25}{10 \cdot 18}\right)$$

$$= -\left(\frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{5}} \cdot 5}{2 \cdot \underset{1}{\cancel{5}} \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot 3}\right) = -\frac{5}{12}$$

- The signs are different, so the quotient is negative.

To multiply decimals, multiply as with integers. Write the decimal point in the product in such a way that the number of decimal places in the product equals the sum of the number of decimal places in the factors.

Multiply: −6.89 × 0.00035

$$\begin{array}{r l}
6.89 & \text{2 decimal places} \\
\times\ 0.00035 & \text{5 decimal places} \\
\hline
3445 & \\
2067 & \\
\hline
0.0024115 & \text{7 decimal places}
\end{array}$$

- Multiply the absolute values.

$-6.89 \times 0.00035 = -0.0024115$

- The signs are different. The product is negative.

To divide decimals, move the decimal point in the divisor to the right so that the divisor becomes a whole number. Move the decimal point in the dividend the same number of places to the right. Place the decimal point in the quotient directly over the decimal point in the dividend. Then divide as with whole numbers.

TAKE NOTE
Moving the decimal point in the numerator and denominator is the same as multiplying the numerator and the denominator by the same number. For the problem at the right, we have

$$1.32 \div 0.27 = \frac{1.32}{0.27}$$
$$= \frac{1.32}{0.27} \cdot \frac{100}{100} = \frac{132}{27}$$

Divide: 1.32 ÷ 0.27. Round to the nearest tenth.

$$\begin{array}{r} 4.88 \approx 4.9 \\ 0.27.\overline{)1.32.00} \\ -108 \\ \hline 240 \\ -216 \\ \hline 240 \\ -216 \\ \hline 24 \end{array}$$

- Move the decimal point 2 places to the right in the divisor and then in the dividend. Place the decimal point in the quotient directly over the decimal point in the dividend.
- Also note that the symbol ≈ is used to indicate that the quotient is an approximate value that has been rounded off.

Example 9

Multiply: $\frac{2}{3}\left(-\frac{9}{10}\right)$

Solution

The product is negative.

$$\frac{2}{3}\left(-\frac{9}{10}\right) = -\left(\frac{2}{3} \cdot \frac{9}{10}\right) = -\frac{2 \cdot 9}{3 \cdot 10}$$
$$= -\frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot 3}{\underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{2}} \cdot 5} = -\frac{3}{5}$$

You Try It 9

Multiply: $-\frac{7}{12}\left(\frac{9}{14}\right)$

Your solution

Example 10

Divide: $-\frac{5}{8} \div \left(-\frac{5}{40}\right)$

Solution

The quotient is positive.

$$-\frac{5}{8} \div \left(-\frac{5}{40}\right) = \frac{5}{8} \div \frac{5}{40} = \frac{5}{8} \cdot \frac{40}{5} = \frac{5 \cdot 40}{8 \cdot 5}$$
$$= \frac{\overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 5}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{5}}} = \frac{5}{1} = 5$$

You Try It 10

Divide: $-\frac{3}{8} \div \left(-\frac{5}{12}\right)$

Your solution

Example 11

Multiply: -4.29×8.2

Solution

Multiply the absolute values.
The product is negative.

$-4.29 \times 8.2 = -35.178$

You Try It 11

Multiply: -5.44×3.8

Your solution

Example 12

Divide: $-0.0792 \div (-0.42)$
Round to the nearest hundredth.

Solution

Divide the absolute values.
The quotient is positive.

$-0.0792 \div (-0.42) \approx 0.19$

You Try It 12

Divide: $-0.394 \div 1.7$
Round to the nearest hundredth.

Your solution

Solutions on p. S2

To solve application problems

One of the applications of percent is to express a portion of a total as a percent. For instance, a recent survey of 450 mall shoppers found that 270 preferred the mall closest to their home even though it did not have the same variety of stores as a mall farther from home. The percent of shoppers who preferred the mall closest to home can be found by converting a fraction to a percent.

$$\frac{\text{Portion preferring mall closest to home}}{\text{Total number surveyed}} = \frac{270}{450}$$
$$= 0.60 = 60\%$$

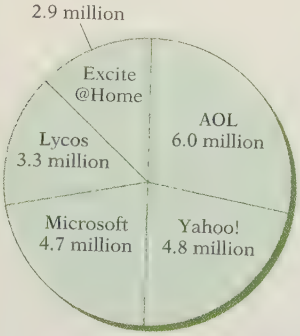
According to the Congressional Budget Office, the U.S. federal budget surplus in 2000 was $232.3 billion. The number 232.3 billion means

$$232.3 \times \underbrace{1{,}000{,}000{,}000}_{\text{1 billion}} = 232{,}300{,}000{,}000$$

Numbers such as 232.3 billion are used in many instances because they are easy to read and offer an approximation to the actual number. Such numbers are used in Example 13 and You Try It 13.

Example 13

The circle graph shows the top five web properties in terms of the number of unique visitors during one week in the year 2000. What percent of the visitors accessed the Yahoo! site? Round to the nearest tenth of a percent.



Source: Media Metrix, March 2000

Strategy

To find the percent of web visitors who accessed Yahoo!:

- Find the total number of visitors to the top five web sites.
- Divide the number of people who visited Yahoo! (4.8 million) by the total number of visitors.

Solution

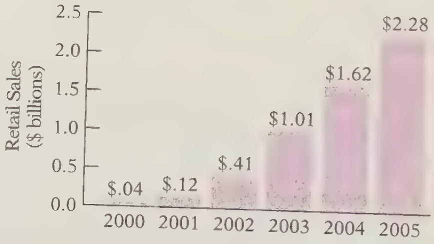
$6.0 + 4.8 + 4.7 + 3.3 + 2.9 = 21.7$

$\frac{4.8}{21.7} \approx 0.221 = 22.1\%$

Of the top five web sites, 22.1% visited Yahoo! during the given week.

You Try It 13

The bar graph below shows the projected retail sales of e-books through 2005. Find the difference between projected e-book sales in 2005 and in 2002.



Source: Andersen Consulting, *Wall Street Journal*, June 21, 2000.

Your strategy

Your solution

Solution on p. S2

1.4 Exercises

Objective A

Write each fraction in simplest form.

1. $\frac{7}{21}$ **2.** $\frac{10}{15}$ **3.** $\frac{8}{22}$ **4.** $\frac{8}{60}$ **5.** $\frac{50}{75}$

6. $\frac{20}{44}$ **7.** $\frac{12}{8}$ **8.** $\frac{36}{9}$ **9.** $\frac{0}{36}$ **10.** $\frac{12}{18}$

11. $\frac{60}{100}$ **12.** $\frac{14}{45}$ **13.** $\frac{44}{60}$ **14.** $\frac{19}{51}$ **15.** $\frac{23}{46}$

Write as a decimal. Place a bar over the repeating digits of a repeating decimal.

16. $\frac{4}{5}$ **17.** $\frac{1}{6}$ **18.** $\frac{5}{6}$ **19.** $\frac{1}{8}$ **20.** $\frac{7}{8}$

21. $\frac{2}{9}$ **22.** $\frac{8}{9}$ **23.** $\frac{5}{11}$ **24.** $\frac{7}{12}$ **25.** $\frac{11}{12}$

26. $\frac{9}{16}$ **27.** $\frac{15}{16}$ **28.** $\frac{7}{18}$ **29.** $\frac{17}{18}$ **30.** $\frac{1}{20}$

Objective B

31. Explain how to write a percent as a fraction.

32. Explain how to write a fraction as a percent.

Write as a fraction and a decimal.

33. 75% **34.** 40% **35.** 64% **36.** 88% **37.** 175%

38. 160% **39.** 19% **40.** 87% **41.** 5% **42.** 8%

Write as a fraction.

43. $11\frac{1}{9}\%$
44. $4\frac{2}{7}\%$
45. $12\frac{1}{2}\%$
46. $37\frac{1}{2}\%$
47. $66\frac{2}{3}\%$

48. $\frac{1}{4}\%$
49. $\frac{1}{2}\%$
50. $6\frac{1}{4}\%$
51. $83\frac{1}{3}\%$
52. $5\frac{3}{4}\%$

Write as a decimal.

53. 7.3%
54. 9.1%
55. 15.8%
56. 16.7%
57. 0.3%

58. 0.9%
59. 9.9%
60. 9.15%
61. 121.2%
62. 18.23%

Write as a percent.

63. 0.15
64. 0.37
65. 0.05
66. 0.02
67. 0.175

68. 0.125
69. 1.15
70. 1.36
71. 0.008
72. 0.004

73. $\frac{27}{50}$
74. $\frac{83}{100}$
75. $\frac{1}{3}$
76. $\frac{3}{8}$
77. $\frac{5}{11}$

78. $\frac{4}{9}$
79. $\frac{7}{8}$
80. $\frac{9}{20}$
81. $1\frac{2}{3}$
82. $2\frac{1}{2}$

Objective C

Simplify.

83. $\frac{2}{3} + \frac{5}{12}$
84. $\frac{1}{2} + \frac{3}{8}$
85. $\frac{5}{8} - \frac{5}{6}$
86. $\frac{1}{9} - \frac{5}{27}$

87. $-\frac{5}{12} - \frac{3}{8}$

88. $-\frac{5}{6} - \frac{5}{9}$

89. $-\frac{6}{13} + \frac{17}{26}$

90. $-\frac{7}{12} + \frac{5}{8}$

91. $\frac{5}{8} - \left(-\frac{3}{4}\right)$

92. $\frac{3}{5} - \frac{11}{12}$

93. $\frac{11}{12} - \frac{5}{6}$

94. $-\frac{2}{3} - \left(-\frac{11}{18}\right)$

95. $-\frac{5}{8} - \left(-\frac{11}{12}\right)$

96. $\frac{1}{3} + \frac{5}{6} - \frac{2}{9}$

97. $\frac{1}{2} - \frac{2}{3} + \frac{1}{6}$

98. $-\frac{3}{8} - \frac{5}{12} - \frac{3}{16}$

99. $-\frac{5}{16} + \frac{3}{4} - \frac{7}{8}$

100. $\frac{1}{2} - \frac{3}{8} - \left(-\frac{1}{4}\right)$

101. $\frac{3}{4} - \left(-\frac{7}{12}\right) - \frac{7}{8}$

102. $\frac{1}{3} - \frac{1}{4} - \frac{1}{5}$

103. $\frac{2}{3} - \frac{1}{2} + \frac{5}{6}$

104. $\frac{5}{16} + \frac{1}{8} - \frac{1}{2}$

105. $\frac{5}{8} - \left(-\frac{5}{12}\right) + \frac{1}{3}$

106. $\frac{1}{8} - \frac{11}{12} + \frac{1}{2}$

107. $7.56 + 0.462$

108. $1.09 + 6.2$

109. $-32.1 - 6.7$

110. $5.13 - 8.179$

111. $-13.092 + 6.9$

112. $2.54 - 3.6$

113. $5.43 + 7.925$

114. $-16.92 - 6.925$

115. $-3.87 + 8.546$

116. $6.9027 - 17.692$

117. $2.09 - 6.72 - 5.4$

118. $-18.39 + 4.9 - 23.7$

119. $19 - (-3.72) - 82.75$

120. $-3.07 - (-2.97) - 17.4$

121. $16.4 - (-3.09) - 7.93$

122. $-3.09 - 4.6 - (-27.3)$

123. $2.66 - (-4.66) - 8.2$

124. $-0.34 - (-4.35) - 3.2$

125. $7.5 - 12.8 - (-0.57)$

126. $2.8 - (-3.44) + 2.3$

[illegible]

Simplify.

127. $\frac{1}{2} \times \left(-\frac{3}{4}\right)$

128. $-\frac{2}{9} \times \left(-\frac{3}{14}\right)$

129. $\left(-\frac{3}{8}\right)\left(-\frac{4}{15}\right)$

130. $\left(-\frac{3}{4}\right)\left(-\frac{8}{27}\right)$

131. $-\frac{1}{2}\left(\frac{8}{9}\right)$

132. $\frac{5}{12}\left(-\frac{8}{15}\right)$

133. $\frac{5}{8}\left(-\frac{7}{12}\right)\frac{16}{25}$

134. $\left(\frac{5}{12}\right)\left(-\frac{8}{15}\right)\left(-\frac{1}{3}\right)$

135. $\frac{1}{2}\left(-\frac{3}{4}\right)\left(-\frac{5}{8}\right)$

136. $\frac{3}{8} \div \frac{1}{4}$

137. $\frac{5}{6} \div \left(-\frac{3}{4}\right)$

138. $-\frac{5}{12} \div \frac{15}{32}$

139. $-\frac{7}{8} \div \frac{4}{21}$

140. $\frac{7}{10} \div \frac{2}{5}$

141. $-\frac{15}{64} \div \left(-\frac{3}{40}\right)$

142. $\frac{1}{8} \div \left(-\frac{5}{12}\right)$

143. $-\frac{4}{9} \div \left(-\frac{2}{3}\right)$

144. $-\frac{6}{11} \div \frac{4}{9}$

145. $1.2(3.47)$

146. $(-0.8)6.2$

147. $(-1.89)(-2.3)$

148. $(6.9)(-4.2)$

149. $1.06(-3.8)$

150. $-2.7(-3.5)$

151. $1.2(-0.5)(3.7)$

152. $-2.4(6.1)(0.9)$

153. $2.3(-0.6)(0.8)$

154. $-1.2(-0.55)(1.9)$

155. $0.44(-2.3)(-0.5)$

156. $-3.4(-22.1)(-0.5)$

157. $1.8(0.33)(-0.4)$

158. $4.5(-0.22)(-0.8)$

159. $-24.7 \div 0.09$

Simplify. Round to the nearest hundredth.

160. $-1.27 \div (-1.7)$

161. $9.07 \div (-3.5)$

162. $0.0976 \div 0.042$

163. $-6.904 \div 1.35$

164. $-7.894 \div (-2.06)$

165. $-354.2086 \div 0.1719$

Objective E Application Problems

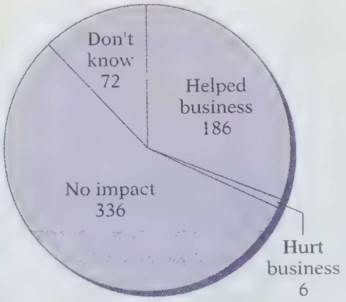
166. When a U.S. company does business with another country, it is necessary to convert U.S. currency into the currency of the other country. These *exchange rates* are determined by various factors. The exchange rates for some currencies during one day in December 2000 are shown in the table at the right.

Foreign Currency per U.S. Dollar	
Chinese renmimbi	8.27750
European monetary unit (euro)	1.14090
Irish punt	0.89853

a. If you sold goods worth 1.2 million U.S. dollars to China, how many renmimbi would you receive?

b. How many U.S. dollars, to the nearest cent, are equivalent to 1 Irish punt?

167. A survey asked 600 small business owners about the impact of the Internet on their businesses. The results are shown in the graph at the right. What percent of the owners responded that the Internet has helped business?



Source: Dun & Bradstreet in *Infoworld*, July 26, 2000

168. The table below shows the first-quarter profits and losses for 2000 for three companies in the telecommunications industry. Profits are shown as positive numbers; losses are shown as negative numbers. One-quarter of a year is 3 months.

a. If earnings were to continue throughout the year at the same level, what would the annual earnings or losses be for Exodus Communications, Inc.?

b. For the quarter shown, what was the average monthly profit or loss for e.spire Communications, Inc.? Round to the nearest hundred thousand.

Telecommunications Company	*First Quarter 2000 Profits ($ millions)*
Exodus Communications, Inc.	−58.4
e.spire Communications, Inc.	−82.1
Optelecom, Inc.	−0.3

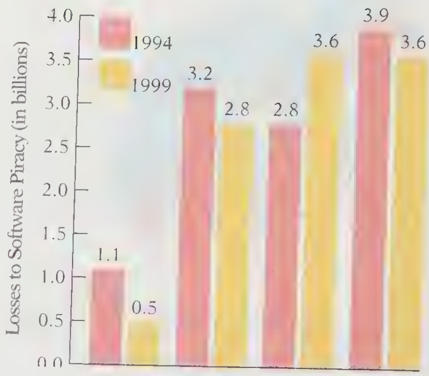
Source: Income statements from individual companies

169. A May 2000 report by the Business Software Alliance and the Software & Information Industry Association detailed the estimated loss to piracy of software profits. The graph at the right shows the losses by region in billions of dollars.

a. What was the percent increase in losses due to software piracy between 1994 and 1999 for Western Europe? Round to the nearest tenth of a percent.

b. What was the percent decrease in losses due to software piracy between 1994 and 1999 for North America? Round to the nearest tenth of a percent.

c. Which region had more than a 50% decrease in software piracy between 1994 and 1999?



170. According to the Federal Highway Administration, the average car is driven approximately 10,300 mi per year and uses approximately 495 gal of gas. Assuming that the average cost of gasoline is \$1.097 per gallon, which includes \$.443 for all taxes, and that there are 1.53 million cars on the road, determine how much total tax is paid for gasoline in one year. (You may not need all the data given in this problem.)

APPLYING THE CONCEPTS

171. A magic square is one in which the numbers in every row, column, and diagonal sum to the same number. Complete the magic square at the right.

$\frac{2}{3}$		
	$\frac{1}{6}$	$\frac{5}{6}$
		$-\frac{1}{3}$

172. If a and b are rational numbers and $a < b$, is it always possible to find a rational number c such that $a < c < b$? If not, explain why. If so, show how to find one.

173. For each part below, find a rational number r that satisfies the condition.
a. $r^2 < r$ **b.** $r^2 = r$ **c.** $r^2 > r$

174. In a survey of consumers, approximately 43% said they would be willing to pay between \$1000 and \$2000 more for a new car if the car had an EPA rating of 80 mpg. If your car now gets 28 mpg and you drive approximately 10,000 mi per year, in how many months would your savings on gasoline pay for the increased cost of such a car? Assume the average cost for gasoline is \$1.56 per gallon.

175. Find three different natural numbers a, b, and c such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ is a natural number.

1.5 Exponents and the Order of Operations Agreement

Objective A

To evaluate exponential expressions

Repeated multiplication of the same factor can be written using an exponent.

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = \underset{\text{Base}}{2}^{5 \leftarrow \text{Exponent}} \qquad a \cdot a \cdot a \cdot a = \underset{\text{Base}}{a}^{4 \leftarrow \text{Exponent}}$$

The **exponent** indicates how many times the factor, which is called the **base,** occurs in the multiplication. The multiplication $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ is in **factored form.** The exponential expression 2^5 is in **exponential form.**

Point of Interest

Rene Descartes (1596–1650) was the first mathematician to use exponential notation extensively as it is used today. However, for some unknown reason, he always used *xx* for x^2.

2^1 is read "2 to the first power" or just "2." Usually the exponent 1 is not written.

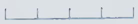
2^2 is read "2 to the second power" or "2 squared."

2^3 is read "2 to the third power" or "2 cubed."

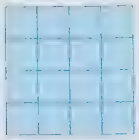
2^4 is read "2 to the fourth power."

a^4 is read "*a* to the fourth power."

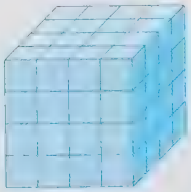
There is a geometric interpretation of the first three natural-number powers.



$4^1 = 4$
Length: 4 ft



$4^2 = 16$
Area: 16 ft^2



$4^3 = 64$
Volume: 64 ft^3

To evaluate an exponential expression, write each factor as many times as indicated by the exponent. Then multiply.

Evaluate $(-2)^4$.

$$(-2)^4 = (-2)(-2)(-2)(-2)$$ • Write −2 as a factor 4 times.

$$= 16$$ • Multiply.

Evaluate -2^4.

$$-2^4 = -(2 \cdot 2 \cdot 2 \cdot 2)$$ • Write 2 as a factor 4 times.

$$= -16$$ • Multiply.

TAKE NOTE

Note the difference between $(-2)^4$ and -2^4.

$(-2)^4 = 16$
$-2^4 = -16$

Example 1 Evaluate -5^3.

Solution $-5^3 = -(5 \cdot 5 \cdot 5) = -125$

Example 2 Evaluate $(-4)^4$.

Solution $(-4)^4 = (-4)(-4)(-4)(-4)$
$= 256$

Example 3 Evaluate $(-3)^2 \cdot 2^3$.

Solution $(-3)^2 \cdot 2^3 = (-3)(-3) \cdot (2)(2)(2)$
$= 9 \cdot 8 = 72$

Example 4

Evaluate $\left(-\frac{2}{3}\right)^3$.

Solution $\left(-\frac{2}{3}\right)^3 = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)$
$= -\frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3} = -\frac{8}{27}$

Example 5 Evaluate $-4(0.7)^2$.

Solution $-4(0.7)^2 = -4(0.7)(0.7)$
$= -2.8(0.7) = -1.96$

You Try It 1 Evaluate -6^3.

Your solution

You Try It 2 Evaluate $(-3)^4$.

Your solution

You Try It 3 Evaluate $(3^3)(-2)^3$.

Your solution

You Try It 4

Evaluate $\left(-\frac{2}{5}\right)^2$.

Your solution

You Try It 5 Evaluate $-3(0.3)^3$.

Your solution

Solutions on p. S2

Objective B

To use the Order of Operations Agreement to simplify expressions

Let's evaluate $2 + 3 \cdot 5$.

There are two arithmetic operations, addition and multiplication, in this expression. The operations could be performed in different orders.

Multiply first. $2 + \underbrace{3 \cdot 5}$

Then add. $\underbrace{2 + 15}_{17}$

Add first. $\underbrace{2 + 3} \cdot 5$

Then multiply. $\underbrace{5 \cdot 5}_{25}$

To prevent there being more than one answer for a numerical expression, an Order of Operations Agreement has been established.

The Order of Operations Agreement

Step 1 Perform operations inside grouping symbols. Grouping symbols include parentheses ( ), brackets [ ], braces { }, and the fraction bar.

Step 2 Simplify exponential expressions.

Step 3 Do multiplication and division as they occur from left to right.

Step 4 Do addition and subtraction as they occur from left to right.

Evaluate $12 - 24(8 - 5) \div 2^2$.

$12 - 24(8 - 5) \div 2^2 = 12 - 24(3) \div 2^2$

- Perform operations inside grouping symbols.

$= 12 - 24(3) \div 4$

- Simplify exponential expressions.

$= 12 - 72 \div 4$

- Do multiplication and division as they occur from left to right.

$= 12 - 18$

$= -6$

- Do addition and subtraction as they occur from left to right.

One or more of the above steps may not be needed to evaluate an expression. In that case, proceed to the next step in the Order of Operations Agreement.

Evaluate $\frac{4 + 8}{2 + 1} - (3 - 1) + 2$.

$\frac{4 + 8}{2 + 1} - (3 - 1) + 2 = \frac{12}{3} - 2 + 2$

- Perform operations above and below the fraction bar and inside parentheses.

$= 4 - 2 + 2$

- Do multiplication and division as they occur from left to right.

$= 2 + 2$

- Do addition and subtraction as they occur from left to right.

$= 4$

When an expression has grouping symbols inside grouping symbols, perform the operations inside the inner grouping symbols first.

Evaluate $6 \div [4 - (6 - 8)] + 2^2$.

$6 \div [4 - (6 - 8)] + 2^2 = 6 \div [4 - (-2)] + 2^2$

- Perform operations inside grouping symbols.

$= 6 \div 6 + 2^2$

$= 6 \div 6 + 4$

- Simplify exponential expressions.

$= 1 + 4$

- Do multiplication and division as they occur from left to right.

$= 5$

- Do addition and subtraction as they occur from left to right.

Example 6

Evaluate $(1.75 - 1.3)^2 \div 0.025 + 6.1$.

Solution

$(1.75 - 1.3)^2 \div 0.025 + 6.1$
$= (0.45)^2 \div 0.025 + 6.1$
$= 0.2025 \div 0.025 + 6.1$
$= 8.1 + 6.1$
$= 14.2$

You Try It 6

Evaluate $(6.97 - 4.72)^2 \cdot 4.5 \div 0.05$.

Your solution

Solution on p. S2

Example 7

Evaluate $4 - 3[4 - 2(6 - 3)] \div 2$.

Solution

$$\begin{aligned} &4 - 3[4 - 2(6 - 3)] \div 2 \\ &\quad = 4 - 3[4 - 2 \cdot 3] \div 2 \\ &\quad = 4 - 3[4 - 6] \div 2 \\ &\quad = 4 - 3[-2] \div 2 \\ &\quad = 4 + 6 \div 2 \\ &\quad = 4 + 3 \\ &\quad = 7 \end{aligned}$$

You Try It 7

Evaluate $18 - 5[8 - 2(2 - 5)] \div 10$.

Your solution

Example 8

Evaluate $27 \div (5 - 2)^2 + (-3)^2 \cdot 4$.

Solution

$$\begin{aligned} &27 \div (5 - 2)^2 + (-3)^2 \cdot 4 \\ &\quad = 27 \div 3^2 + (-3)^2 \cdot 4 \\ &\quad = 27 \div 9 + 9 \cdot 4 \\ &\quad = 3 + 9 \cdot 4 \\ &\quad = 3 + 36 \\ &\quad = 39 \end{aligned}$$

You Try It 8

Evaluate $36 \div (8 - 5)^2 - (-3)^2 \cdot 2$.

Your solution

Example 9

Evaluate $\frac{5}{8} - \left(\frac{2}{5} - \frac{1}{2}\right) \div \left(\frac{2}{3}\right)^2$.

Solution

$$\begin{aligned} &\frac{5}{8} - \left(\frac{2}{5} - \frac{1}{2}\right) \div \left(\frac{2}{3}\right)^2 \\ &\quad = \frac{5}{8} - \left(-\frac{1}{10}\right) \div \left(\frac{2}{3}\right)^2 \\ &\quad = \frac{5}{8} + \frac{1}{10} \div \frac{4}{9} \\ &\quad = \frac{5}{8} + \frac{1}{10} \cdot \frac{9}{4} \\ &\quad = \frac{5}{8} + \frac{9}{40} \\ &\quad = \frac{25}{40} + \frac{9}{40} \\ &\quad = \frac{34}{40} = \frac{17}{20} \end{aligned}$$

You Try It 9

Evaluate $\frac{5}{8} \div \left(\frac{1}{3} - \frac{3}{4}\right) + \frac{7}{12}$.

Your solution

Solutions on pp. S2–S3

1.5 Exercises

[illegible]

Evaluate.

1. 6^2

2. 7^4

3. -7^2

4. -4^3

5. $(-3)^2$

6. $(-2)^3$

7. $(-3)^4$

8. $(-5)^3$

9. $\left(\frac{1}{2}\right)^2$

10. $\left(-\frac{3}{4}\right)^3$

11. $(0.3)^2$

12. $(1.5)^3$

13. $\left(\frac{2}{3}\right)^2 \cdot 3^3$

14. $\left(-\frac{1}{2}\right)^3 \cdot 8$

15. $(0.3)^3 \cdot 2^3$

16. $(0.5)^2 \cdot 3^3$

17. $(-3) \cdot 2^2$

18. $(-5) \cdot 3^4$

19. $(-2) \cdot (-2)^3$

20. $(-2) \cdot (-2)^2$

21. $2^3 \cdot 3^3 \cdot (-4)$

22. $(-3)^3 \cdot 5^2 \cdot 10$

23. $(-7) \cdot 4^2 \cdot 3^2$

24. $(-2) \cdot 2^3 \cdot (-3)^2$

25. $\left(\frac{2}{3}\right)^2 \cdot \frac{1}{4} \cdot 3^3$

26. $\left(\frac{3}{4}\right)^2 \cdot (-4) \cdot 2^3$

27. $8^2 \cdot (-3)^5 \cdot 5$

[illegible]

28. Explain why an Order of Operations Agreement is necessary.

29. Write out the steps of the Order of Operations Agreement.

Evaluate by using the Order of Operations Agreement.

30. $4 - 8 \div 2$

31. $2^2 \cdot 3 - 3$

32. $2(3 - 4) - (-3)^2$

33. $16 - 32 \div 2^3$

34. $24 - 18 \div 3 + 2$

35. $8 - (-3)^2 - (-2)$

36. $8 - 2(3)^2$

37. $16 - 16 \cdot 2 \div 4$

38. $12 + 16 \div 4 \cdot 2$

39. $16 - 2 \cdot 4^2$

40. $27 - 18 \div (-3^2)$

41. $4 + 12 \div 3 \cdot 2$

42. $16 + 15 \div (-5) - 2$

43. $14 - 2^2 - (4 - 7)$

44. $3 - 2[8 - (3 - 2)]$

45. $-2^2 + 4[16 \div (3 - 5)]$

46. $6 + \dfrac{16 - 4}{2^2 + 2} - 2$

47. $24 \div \dfrac{3^2}{8 - 5} - (-5)$

48. $96 \div 2[12 + (6 - 2)] - 3^2$

49. $4[16 - (7 - 1)] \div 10$

50. $18 \div 2 - 4^2 - (-3)^2$

51. $18 \div (9 - 2^3) + (-3)$

52. $16 - 3(8 - 3)^2 \div 5$

53. $4(-8) \div [2(7 - 3)^2]$

54. $\dfrac{(-19) + (-2)}{6^2 - 30} \div (2 - 4)$

55. $16 - 4 \cdot \dfrac{3^3 - 7}{2^3 + 2} - (-2)^2$

56. $(0.2)^2 \cdot (-0.5) + 1.72$

57. $0.3(1.7 - 4.8) + (1.2)^2$

58. $(1.8)^2 - 2.52 \div 1.8$

59. $(1.65 - 1.05)^2 \div 0.4 + 0.8$

60. $\dfrac{3}{8} \div \left(\dfrac{5}{6} + \dfrac{2}{3}\right)$

61. $\left(\dfrac{5}{12} - \dfrac{9}{16}\right)\dfrac{3}{7}$

62. $\left(\dfrac{3}{4}\right)^2 - \left(\dfrac{1}{2}\right)^3 \div \dfrac{3}{5}$

APPLYING THE CONCEPTS

63. The following was offered as the simplification of $6 + 2(4 - 9)$.

$$\begin{aligned} 6 + 2(4 - 9) &= 6 + 2(-5) \\ &= 8(-5) \\ &= -40 \end{aligned}$$

If this is a correct simplification, write yes for the answer. If it is incorrect, write no and explain the incorrect step.

64. The following was offered as the simplification of $2 \cdot 3^3$.

$$2 \cdot 3^3 = 6^3 = 216$$

If this is a correct simplification, write yes for the answer. If it is incorrect, write no and explain the incorrect step.

Focus on Problem Solving

Inductive Reasoning

Suppose you take 9 credit hours each semester. The total number of credit hours you have taken at the end of each semester can be described by a list of numbers.

$$9, 18, 27, 36, 45, 54, 63, \ldots$$

The list of numbers that indicates the total credit hours is an ordered list of numbers called a **sequence.** Each number in a sequence is called a **term** of the sequence. The list is ordered because the position of a number in the list indicates the semester in which that number of credit hours has been taken. For example, the 7th term of the sequence is 63, and a total of 63 credit hours have been taken after the 7th semester.

Assuming the pattern is continued, find the next three numbers in the pattern

$$6, -10, -14, -18, \ldots$$

This list of numbers is a sequence. The first step in solving this problem is to observe the pattern in the list of numbers. In this case, each number in the list is 4 less than the previous number. The next three numbers are -22, -26, -30.

This process of discovering the pattern in a list of numbers is inductive reasoning. **Inductive reasoning** involves making generalizations from specific examples; in other words, we reach a conclusion by making observations about particular facts or cases.

Try the following exercises. Each exercise requires inductive reasoning.

Name the next two terms in the sequence.

1. 1, 3, 5, 7, 1, 3, 5, 7, 1, …

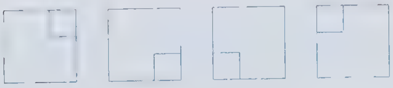
2. 1, 4, 2, 5, 3, 6, 4, …

3. 1, 2, 4, 7, 11, 16, …

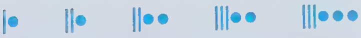
4. A, B, C, G, H, I, M, …

Draw the next shape in the sequence.

5.



6.



Solve.

7. Convert $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}$, and $\frac{5}{11}$ to decimals. Then use the pattern you observe to convert $\frac{6}{11}, \frac{7}{11}$, and $\frac{9}{11}$ to decimals.

8. Convert $\frac{1}{33}, \frac{2}{33}, \frac{4}{33}, \frac{5}{33}$, and $\frac{7}{33}$ to decimals. Then use the pattern you observe to convert $\frac{8}{33}, \frac{13}{33}$, and $\frac{19}{33}$ to decimals.

Projects and Group Activities

The $\boxed{+/-}$ Key on a Calculator

Using your calculator to simplify numerical expressions sometimes requires use of the $\boxed{+/-}$ key or, on some calculators, the negative key, which is frequently shown as $\boxed{(-)}$. These keys change the sign of the number currently in the display. To enter -4:

- For those calculators with $\boxed{+/-}$, press 4 and then $\boxed{+/-}$.
- For those calculators with $\boxed{(-)}$, press $\boxed{(-)}$ and then 4.

Here are the keystrokes for evaluating the expression $3(-4) - (-5)$.

Calculators with $\boxed{+/-}$ key: $3 \boxed{\times} 4 \boxed{+/-} - 5 \boxed{+/-} \boxed{=}$

Calculators with $\boxed{(-)}$ key: $3 \boxed{\times} \boxed{(-)} 4 \boxed{-} \boxed{(-)} 5 \boxed{=}$

This example illustrates that calculators make a distinction between negative and minus. To perform the operation $3 - (-3)$, you cannot enter $3 \boxed{-} \boxed{-} 3$. This would result in 0, which is not the correct answer. You must enter

$$3 \boxed{-} 3 \boxed{+/-} \boxed{=} \quad \text{or} \quad 3 \boxed{-} \boxed{(-)} 3 \boxed{=}$$

Use a calculator to evaluate each of the following exercises.

1. $-16 \div 2$
2. $3(-8)$
3. $47 - (-9)$
4. $-50 - (-14)$
5. $4 - (-3)^2$
6. $-8 + (-6)^2 - 7$

Moving Averages

Objective 1.3C on page 20 describes how to find the moving average of a stock. Use this method to calculate the 5-day moving average for at least three different stocks. Discuss and compare the results for the different stocks.

Stock	Div.	Sales	Last	Chg.
AAR	.34	41	15.30	−.20
ABB Ltd n		7	13.62	−.42
ABM	.66	46	37.05	+.90
ABN Amro	79e	121	17.42	−.51
ABN pfA	1.88	34	25.25	−.08
ABN pfB	1.78	75	24.50	.07
ACE Ltd	.52	362	35.97	+.13
ACMIn	.84	516	8.32	−.07
ACM Op	.72	8	8.28	
ACMMD	1.02	35	7.51	-.04
ACM MI	.51	35	4.60	+.01
ACMMu	.87	6	12.92	−.06
▼ AES Cp	AES Cp	3445	37.99	−.43
AFLAC s	.20	992	31.11	−.19
AGCO	.01j	199	9.60	−.30
AGLCap pf	2.00	14	25.30	+.04
AGL Res	1.08	131	23.32	+.05
AK Steel	.25	169	11.90	−.13
AMB Pr	1.58	114	25.52	+.02
AMLI Rs	1.88	38	23.71	−.11
AMR		1055	37.26	−.68
AMR Cp 39	1.97	7	24.85	+.07

For this project, you will need to use stock tables, which are printed in the business section of major newspapers. Your college library should have copies of these publications. In a stock table, the column headed "Chg." provides the change in the price of a share of the stock; that is, it gives the difference between the closing price for the day shown and the closing price for the previous day. The symbol + indicates that the change was an increase in price; the symbol − indicates that the change was a decrease in price.

www.fedstats.gov Information regarding the history of the federal budget can be found on the World Wide Web. Go to the web site **www.fedstats.gov.** Click on "Fast Facts" and then on "Frequently Requested Tables." In the list of tables printed to the screen, find the table entitled "Federal Budget - Summary." Click on it. When the federal budget table appears on the screen, look for the column that lists each year's surplus or deficit. You will see that a negative sign (−) is used to show a deficit. Note that it states near the top of the screen that the figures in the table are in millions of dollars.

1. During which years shown in the table was there a surplus?
2. During which year was the deficit the greatest?
3. Find the difference between the surplus or deficit this year and the surplus or deficit 5 years ago.
4. What is the difference between the surplus or deficit this year and the surplus or deficit a decade ago?
5. Determine what two numbers in the table are being subtracted in each row in order to arrive at the number in the surplus or deficit column.
6. Describe the trend of the federal deficit over the last 10 years.

Chapter Summary

Key Words

A *set* is a collection of objects. The objects in a set are called the *elements* of the set. [p. 3]

The *roster method* of writing sets encloses a list of the elements in braces. [p. 3]

The set of *natural numbers* is $\{1, 2, 3, 4, 5, 6, 7, \ldots\}$. [p. 3]

The set of *integers* is $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$. [p. 3]

A number *a is less than* another number b, written $a < b$, if a is to the left of b on the number line. [p. 4]

A number *a is greater than* another number b, written $a > b$, if a is to the right of b on the number line. [p. 4]

The symbol $\le$ means *is less than or equal to*. The symbol $\ge$ means *is greater than or equal to*. [p. 4]

Two numbers that are the same distance from zero on the number line but on opposite sides of zero are *opposite numbers*, or *opposites*. The opposite of a number is also called its *additive inverse*. [p. 5]

The *absolute value* of a number is its distance from zero on the number line. [pp. 5–6]

A *rational number* is a number of the form $\frac{a}{b}$, where a and b are integers and b is not equal to zero. A rational number written in this form is commonly called a fraction. [p. 24]

A fraction is in *simplest form* when there are no common factors in the numerator and the denominator. [p. 24]

The *reciprocal* of a fraction is the fraction with the numerator and denominator interchanged. [p. 28]

An *irrational number* is a number that has a decimal representation that never terminates or repeats. [p. 24]

The rational numbers and the irrational numbers taken together are called the *real numbers*. [p. 24]

Percent means "parts of 100." Thus 63% means 63 parts of 100 parts. [p. 25]

An expression of the form a^n is in *exponential form*, where a is the *base* and n is the *exponent*. [p. 37]

Essential Rules

To add two numbers with the same sign, add the absolute values of the numbers. Then attach the sign of the addends. [p. 9]

To add two numbers with different signs, find the absolute value of each number. Subtract the smaller of the two numbers from the larger. Then attach the sign of the number with the larger absolute value. [p. 9]

To subtract two integers, add the opposite of the second integer to the first integer. [p. 10]

To multiply two numbers with the same sign, multiply the absolute values of the factors. The product is positive. [p. 17]

To multiply two numbers with different signs, multiply the absolute values of the factors. The product is negative. [p. 17]

To divide two numbers with the same sign, divide the absolute values of the numbers. The quotient is positive. [p. 18]

To divide two numbers with different signs, divide the absolute values of the numbers. The quotient is negative. [p. 18]

Properties of Zero and One in Division [p. 19]

If $a \neq 0$, $0 \div a = 0$.
If $a \neq 0$, $a \div a = 1$.
$a \div 1 = a$.
$a \div 0$ is undefined.

To convert a percent to a decimal, remove the percent sign and multiply by 0.01. [p. 25]

To convert a percent to a fraction, remove the percent sign and multiply by $\frac{1}{100}$. [p. 25]

To convert a fraction to a percent, multiply by 100%. [p. 25]

To convert a decimal to a percent, multiply by 100%. [p. 25]

The Order of Operations Agreement [p. 38]

Step 1 Perform all operations inside grouping symbols. Grouping symbols include parentheses, brackets, braces, and the fraction bar.

Step 2 Simplify any numerical expressions that contain exponents.

Step 3 Do multiplication and division as they occur from left to right.

Step 4 Do addition and subtraction as they occur from left to right.

Chapter Review

1. Add: $-13 + 7$

2. Write $\frac{7}{25}$ as a decimal.

3. Evaluate -5^2.

4. Evaluate $5 - 2^2 + 9$.

5. Place the correct symbol, $<$ or $>$, between the two numbers.
$-4 \quad 2$

6. Write 6.2% as a decimal.

7. Multiply: $(-6)(7)$

8. Simplify: $\frac{1}{3} - \frac{1}{6} + \frac{5}{12}$

9. Evaluate $15 \cdot (6 - 4)^2$.

10. Given $A = \{-4, 0, 11\}$, which elements of set A are less than -1?

11. Subtract: $5.17 - 6.238$

12. Write $\frac{5}{8}$ as a percent.

13. Write $\frac{2}{15}$ as a decimal. Place a bar over the repeating digits of the decimal.

14. Subtract: $9 - 13$

15. Find the additive inverse of -4.

16. Divide: $-32 \div (-4)$

17. Divide: $-100 \div 5$

18. Write $79\frac{1}{2}\%$ as a fraction.

19. Evaluate $-3^2 + 4[18 + (12 - 20)]$.

20. Add: $-3 + (-12) + 6 + (-4)$

21. Write $\frac{19}{35}$ as a percent. Write the remainder in fractional form.

22. Evaluate $\left(\frac{2}{3}\right)^4$.

23. Multiply: $4.32(-1.07)$

24. Evaluate $-|-5|$.

25. Subtract: $16 - (-3) - 18$

26. Divide: $-\frac{18}{35} \div \frac{27}{28}$

27. Given $C = \{-7, 0, 9\}$, find the absolute value of each element of set C.

28. Multiply: $-9(-9)$

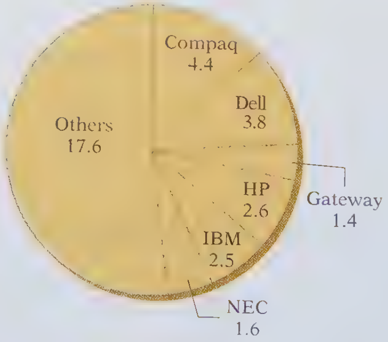
29. Place the correct symbol, $>$ or $<$, between the two numbers.
$-|6| \quad |-10|$

30. Evaluate $\frac{5^2 + 11}{2^2 + 5} \div (2^3 - 2^2)$.

31. To discourage random guessing on a multiple-choice exam, a professor assigns 6 points for a correct answer, -4 points for an incorrect answer, and -2 points for leaving a question blank. What is the score for a student who had 21 correct answers, 5 incorrect answers, and left 4 questions blank?

32. The graph at the right shows the number (in millions) of personal computers shipped worldwide in the third quarter of 2000. What percent of the total number of personal computers shipped did IBM ship? Round to the nearest tenth of a percent.

Worldwide Third-Quarter PC Shipments in 2000 (in millions)



Source: *Dataquest*, October 2000

33. The temperature at which mercury boils is 357°C. The temperature at which mercury freezes is -39°C. Find the difference between the boiling point and the freezing point of mercury.

Chapter Test

1. Divide: $-561 \div (-33)$

2. Write $\frac{5}{6}$ as a percent. Write the remainder in fractional form.

3. Evaluate $\frac{3}{4} \cdot (4)^2$.

4. Multiply: $6.02(-0.89)$

5. Subtract: $16 - 30$

6. Write $37\frac{1}{2}\%$ as a fraction.

7. Add: $-\frac{2}{5} + \frac{7}{15}$

8. Evaluate $\frac{-10 + 2}{2 + (-4)} \div 2 + 6$.

9. Multiply: $-5(-6)(3)$

10. Find the opposite of -4.

11. Evaluate $(-3^3) \cdot 2^2$.

12. Given $B = \{-5, -3, 0, 4\}$, which elements of set B are greater than 2?

13. Place the correct symbol, $<$ or $>$, between the two numbers.
$-2 \quad -40$

14. Evaluate $-|-4|$.

15. Write 45% as a fraction and as a decimal.

16. Add: $-22 + 14 + (-8)$

17. Multiply: $-4 \cdot 12$

18. Evaluate $16 \div 2[8 - 3(4 - 2)] + 1$.

19. Subtract: $16 - (-30) - 42$

20. Divide: $\frac{5}{12} \div \left(-\frac{5}{6}\right)$

21. Write 1.025 as a percent.

22. Evaluate $3^2 - 4 + 20 \div 5$.

23. Write $\frac{7}{9}$ as a decimal. Place a bar over the repeating digit of the decimal.

24. The table below shows the first-quarter profits and losses for 2000 for five companies in the health care industry. Profits are shown as positive numbers; losses are shown as negative numbers. One-quarter of a year is 3 months.
   a. If earnings were to continue throughout the year at the same level, what would be the annual earnings for Genentech, Inc.?
   b. For the quarter shown, what was the average monthly earnings for Isis Pharmaceuticals, Inc.?

Pharmaceutical Company	*First Quarter 2000 Profits ($ millions)*
Boston Scientific Corporation	106.0
Celltech Group, plc	−305.9
Genentech, Inc.	−25.9
Incyte Genomics, Inc.	−8.2
Isis Pharmaceuticals, Inc.	−18.3

Source: Income statements from individual companies

25. At the end of the trading day on February 12, the price of one share of IBM stock was $114.90. The change in the price of a share of IBM stock on February 13 was −$1.15. What was the value of 100 shares of IBM stock on February 13?

Chapter 2 Variable Expressions

Suppose you have received a job opportunity in another city and you have been offered a salary that is almost twice what you are earning now. Is this too good to be true? Perhaps so, if the cost of living in that city is more than twice the cost of living in the city where you live now. One way to determine how expensive it is to live in a city is to check out the prices of homes in that area. **Exercise 67 on page 74** shows how variable expressions can be used to compare and calculate such data.

Objectives

Section 2.1

A To evaluate a variable expression

Section 2.2

A To simplify a variable expression using the Properties of Addition

B To simplify a variable expression using the Properties of Multiplication

C To simplify a variable expression using the Distributive Property

D To simplify general variable expressions

Section 2.3

A To translate a verbal expression into a variable expression, given the variable

B To translate a verbal expression into a variable expression and then simplify

C To translate application problems

Need help? For online student resources, such as section quizzes, visit this textbook's website at **math.college.hmco.com/students.**

Prep Test

1. Subtract: $-12 - (-15)$

2. Divide: $-36 \div (-9)$

3. Add: $-\frac{3}{4} + \frac{5}{6}$

4. What is the reciprocal of $-\frac{9}{4}$?

5. Divide: $-\frac{3}{4} \div \left(-\frac{5}{2}\right)$

6. Evaluate: -2^4

7. Evaluate: $\left(\frac{2}{3}\right)^3$

8. Evaluate: $3 \cdot 4^2$

9. Evaluate: $7 - 2 \cdot 3$

10. Evaluate: $5 - 7(3 - 2^2)$

Go Figure

Two fractions are inserted between $\frac{1}{4}$ and $\frac{1}{2}$ so that the difference between any two successive fractions is the same. Find the sum of the four fractions.

2.1 Evaluating Variable Expressions

Objective A

To evaluate a variable expression 2

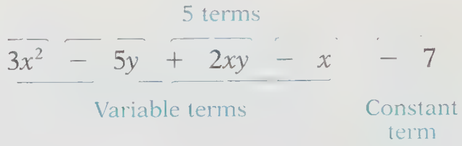
Often we discuss a quantity without knowing its exact value—for example, the price of gold next month, the cost of a new automobile next year, or the tuition cost for next semester. Recall that a letter of the alphabet is used to stand for a quantity that is unknown or that can change, or *vary*. The letter is called a variable. An expression that contains one or more variables is called a **variable expression.**

A variable expression is shown at the right. The expression can be rewritten by writing subtraction as the addition of the opposite.

$$3x^2 - 5y + 2xy - x - 7$$

$$3x^2 + (-5y) + 2xy + (-x) + (-7)$$

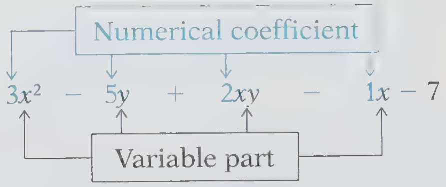
Note that the expression has 5 addends. The **terms** of a variable expression are the addends of the expression. The expression has 5 terms.



The terms $3x^2$, $-5y$, $2xy$, and $-x$ are **variable terms.**

The term -7 is a **constant term,** or simply a **constant.**

Each variable term is composed of a **numerical coefficient** and a **variable part** (the variable or variables and their exponents).



When the numerical coefficient is 1 or -1, the 1 is usually not written ($x = 1x$ and $-x = -1x$).

Point of Interest

Historical manuscripts indicate that mathematics is at least 4000 years old. Yet it was only 400 years ago that mathematicians started using variables to stand for numbers. The idea that a letter can stand for some number was a critical turning point in mathematics. Today, *x* is used by most nations as the standard letter for a single unknown. In fact, *x*-rays were so named because the scientists who discovered them did not know what they were and thus labeled them the "unknown rays" or *x*-rays.

Variable expressions occur naturally in science. In a physics lab, a student may discover that a weight of 1 pound will stretch a spring $\frac{1}{2}$ inch. Two pounds will stretch the spring 1 inch. By experimenting, the student can discover that the distance the spring will stretch is found by multiplying the weight by $\frac{1}{2}$. By letting W represent the weight attached to the spring, the student can represent the distance the spring stretches by the variable expression $\frac{1}{2}W$.

With a weight of W pounds, the spring will stretch $\frac{1}{2} \cdot W = \frac{1}{2}W$ inches.

With a weight of 10 pounds, the spring will stretch $\frac{1}{2} \cdot 10 = 5$ inches. The number 10 is called the **value** of the variable W.

With a weight of 3 pounds, the spring will stretch $\frac{1}{2} \cdot 3 = 1\frac{1}{2}$ inches.

Replacing each variable by its value and then simplifying the resulting numerical expression is called **evaluating the variable expression.**

See the appendix "Guidelines for Using Graphing Calculators" for instructions on using a graphing calculator to evaluate variable expressions.

Evaluate $ab - b^2$ when $a = 2$ and $b = -3$.

Replace each variable in the expression by its value. Then use the Order of Operations Agreement to simplify the resulting numerical expression.

$$ab - b^2$$

$$\begin{aligned} 2(-3) - (-3)^2 &= -6 - 9 \\ &= -15 \end{aligned}$$

When $a = 2$ and $b = -3$, the value of $ab - b^2$ is -15.

Example 1 Name the variable terms of the expression $2a^2 - 5a + 7$.

Solution $2a^2$ and $-5a$

You Try It 1 Name the constant term of the expression $6n^2 + 3n - 4$.

Your solution

Example 2 Evaluate $x^2 - 3xy$ when $x = 3$ and $y = -4$.

Solution

$$x^2 - 3xy$$

$$\begin{aligned} 3^2 - 3(3)(-4) &= 9 - 3(3)(-4) \\ &= 9 - 9(-4) \\ &= 9 - (-36) \\ &= 9 + 36 = 45 \end{aligned}$$

You Try It 2 Evaluate $2xy + y^2$ when $x = -4$ and $y = 2$.

Your solution

Example 3

Evaluate $\frac{a^2 - b^2}{a - b}$ when $a = 3$ and $b = -4$.

Solution

$$\frac{a^2 - b^2}{a - b}$$

$$\begin{aligned} \frac{3^2 - (-4)^2}{3 - (-4)} &= \frac{9 - 16}{3 - (-4)} \\ &= \frac{-7}{7} = -1 \end{aligned}$$

You Try It 3

Evaluate $\frac{a^2 + b^2}{a + b}$ when $a = 5$ and $b = -3$.

Your solution

Example 4 Evaluate $x^2 - 3(x - y) - z^2$ when $x = 2$, $y = -1$, and $z = 3$.

Solution

$$x^2 - 3(x - y) - z^2$$

$$\begin{aligned} &2^2 - 3[2 - (-1)] - 3^2 \\ &= 2^2 - 3(3) - 3^2 \\ &= 4 - 3(3) - 9 \\ &= 4 - 9 - 9 \\ &= -5 - 9 \\ &= -14 \end{aligned}$$

You Try It 4 Evaluate $x^3 - 2(x + y) + z^2$ when $x = 2$, $y = -4$, and $z = -3$.

Your solution

Solutions on p. S3

2.1 Exercises

[illegible]

Name the terms of the variable expression. Then underline the constant term.

1. $2x^2 + 5x - 8$

2. $-3n^2 - 4n + 7$

3. $6 - a^4$

Name the variable terms of the expression. Then underline the variable part of each term.

4. $9b^2 - 4ab + a^2$

5. $7x^2y + 6xy^2 + 10$

6. $5 - 8n - 3n^2$

Name the coefficients of the variable terms.

7. $x^2 - 9x + 2$

8. $12a^2 - 8ab - b^2$

9. $n^3 - 4n^2 - n + 9$

10. What is the numerical coefficient of a variable term?

11. Explain the meaning of the phrase "evaluate a variable expression."

Evaluate the variable expression when $a = 2$, $b = 3$, and $c = -4$.

12. $3a + 2b$

13. $a - 2c$

14. $-a^2$

15. $2c^2$

16. $-3a + 4b$

17. $3b - 3c$

18. $b^2 - 3$

19. $-3c + 4$

20. $16 \div (2c)$

21. $6b \div (-a)$

22. $bc \div (2a)$

23. $b^2 - 4ac$

24. $a^2 - b^2$

25. $b^2 - c^2$

26. $(a + b)^2$

27. $a^2 + b^2$

28. $2a - (c + a)^2$

29. $(b - a)^2 + 4c$

30. $b^2 - \dfrac{ac}{8}$

31. $\dfrac{5ab}{6} - 3cb$

32. $(b - 2a)^2 + bc$

Evaluate the variable expression when $a = -2$, $b = 4$, $c = -1$, and $d = 3$.

33. $\dfrac{b + c}{d}$

34. $\dfrac{d - b}{c}$

35. $\dfrac{2d + b}{-a}$

36. $\dfrac{b + 2d}{b}$

37. $\dfrac{b - d}{c - a}$

38. $\dfrac{2c - d}{-ad}$

39. $(b + d)^2 - 4a$

40. $(d - a)^2 - 3c$

41. $(d - a)^2 \div 5$

42. $3(b - a) - bc$

43. $\dfrac{b - 2a}{bc^2 - d}$

44. $\dfrac{b^2 - a}{ad + 3c}$

45. $\dfrac{1}{3}d^2 - \dfrac{3}{8}b^2$

46. $\dfrac{5}{8}a^4 - c^2$

47. $\dfrac{-4bc}{2a - b}$

48. $-\dfrac{3}{4}b + \dfrac{1}{2}(ac + bd)$

49. $-\dfrac{2}{3}d - \dfrac{1}{5}(bd - ac)$

50. $(b - a)^2 - (d - c)^2$

51. The value of z is the value of $a^2 - 2a$ when $a = -3$. Find the value of z^2.

52. The value of a is the value of $3x^2 - 4x - 5$ when $x = -2$. Find the value of $3a - 4$.

53. The value of c is the value of $a^2 + b^2$ when $a = 2$ and $b = -2$. Find the value of $c^2 - 4$.

APPLYING THE CONCEPTS

Evaluate the following expressions for $x = 2$, $y = 3$, and $z = -2$.

54. $3^x - x^3$

55. z^x

56. $x^x - y^y$

57. $y^{(x^2)}$

58. For each of the following, determine the first natural number x, greater than 1, for which the second expression is larger than the first.

a. $x^3, 3^x$ **b.** $x^4, 4^x$ **c.** $x^5, 5^x$ **d.** $x^6, 6^x$

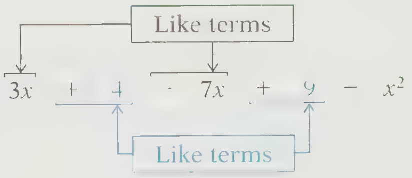
59. On the basis of your answer to Exercise 58, make a conjecture that appears to be true about the expressions x^n and n^x, where $n = 3, 4, 5, 6, 7, \ldots$ and x is a natural number greater than 1.

2.2 Simplifying Variable Expressions

[illegible]

To simplify a variable expression using the Properties of Addition

Like terms of a variable expression are terms with the same variable part. (Because $x^2 = x \cdot x$, x^2 and x are not like terms.)



Constant terms are like terms. 4 and 9 are like terms.

To simplify a variable expression, use the Distributive Property to combine like terms by adding the numerical coefficients. The variable part remains unchanged.

TAKE NOTE

Here is an example of the Distributive Property with just numbers.

$$2(5 + 9) = 2(5) + 2(9)$$
$$= 10 + 18 = 28$$

This is the same result we would obtain using the Order of Operations Agreement.

$$2(5 + 9) = 2(14) = 28$$

The usefulness of the Distributive Property will become more apparent as we explore variable expressions.

Distributive Property

If a, b, and c are real numbers, then $a(b + c) = ab + ac$.

The Distributive Property can also be written $ba + ca = (b + c)a$. This form is used to simplify a variable expression.

To simplify $2x + 3x$, use the Distributive Property to add the numerical coefficients of the like variable terms. This is called **combining like terms.**

$$2x + 3x = (2 + 3)x$$
$$= 5x$$

Simplify: $5y - 11y$

$$5y - 11y = (5 - 11)y$$
$$= -6y$$

- **Use the Distributive Property.**

Simplify: $5 + 7p$

The terms 5 and $7p$ are not like terms.

The expression $5 + 7p$ is in simplest form.

TAKE NOTE

Simplifying an expression means combining like terms. A constant term (5) and a variable term ($7p$) are not like terms and therefore cannot be combined.

The Associative Property of Addition

If a, b, and c are real numbers, then $(a + b) + c = a + (b + c)$.

When three or more terms are added, the terms can be grouped (with parentheses, for example) in any order. The sum is the same. For example,

$$(5 + 7) + 15 = 5 + (7 + 15)$$
$$12 + 15 = 5 + 22$$
$$27 = 27$$

$$(3x + 5x) + 9x = 3x + (5x + 9x)$$
$$8x + 9x = 3x + 14x$$
$$17x = 17x$$

The Commutative Property of Addition

If a and b are real numbers, then $a + b = b + a$.

When two like terms are added, the terms can be added in either order. The sum is the same. For example,

$$15 + (-28) = (-28) + 15 \qquad 2x + (-4x) = -4x + 2x$$
$$-13 = -13 \qquad -2x = -2x$$

The Addition Property of Zero

If a is a real number, then $a + 0 = 0 + a = a$.

The sum of a term and zero is the term. For example,

$$-9 + 0 = 0 + (-9) = -9 \qquad 5x + 0 = 0 + 5x = 5x$$

The Inverse Property of Addition

If a is a real number, then $a + (-a) = (-a) + a = 0$.

The sum of a term and its opposite is zero. The opposite of a number is called its **additive inverse.**

$$12 + (-12) = (-12) + 12 = 0 \qquad 7x + (-7x) = -7x + 7x = 0$$

⇨ Simplify: $8x + 4y - 8x + y$

$$8x + 4y - 8x + y$$
$$= (8x - 8x) + (4y + y)$$
$$= 0 + 5y = 5y$$

- Use the Commutative and Associative Properties of Addition to rearrange and group like terms.
- Combine like terms.

⇨ Simplify: $4x^2 + 5x - 6x^2 - 2x + 1$

$$4x^2 + 5x - 6x^2 - 2x + 1$$
$$= (4x^2 - 6x^2) + (5x - 2x) + 1$$
$$= -2x^2 + 3x + 1$$

- Use the Commutative and Associative Properties of Addition to rearrange and group like terms.
- Combine like terms.

Example 1 Simplify: $3x + 4y - 10x + 7y$

Solution $3x + 4y - 10x + 7y = -7x + 11y$

You Try It 1 Simplify: $3a - 2b - 5a + 6b$

Your solution

Example 2 Simplify: $x^2 - 7 + 4x^2 - 16$

Solution $x^2 - 7 + 4x^2 - 16 = 5x^2 - 23$

You Try It 2 Simplify: $-3y^2 + 7 + 8y^2 - 14$

Your solution

Solutions on p. S3

[illegible]

To simplify a variable expression using the Properties of Multiplication

In simplifying variable expressions, the following Properties of Multiplication are used.

The Associative Property of Multiplication

If a, b, and c are real numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

TAKE NOTE
The Associative Property of Multiplication allows us to multiply a coefficient by a number. Without this property, the expression $2(3x)$ could not be changed.

When three or more factors are multiplied, the factors can be grouped in any order. The product is the same.

$$\begin{aligned} 3(5 \cdot 6) &= (3 \cdot 5)6 \\ 3(30) &= (15)6 \\ 90 &= 90 \end{aligned} \qquad \begin{aligned} 2(3x) &= (2 \cdot 3)x \\ 6x &= 6x \end{aligned}$$

The Commutative Property of Multiplication

If a and b are real numbers, then $a \cdot b = b \cdot a$.

TAKE NOTE
The Commutative Property of Multiplication allows us to rearrange factors. This property, along with the Associative Property of Multiplication, allows us to simplify some variable expressions.

Two factors can be multiplied in either order. The product is the same.

$$\begin{aligned} 5(-7) &= -7(5) \\ -35 &= -35 \end{aligned} \qquad \begin{aligned} (5x) \cdot 3 &= 3 \cdot (5x) \\ &= (3 \cdot 5)x \\ &= 15x \end{aligned}$$

- Commutative Property of Multiplication
- Associative Property of Multiplication

The Multiplication Property of One

If a is a real number, then $a \cdot 1 = 1 \cdot a = a$.

The product of a term and one is the term.

$$9 \cdot 1 = 1 \cdot 9 = 9 \qquad (8x) \cdot 1 = 1 \cdot (8x) = 8x$$

The Inverse Property of Multiplication

If a is a real number, and a is not equal to zero, then

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$$

TAKE NOTE
We must state that $x \neq 0$ because division by zero is undefined.

$\frac{1}{a}$ is called the **reciprocal** of a. $\frac{1}{a}$ is also called the **multiplicative inverse** of a. The product of a number and its reciprocal is one.

$$7 \cdot \frac{1}{7} = \frac{1}{7} \cdot 7 = 1 \qquad x \cdot \frac{1}{x} = \frac{1}{x} \cdot x = 1, \quad x \neq 0$$

The multiplication properties just discussed are used to simplify variable expressions.

➡ Simplify: $2(-x)$

$$\begin{aligned} 2(-x) &= 2(-1 \cdot x) \\ &= [2(-1)]x \\ &= -2x \end{aligned}$$

- Use the Associative Property of Multiplication to group factors.

Simplify: $\frac{3}{2}\left(\frac{2x}{3}\right)$

$$\frac{3}{2}\left(\frac{2x}{3}\right) = \frac{3}{2}\left(\frac{2}{3}x\right)$$

- Note that $\frac{2x}{3} = \frac{2}{3}x$.

$$= \left(\frac{3}{2} \cdot \frac{2}{3}\right)x$$

- Use the Associative Property of Multiplication to group factors.

$$= 1 \cdot x$$

$$= x$$

Simplify: $(16x)2$

$$(16x)2 = 2(16x)$$
$$= (2 \cdot 16)x$$
$$= 32x$$

- Use the Commutative and Associative Properties of Multiplication to rearrange and group factors.

Example 3 Simplify: $-2(3x^2)$

Solution $-2(3x^2) = -6x^2$

You Try It 3 Simplify: $-5(4y^2)$

Your solution

Example 4 Simplify: $-5(-10x)$

Solution $-5(-10x) = 50x$

You Try It 4 Simplify: $-7(-2a)$

Your solution

Example 5 Simplify: $(6x)(-4)$

Solution $(6x)(-4) = -24x$

You Try It 5 Simplify: $(-5x)(-2)$

Your solution

Solutions on p. S3

Objective C

To simplify a variable expression using the Distributive Property

Recall that the Distributive Property states that if a, b, and c are real numbers, then

$$a(b + c) = ab + ac$$

The Distributive Property is used to remove parentheses from a variable expression.

Simplify: $3(2x + 7)$

$$3(2x + 7) = 3(2x) + 3(7)$$
$$= 6x + 21$$

- Use the Distributive Property. Multiply each term inside the parentheses by 3.

Simplify: $-5(4x + 6)$

$$\begin{aligned} -5(4x + 6) &= -5(4x) + (-5)(6) \\ &= -20x - 30 \end{aligned}$$

• Use the Distributive Property.

Simplify: $-(2x - 4)$

$$\begin{aligned} -(2x - 4) &= -1(2x - 4) \\ &= -1(2x) - (-1)(4) \\ &= -2x + 4 \end{aligned}$$

• Use the Distributive Property.

Note: When a negative sign immediately precedes the parentheses, the sign of each term inside the parentheses is changed.

Simplify: $-\frac{1}{2}(8x - 12y)$

$$\begin{aligned} -\frac{1}{2}(8x - 12y) &= -\frac{1}{2}(8x) - \left(-\frac{1}{2}\right)(12y) \\ &= -4x + 6y \end{aligned}$$

• Use the Distributive Property.

An extension of the Distributive Property is used when an expression contains more than two terms.

Simplify: $3(4x - 2y - z)$

$$\begin{aligned} 3(4x - 2y - z) &= 3(4x) - 3(2y) - 3(z) \\ &= 12x - 6y - 3z \end{aligned}$$

• Use the Distributive Property.

Example 6

Simplify: $7(4 + 2x)$

Solution

$7(4 + 2x) = 28 + 14x$

You Try It 6

Simplify: $5(3 + 7b)$

Your solution

Example 7

Simplify: $(2x - 6)2$

Solution

$(2x - 6)2 = 4x - 12$

You Try It 7

Simplify: $(3a - 1)5$

Your solution

Example 8

Simplify: $-3(-5a + 7b)$

Solution

$-3(-5a + 7b) = 15a - 21b$

You Try It 8

Simplify: $-8(-2a + 7b)$

Your solution

Solutions on p. S3

Example 9 Simplify: $3(x^2 - x - 5)$

Solution $3(x^2 - x - 5) = 3x^2 - 3x - 15$

You Try It 9 Simplify: $3(12x^2 - x + 8)$

Your solution

Example 10 Simplify: $-2(x^2 + 5x - 4)$

Solution
$$\begin{aligned} &-2(x^2 + 5x - 4)\\ &= -2x^2 - 10x + 8 \end{aligned}$$

You Try It 10 Simplify: $3(-a^2 - 6a + 7)$

Your solution

Solutions on p. S3

[illegible] To simplify general variable expressions

When simplifying variable expressions, use the Distributive Property to remove parentheses and brackets used as grouping symbols.

➡ Simplify: $4(x - y) - 2(-3x + 6y)$

$$\begin{aligned} &4(x - y) - 2(-3x + 6y)\\ &= 4x - 4y + 6x - 12y &&\text{• Use the Distributive Property.}\\ &= 10x - 16y &&\text{• Combine like terms.} \end{aligned}$$

Example 11 Simplify: $2x - 3(2x - 7y)$

Solution
$$\begin{aligned} 2x - 3(2x - 7y) &= 2x - 6x + 21y\\ &= -4x + 21y \end{aligned}$$

You Try It 11 Simplify: $3y - 2(y - 7x)$

Your solution

Example 12 Simplify:
$7(x - 2y) - (-x - 2y)$

Solution
$$\begin{aligned} &7(x - 2y) - (-x - 2y)\\ &= 7x - 14y + x + 2y\\ &= 8x - 12y \end{aligned}$$

You Try It 12 Simplify:
$-2(x - 2y) - (-x + 3y)$

Your solution

Example 13 Simplify:
$2x - 3[2x - 3(x + 7)]$

Solution
$$\begin{aligned} &2x - 3[2x - 3(x + 7)]\\ &= 2x - 3[2x - 3x - 21]\\ &= 2x - 3[-x - 21]\\ &= 2x + 3x + 63\\ &= 5x + 63 \end{aligned}$$

You Try It 13 Simplify:
$3y - 2[x - 4(2 - 3y)]$

Your solution

Solutions on p. S3

2.2 Exercises

Objective A

1. What are *like terms*? Give an example of two like terms. Give an example of two terms that are not like terms.

2. Explain the meaning of the phrase "simplify the variable expression."

Simplify.

3. $6x + 8x$

4. $12x + 13x$

5. $9a - 4a$

6. $12a - 3a$

7. $4y + (-10y)$

8. $8y + (-6y)$

9. $7 - 3b$

10. $5 + 2a$

11. $-12a + 17a$

12. $-3a + 12a$

13. $5ab - 7ab$

14. $9ab - 3ab$

15. $-12xy + 17xy$

16. $-15xy + 3xy$

17. $-3ab + 3ab$

18. $-7ab + 7ab$

19. $-\frac{1}{2}x - \frac{1}{3}x$

20. $-\frac{2}{5}y + \frac{3}{10}y$

21. $2.3x + 4.2x$

22. $6.1y - 9.2y$

23. $x - 0.55x$

24. $0.65A - A$

25. $5a - 3a + 5a$

26. $10a - 17a + 3a$

27. $-5x^2 - 12x^2 + 3x^2$

28. $-y^2 - 8y^2 + 7y^2$

29. $7x + (-8x) + 3y$

30. $8y + (-10x) + 8x$

31. $7x - 3y + 10x$

32. $8y + 8x - 8y$

33. $3a + (-7b) - 5a + b$

34. $-5b + 7a - 7b + 12a$

35. $3x + (-8y) - 10x + 4x$

36. $3y + (-12x) - 7y + 2y$

37. $x^2 - 7x + (-5x^2) + 5x$

38. $3x^2 + 5x - 10x^2 - 10x$

[illegible]

Simplify.

39. $4(3x)$

40. $12(5x)$

41. $-3(7a)$

42. $-2(5a)$

43. $-2(-3y)$

44. $-5y(-6y)$

45. $(4x)2$

46. $(6x)12$

47. $(3a)(-2)$

48. $(7a)(-4)$

49. $(-3b)(-4)$

50. $(-12b)(-9)$

51. $-5(3x^2)$

52. $-8(7x^2)$

53. $\frac{1}{3}(3x^2)$

54. $\frac{1}{6}(6x^2)$

55. $\frac{1}{5}(5a)$

56. $\frac{1}{8}(8x)$

57. $-\frac{1}{2}(-2x)$

58. $-\frac{1}{4}(-4a)$

59. $-\frac{1}{7}(-7n)$

60. $-\frac{1}{9}(-9b)$

61. $(3x)\left(\frac{1}{3}\right)$

62. $(12x)\left(\frac{1}{12}\right)$

63. $(-6y)\left(-\frac{1}{6}\right)$

64. $(-10n)\left(-\frac{1}{10}\right)$

65. $\frac{1}{3}(9x)$

66. $\frac{1}{7}(14x)$

67. $-\frac{1}{5}(10x)$

68. $-\frac{1}{8}(16x)$

69. $-\frac{2}{3}(12a^2)$

70. $-\frac{5}{8}(24a^2)$

71. $-\frac{1}{2}(-16y)$

72. $-\frac{3}{4}(-8y)$

73. $(16y)\left(\frac{1}{4}\right)$

74. $(33y)\left(\frac{1}{11}\right)$

75. $(-6x)\left(\frac{1}{3}\right)$

76. $(-10x)\left(\frac{1}{5}\right)$

77. $(-8a)\left(-\frac{3}{4}\right)$

78. $(21y)\left(-\frac{3}{7}\right)$

[illegible]

Simplify.

79. $-(x + 2)$

80. $-(x + 7)$

81. $2(4x - 3)$

82. $5(2x - 7)$

83. $-2(a + 7)$

84. $-5(a + 16)$

85. $-3(2y - 8)$

86. $-5(3y - 7)$

87. $(5 - 3b)7$

88. $(10 - 7b)2$

89. $\frac{1}{3}(6 - 15y)$

90. $\frac{1}{2}(-8x + 4y)$

91. $3(5x^2 + 2x)$

92. $6(3x^2 + 2x)$

93. $-2(-y + 9)$

94. $-5(-2x + 7)$

95. $(-3x - 6)5$

96. $(-2x + 7)7$

97. $2(-3x^2 - 14)$

98. $5(-6x^2 - 3)$

99. $-3(2y^2 - 7)$

100. $-8(3y^2 - 12)$

101. $3(x^2 - y^2)$

102. $5(x^2 + y^2)$

103. $-\frac{2}{3}(6x - 18y)$

104. $-\frac{1}{2}(x - 4y)$

105. $-(6a^2 - 7b^2)$

106. $3(x^2 + 2x - 6)$

107. $4(x^2 - 3x + 5)$

108. $-2(y^2 - 2y + 4)$

109. $\frac{1}{2}(2x - 6y + 8)$

110. $-\frac{1}{3}(6x - 9y + 1)$

111. $4(-3a^2 - 5a + 7)$

112. $-5(-2x^2 - 3x + 7)$

113. $-3(-4x^2 + 3x - 4)$

114. $3(2x^2 + xy - 3y^2)$

115. $5(2x^2 - 4xy - y^2)$

116. $-(3a^2 + 5a - 4)$

117. $-(8b^2 - 6b + 9)$

[illegible]

Simplify.

118. $4x - 2(3x + 8)$

119. $6a - (5a + 7)$

120. $9 - 3(4y + 6)$

121. $10 - (11x - 3)$

122. $5n - (7 - 2n)$

123. $8 - (12 + 4y)$

124. $3(x+2)-5(x-7)$

125. $2(x-4)-4(x+2)$

126. $12(y-2)+3(7-3y)$

127. $6(2y-7)-(3-2y)$

128. $3(a-b)-(a+b)$

129. $2(a+2b)-(a-3b)$

130. $4[x-2(x-3)]$

131. $2[x+2(x+7)]$

132. $-2[3x+2(4-x)]$

133. $-5[2x+3(5-x)]$

134. $-3[2x-(x+7)]$

135. $-2[3x-(5x-2)]$

136. $2x-3[x-(4-x)]$

137. $-7x+3[x-(3-2x)]$

138. $-5x-2[2x-4(x+7)]-6$

APPLYING THE CONCEPTS

139. Determine whether the statement is true or false. If the statement is false, give an example that illustrates that it is false.

a. Division is a commutative operation.
b. Division is an associative operation.
c. Subtraction is an associative operation.
d. Subtraction is a commutative operation.

140. Is the statement "any number divided by itself is 1" a true statement? If not, for what number or numbers is the statement not true?

141. Does every number have a multiplicative inverse? If not, which real number or numbers do not have a multiplicative inverse?

142. In your own words, explain the Distributive Property.

143. Give examples of two operations that occur in everyday experience that are not commutative (for example, putting on socks and then shoes).

144. Define an operation $\otimes$ as $a \otimes b = (a \cdot b) - (a + b)$. For example, $7 \otimes 5 = (7 \cdot 5) - (7 + 5) = 35 - 12 = 23$.

a. Is $\otimes$ a commutative operation? Support your answer.

b. Is $\otimes$ an associative operation? Support your answer.

2.3 Translating Verbal Expressions into Variable Expressions

To translate a verbal expression into a variable expression, given the variable

One of the major skills required in applied mathematics is to translate a verbal expression into a variable expression. This requires recognizing the verbal phrases that translate into mathematical operations. A partial list of the verbal phrases used to indicate the different mathematical operations follows.

Point of Interest

The way in which expressions are symbolized has changed over time. Here are how some of the expressions shown at the right may have appeared in the early 16th century.

R p. 9 for $x + 9$. The symbol R was used for a variable to the first power. The symbol p. was used for plus.

R m. 3 for $x - 3$. The symbol R is still used for the variable. The symbol m. was used for minus.

The square of a variable was designated by Q and the cube was designated by C. The expression $x^2 + x^3$ was written **Q p. C**.

Addition	added to	6 added to y	$y + 6$
	more than	8 more than x	$x + 8$
	the sum of	the sum of x and z	$x + z$
	increased by	t increased by 9	$t + 9$
	the total of	the total of 5 and y	$5 + y$
Subtraction	minus	x minus 2	$x - 2$
	less than	7 less than t	$t - 7$
	decreased by	m decreased by 3	$m - 3$
	the difference between	the difference between y and 4	$y - 4$
	subtract...from...	subtract 9 from z	$z - 9$
Multiplication	times	10 times t	$10t$
	twice	twice w	$2w$
	of	one-half of x	$\frac{1}{2}x$
	the product of	the product of y and z	yz
	multiplied by	y multiplied by 11	$11y$
Division	divided by	x divided by 12	$\frac{x}{12}$
	the quotient of	the quotient of y and z	$\frac{y}{z}$
	the ratio of	the ratio of t to 9	$\frac{t}{9}$
Power	the square of	the square of x	x^2
	the cube of	the cube of a	a^3

⇨ Translate "14 less than the cube of x" into a variable expression.

14 *less than* the *cube* of x	• **Identify the words that indicate the mathematical operations.**
$x^3 - 14$	• **Use the identified operations to write the variable expression.**

Translating a phrase that contains the word *sum, difference, product,* or *quotient* can sometimes cause a problem. In the examples at the right, note where the operation symbol is placed.

the *sum* of $x \overset{+}{\text{and}}$ y	$x + y$
the *difference* between $x \overset{-}{\text{and}}$ y	$x - y$
the *product* of $x \overset{\cdot}{\text{and}}$ y	$x \cdot y$
the *quotient* of $x \overset{\div}{\text{and}}$ y	$\frac{x}{y}$

⇨ Translate "the difference between the square of x and the sum of y and z" into a variable expression.

the $\underline{\text{difference between}}$ the $\underline{\text{square}}$ of x and the $\underline{\text{sum}}$ of y and z

- Identify the words that indicate the mathematical operations.

$x^2 - (y + z)$

- Use the identified operations to write the variable expression.

Example 1
Translate "the total of 3 times n and n" into a variable expression.

Solution
the $\underline{\text{total of}}$ 3 $\underline{\text{times}}$ n and n
$3n + n$

You Try It 1
Translate "the difference between twice n and one-third of n" into a variable expression.

Your solution

Example 2
Translate "m decreased by the sum of n and 12" into a variable expression.

Solution
m $\underline{\text{decreased by}}$ the $\underline{\text{sum}}$ of n and 12
$m - (n + 12)$

You Try It 2
Translate "the quotient of 7 less than b and 15" into a variable expression.

Your solution

Solutions on p. S3

Objective B

To translate a verbal expression into a variable expression and then simplify

In most applications that involve translating phrases into variable expressions, the variable to be used is not given. To translate these phrases, a variable must be assigned to an unknown quantity before the variable expression can be written.

Translate "a number multiplied by the total of six and the cube of the number" into a variable expression.

the unknown number: n

- Assign a variable to one of the unknown quantities.

the cube of the number: n^3
the total of six and the cube of the number: $6 + n^3$

- Use the assigned variable to write an expression for any other unknown quantity.

$n(6 + n^3)$

- Use the assigned variable to write the variable expression.

Example 3

Translate "a number added to the product of four and the square of the number" into a variable expression.

Solution

the unknown number: n
the square of the number: n^2
the product of four and the square of the number: $4n^2$
$4n^2 + n$

You Try It 3

Translate "negative four multiplied by the total of ten and the cube of a number" into a variable expression.

Your solution

Example 4

Translate "four times the sum of one-half of a number and fourteen" into a variable expression. Then simplify.

Solution

the unknown number: n

one-half of the number: $\frac{1}{2}n$

the sum of one-half of the number and fourteen: $\frac{1}{2}n + 14$

$4\left(\frac{1}{2}n + 14\right)$
$2n + 56$

You Try It 4

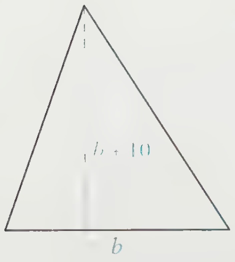
Translate "five times the difference between a number and sixty" into a variable expression. Then simplify.

Your solution

Solutions on p. S4

To translate application problems

Many of the applications of mathematics require that you identify the unknown quantity, assign a variable to that quantity, and then attempt to express other unknown quantities in terms of the variable.



The height of a triangle is 10 ft longer than the base of the triangle. Express the height of the triangle in terms of the base of the triangle.

the base of the triangle: b

- Assign a variable to the base of the triangle.

the height is 10 more than the base: $b + 10$

- Express the height of the triangle in terms of b.

Example 5

The length of a swimming pool is 4 ft less than two times the width. Express the length of the pool in terms of the width.

Solution

the width of the pool: w

the length is 4 ft less than two times the width: $2w - 4$

You Try It 5

The speed of a new jet plane is twice the speed of an older model. Express the speed of the new model in terms of the speed of the older model.

Your solution

Example 6

A banker divided \$5000 between two accounts, one paying 10% annual interest and the second paying 8% annual interest. Express the amount invested in the 10% account in terms of the amount invested in the 8% account.

Solution

the amount invested at 8%: x

the amount invested at 10%: $5000 - x$

You Try It 6

A guitar string 6 ft long was cut into two pieces. Express the length of the shorter piece in terms of the length of the longer piece.

Your solution

Solutions on p. S4

2.3 Exercises

[illegible]

Translate into a variable expression.

1. the sum of 8 and y

2. a less than 16

3. t increased by 10

4. p decreased by 7

5. z added to 14

6. q multiplied by 13

7. 20 less than the square of x

8. 6 times the difference between m and 7

9. the sum of three-fourths of n and 12

10. b decreased by the product of 2 and b

11. 8 increased by the quotient of n and 4

12. the product of -8 and y

13. the product of 3 and the total of y and 7

14. 8 divided by the difference between x and 6

15. the product of t and the sum of t and 16

16. the quotient of 6 less than n and twice n

17. 15 more than one-half of the square of x

18. 19 less than the product of n and -2

19. the total of 5 times the cube of n and the square of n

20. the ratio of 9 more than m to m

21. r decreased by the quotient of r and 3

22. four-fifths of the sum of w and 10

23. the difference between the square of x and the total of x and 17

24. s increased by the quotient of 4 and s

25. the product of 9 and the total of z and 4

26. n increased by the difference between 10 times n and 9

Translate into a variable expression. Then simplify.

27. twelve minus a number

28. a number divided by eighteen

29. two-thirds of a number

30. twenty more than a number

31. the quotient of twice a number and nine

32. ten times the difference between a number and fifty

33. eight less than the product of eleven and a number

34. the sum of five-eighths of a number and six

35. nine less than the total of a number and two

36. the difference between a number and three more than the number

37. the quotient of seven and the total of five and a number

38. four times the sum of a number and nineteen

39. five increased by one-half of the sum of a number and three

40. the quotient of fifteen and the sum of a number and twelve

41. a number added to the difference between twice the number and four

42. the product of two-thirds and the sum of a number and seven

43. the product of five less than a number and seven

44. the difference between forty and the quotient of a number and twenty

45. the quotient of five more than twice a number and the number

46. the sum of the square of a number and twice the number

47. a number decreased by the difference between three times the number and eight

48. the sum of eight more than a number and one-third of the number

49. a number added to the product of three and the number

50. a number increased by the total of the number and nine

51. five more than the sum of a number and six

52. a number decreased by the difference between eight and the number

53. a number minus the sum of the number and ten

54. the difference between one-third of a number and five-eighths of the number

55. the sum of one-sixth of a number and four-ninths of the number

56. two more than the total of a number and five

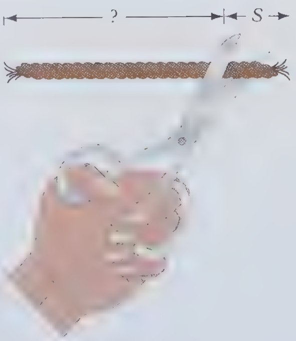
57. the sum of a number divided by three and the number

58. twice the sum of six times a number and seven

[illegible] *Application Problems*

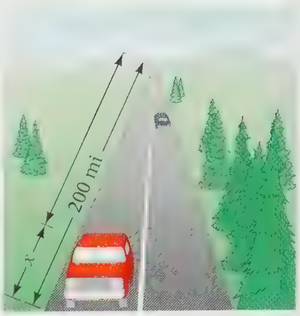
59. According to a report by the FAA (Federal Aviation Authority), the number of near collisions on airport runways in 2000 was 91 more than in 1999. Express the number of near collisions on runways in 2000 in terms of the number of near collisions on runways in 1999.

60. In 1947, phone companies began using area codes. According to information found at **555-1212.com,** at the beginning of 2001 there were 151 more area codes than there were in 1947. Express the number of area codes in 2001 in terms of the number of area codes in 1947.



61. A rope 12 ft long was cut into two pieces of different lengths. Use one variable to express the lengths of the two pieces.

62. Twenty gallons of crude oil were poured into two containers of different sizes. Use one variable to express the amount of oil poured into each container.



63. Two cars start at the same place and travel at different rates in opposite directions. Two hours later the cars are 200 mi apart. Express the distance traveled by the slower car in terms of the distance traveled by the faster car.

64. A survey reported in *USA Today* indicated that almost three-fourths of all Americans think that money should be spent on some kind of Mars exploration program. Express the number of Americans who think that money should be spent on some kind of Mars exploration program in terms of the total number of Americans.

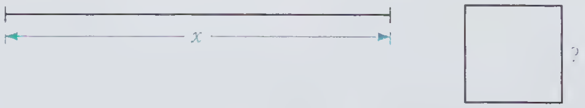
65. According to the American Podiatric Medical Association, the bones in your foot account for one-fourth of all the bones in your body. Express the number of bones in your foot in terms of the total number of bones in your body.

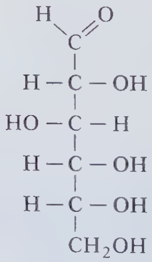
66. The diameter of a basketball is approximately 4 times the diameter of a baseball. Express the diameter of a basketball in terms of the diameter of a baseball.

67. A cost-of-living calculator provided by **HOMECARE.com** shows that a person living in New York City (Manhattan) would need twice the salary of a person living in San Diego. Express the salary needed in New York City in terms of the salary needed in San Diego.

APPLYING THE CONCEPTS

68. A wire whose length is given as x inches is bent into a square. Express the length of a side of the square in terms of x.





69. The chemical formula for glucose (sugar) is $C_6H_{12}O_6$. This formula means that there are 12 hydrogen atoms for every 6 carbon atoms and 6 oxygen atoms in each molecule of glucose (see the figure at the right). If x represents the number of atoms of oxygen in a pound of sugar, express the number of hydrogen atoms in the pound of sugar.

70. Translate the expressions $5x + 8$ and $5(x + 8)$ into phrases.

71. In your own words, explain how variables are used.

72. Explain the similarities and the differences between the expressions "the difference between x and 5" and "5 less than x."

Focus on Problem Solving

From Concrete to Abstract

In your study of algebra, you will find that the problems are less concrete than those you studied in arithmetic. Problems that are concrete provide information pertaining to a specific instance. Algebra is more abstract. Abstract problems are theoretical; they are stated without reference to a specific instance. Let's look at an example of an abstract problem.

How many minutes are in h hours?

A strategy that can be used to solve this problem is to solve the same problem after substituting a number for the variable.

How many minutes are in 5 hours?

You know that there are 60 minutes in 1 hour. To find the number of minutes in 5 hours, multiply 5 by 60.

$60 \cdot 5 = 300$ There are 300 minutes in 5 hours.

Use the same procedure to find the number of minutes in h hours: multiply h by 60.

$60 \cdot h = 60h$ There are $60h$ minutes in h hours.

This problem might be taken a step further:

If you walk 1 mile in x minutes, how far can you walk in h hours?

Consider the same problem using numbers in place of the variables.

If you walk 1 mile in 20 minutes, how far can you walk in 3 hours?

To solve this problem, you need to calculate the number of minutes in 3 hours (multiply 3 by 60), and divide the result by the number of minutes it takes to walk 1 mile (20 minutes).

$\frac{60 \cdot 3}{20} = \frac{180}{20} = 9$ If you walk 1 mile in 20 minutes, you can walk 9 miles in 3 hours.

Use the same procedure to solve the related abstract problem. Calculate the number of minutes in h hours (multiply h by 60), and divide the result by the number of minutes it takes to walk 1 mile (x minutes).

$\frac{60 \cdot h}{x} = \frac{60h}{x}$ If you walk 1 mile in x minutes, you can walk $\frac{60h}{x}$ miles in h hours.

At the heart of the study of algebra is the use of variables. It is the variables in the problems above that make them abstract. But it is variables that allow us to generalize situations and state rules about mathematics.

Try each of the following problems.

1. How many hours are in d days?

2. You earn d dollars an hour. What are your wages for working h hours?
3. If p is the price of one share of stock, how many shares can you purchase with d dollars?
4. A company pays a television station d dollars to air a commercial lasting s seconds. What is the cost per second?
5. After every v videotape rentals, you are entitled to one free rental. You have rented t tapes, where $t < v$. How many more do you need to rent before you are entitled to a free rental?
6. Your car gets g miles per gallon. How many gallons of gasoline does your car consume traveling t miles?
7. If you drink j ounces of juice each day, how many days will q quarts of the juice last?
8. A TV station has m minutes of commercials each hour. How many ads lasting s seconds each can be sold for each hour of programming?
9. A factory worker can assemble p products in m minutes. How many products can the factory worker assemble in h hours?
10. If one candy bar costs n nickels, how many candy bars can be purchased with q quarters?

Projects and Group Activities

Prime and Composite Numbers

A **prime number** is a natural number greater than 1 whose only natural-number factors are itself and 1. The number 11 is a prime number because the only natural-number factors of 11 are 11 and 1.

Eratosthenes, a Greek philosopher and astronomer who lived from 270 to 190 B.C., devised a method of identifying prime numbers. It is called the **Sieve of Eratosthenes.** The procedure is illustrated below.

~~1~~	②	③	~~4~~	⑤	~~6~~	⑦	~~8~~	~~9~~	~~10~~
⑪	~~12~~	⑬	~~14~~	~~15~~	~~16~~	⑰	~~18~~	⑲	~~20~~
~~21~~	~~22~~	㉓	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	㉙	~~30~~
㉛	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	㊲	~~38~~	~~39~~	~~40~~
㊶	~~42~~	㊸	~~44~~	~~45~~	~~46~~	㊼	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	(53)	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	(59)	~~60~~
(61)	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	(67)	~~68~~	~~69~~	~~70~~
(71)	~~72~~	(73)	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	(79)	~~80~~
~~81~~	~~82~~	(83)	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	(89)	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	(97)	~~98~~	~~99~~	~~100~~

List all the natural numbers from 1 to 100. Cross out the number 1, because it is not a prime number. The number 2 is prime; circle it. Cross out all the other multiples of 2 (4, 6, 8,...), because they are not prime. The number 3 is prime; circle it. Cross out all the other multiples of 3 (6, 9, 12,...) that are not already crossed out. The number 4, the next consecutive number in the list, has already been crossed out. The number 5 is prime; circle it. Cross out all the other multiples of 5 that are not already crossed out. Continue in this manner until all the prime numbers less than 100 are circled.

A **composite number** is a natural number greater than 1 that has a natural-number factor other than itself and 1. The number 21 is a composite number because it has factors of 3 and 7. All the numbers crossed out in the preceding table, except the number 1, are composite numbers.

1. Use the Sieve of Eratosthenes to find the prime numbers between 100 and 200.

2. How many prime numbers are even numbers?

3. Find the "twin primes" between 1 and 200. Twin primes are two prime numbers whose difference is 2. For instance, 3 and 5 are twin primes; 5 and 7 are also twin primes.

4. a. List two prime numbers that are consecutive natural numbers.
b. Can there be any other pairs of prime numbers that are consecutive natural numbers?

5. Some primes are the sum of a square and 1. For example, $5 = 2^2 + 1$. Find another prime p such that $p = n^2 + 1$, where n is a natural number.

6. Find a prime number p such that $p = n^2 - 1$, where n is a natural number.

7. a. 4! (which is read "4 factorial") is equal to $4 \cdot 3 \cdot 2 \cdot 1$. Show that $4! + 2$, $4! + 3$, and $4! + 4$ are all composite numbers.
b. 5! (which is read "5 factorial") is equal to $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$. Will $5! + 2$, $5! + 3$, $5! + 4$, and $5! + 5$ generate four consecutive composite numbers?
c. Use the notation 6! to represent a list of five consecutive composite numbers.

Investigation into Operations with Even and Odd Integers

Complete each statement with the word *even* or *odd*.

1. If k is an odd integer, then $k + 1$ is an ______ integer.

2. If k is an odd integer, then $k - 2$ is an ______ integer.

3. If n is an integer, then $2n$ is an ______ integer.

4. If m and n are even integers, then $m - n$ is an ______ integer.

5. If m and n are even integers, then mn is an ______ integer.

6. If m and n are odd integers, then $m + n$ is an ______ integer.

7. If m and n are odd integers, then $m - n$ is an ______ integer.

8. If m and n are odd integers, then mn is an ______ integer.

9. If m is an even integer and n is an odd integer, then $m - n$ is an ______ integer.

10. If m is an even integer and n is an odd integer, then $m + n$ is an ______ integer.

Chapter Summary

Key Words

A *variable* is a letter that is used to stand for a quantity that is unknown. [p.53]

A *variable expression* is an expression that contains one or more variables. [p.53]

The *terms* of a variable expression are the addends of the expression. A *variable term* is composed of a *numerical coefficient* and a *variable part*. A *constant term* has no variable part. [p.53]

Replacing the variables in a variable expression with numbers and then simplifying the numerical expression is called *evaluating the variable expression*. [p.54]

Like terms of a variable expression are terms that have the same variable part. Adding or subtracting the like terms of a variable expression is called *combining like terms*. [p.57]

The *additive inverse* of a number is the opposite of the number. [p.58]

The *multiplicative inverse* of a number is the reciprocal of the number. [p.59]

Essential Rules

The Distributive Property [p.57]
If a, b, and c are real numbers, then $a(b + c) = ab + ac$.

The Associative Property of Addition [p.57]
If a, b, and c are real numbers, then $(a + b) + c = a + (b + c)$.

The Commutative Property of Addition [p.58]
If a and b are real numbers, then $a + b = b + a$.

The Addition Property of Zero [p.58]
If a is a real number, then $a + 0 = 0 + a = a$.

The Inverse Property of Addition [p.58]
If a is a real number, then $a + (-a) = (-a) + a = 0$.

The Associative Property of Multiplication [p.59]
If a, b, and c are real numbers, then $(ab)c = a(bc)$.

The Commutative Property of Multiplication [p.59]
If a and b are real numbers, then $ab = ba$.

The Multiplication Property of One [p.59]
If a is a real number, then $1 \cdot a = a \cdot 1 = a$.

The Inverse Property of Multiplication [p.59]
If a is a nonzero real number, then $a\left(\frac{1}{a}\right) = \left(\frac{1}{a}\right)a = 1$.

Chapter Review

1. Simplify: $3(x^2 - 8x - 7)$

2. Simplify: $7x + 4x$

3. Simplify: $6a - 4b + 2a$

4. Simplify: $(-50n)\left(\frac{1}{10}\right)$

5. Evaluate $(5c - 4a)^2 - b$ when $a = -1$, $b = 2$, and $c = 1$.

6. Simplify: $5(2x - 7)$

7. Simplify: $2(6y^2 + 4y - 5)$

8. Simplify: $\frac{1}{4}(-24a)$

9. Simplify: $-6(7x^2)$

10. Simplify: $-9(7 + 4x)$

11. Simplify: $12y - 17y$

12. Evaluate $2bc \div (a + 7)$ when $a = 3$, $b = -5$, and $c = 4$.

13. Simplify: $6(8y - 3) - 8(3y - 6)$

14. Simplify: $5c + (-2d) - 3d - (-4c)$

15. Simplify: $5(4x)$

16. Simplify: $-4(2x - 9) + 5(3x + 2)$

17. Evaluate $(b - a)^2 + c$ when $a = -2$, $b = 3$, and $c = 4$.

18. Simplify: $-9r + 2s - 6s + 12s$

19. Simplify: $4x - 3x^2 + 2x - x^2$

20. Simplify: $5[2 - 3(6x - 1)]$

21. Simplify: $(7a^2 - 2a + 3)4$

22. Simplify: $18 - (4x - 2)$

23. Evaluate $a^2 - b^2$ when $a = 3$ and $b = 4$.

24. Simplify: $-3(-12y)$

25. Translate "two-thirds of the total of x and 10" into a variable expression.

26. A candy bar contains eight more calories than twice the number of calories in an apple. Express the number of calories in a candy bar in terms of the number of calories in an apple.

27. Translate "three times a number plus the product of five and one less than the number" into a variable expression. Then simplify.

28. Translate "the product of 4 and x" into a variable expression.

29. A club treasurer has some five-dollar bills and some ten-dollar bills. The treasurer has a total of 35 bills. Express the number of five-dollar bills in terms of the number of ten-dollar bills.

30. Translate "the difference between twice a number and one-half of the number" into a variable expression. Then simplify.

31. A baseball card collection contains five times as many National League players' cards as American League players' cards. Express the number of National League players' cards in the collection in terms of the number of American League players' cards.

32. Translate "6 less than x" into a variable expression.

33. Translate "a number plus twice the number" into a variable expression. Then simplify.

Chapter Test

1. Simplify: $3x - 5x + 7x$

2. Simplify: $-3(2x^2 - 7y^2)$

3. Simplify: $2x - 3(x - 2)$

4. Simplify: $2x + 3[4 - (3x - 7)]$

5. Simplify: $3x - 7y - 12x$

6. Evaluate $b^2 - 3ab$ when $a = 3$ and $b = -2$.

7. Simplify: $\frac{1}{5}(10x)$

8. Simplify: $5(2x + 4) - 3(x - 6)$

9. Simplify: $-5(2x^2 - 3x + 6)$

10. Simplify: $3x + (-12y) - 5x - (-7y)$

11. Evaluate $\frac{-2ab}{2b - a}$ when $a = -4$ and $b = 6$.

12. Simplify: $(12x)\left(\frac{1}{4}\right)$

13. Simplify: $-7y^2 + 6y^2 - (-2y^2)$

14. Simplify: $-2(2x - 4)$

15. Simplify: $\frac{2}{3}(-15a)$

16. Simplify: $-2[x - 2(x - y)] + 5y$

17. Simplify: $(-3)(-12y)$

18. Simplify: $5(3 - 7b)$

19. Translate "the difference between the squares of a and b" into a variable expression.

20. Translate "ten times the difference between a number and 3" into a variable expression. Then simplify.

21. Translate "the sum of a number and twice the square of the number" into a variable expression.

22. The speed of a pitcher's fastball is twice the speed of the catcher's return throw. Express the speed of the fastball in terms of the speed of the return throw.

23. Translate "three less than the quotient of six and a number" into a variable expression.

24. Translate "b decreased by the product of b and seven" into a variable expression.

25. A wire is cut into two lengths. The length of the longer piece is 3 in. less than four times the length of the shorter piece. Express the length of the longer piece in terms of the length of the shorter piece.

Cumulative Review

1. Add: $-4 + 7 + (-10)$

2. Subtract: $-16 - (-25) - 4$

3. Multiply: $(-2)(3)(-4)$

4. Divide: $(-60) \div 12$

5. Write $1\frac{1}{4}$ as a decimal.

6. Simplify: $\frac{7}{12} - \frac{11}{16} - \left(-\frac{1}{3}\right)$

7. Simplify: $\frac{5}{12} \div \left(2\frac{1}{2}\right)$

8. Simplify: $\left(-\frac{9}{16}\right) \cdot \left(\frac{8}{27}\right) \cdot \left(-\frac{3}{2}\right)$

9. Write $\frac{3}{4}$ as a percent.

10. Simplify: $-2^5 \div (3 - 5)^2 - (-3)$

11. Simplify: $\left(-\frac{3}{4}\right)^2 - \left(\frac{3}{8} - \frac{11}{12}\right)$

12. Evaluate $a^2 - 3b$ when $a = 2$ and $b = -4$.

13. Simplify: $-2x^2 - (-3x^2) + 4x^2$

14. Simplify: $5a - 10b - 12a$

15. Simplify: $\frac{1}{2}(12a)$

16. Simplify: $\left(-\frac{5}{6}\right)(-36b)$

17. Simplify: $3(8 - 2x)$

18. Simplify: $-2(-3y + 9)$

19. Write $37\frac{1}{2}\%$ as a fraction.

20. Write 1.05% as a decimal.

21. Simplify: $-4(2x^2 - 3y^2)$

22. Simplify: $-3(3y^2 - 3y - 7)$

23. Simplify: $-3x - 2(2x - 7)$

24. Simplify: $4(3x - 2) - 7(x + 5)$

25. Simplify: $2x + 3[x - 2(4 - 2x)]$

26. Simplify: $3[2x - 3(x - 2y)] + 3y$

27. Translate "the sum of one-half of *b* and *b*" into a variable expression.

28. Translate "10 divided by the difference between *y* and 2" into a variable expression.

29. Translate "the difference between eight and the quotient of a number and twelve" into a variable expression.

30. Translate "the sum of a number and two more than the number" into a variable expression. Then simplify.

31. Translate "twelve more than the product of three plus a number and five" into a variable expression. Then simplify.

32. The speed of a DSL (Digital Subscriber Line) Internet connection is ten times faster than that of a dial-up connection. Express the speed of the DSL connection in terms of the speed of the dial-up connection.

Chapter 3 Solving Equations

By surveying television viewers, Nielsen Media Research can determine the number of people watching particular TV shows. The larger the audience, the greater the number of rating points the show receives. Programs with a high number of rating points can demand and receive higher fees from sponsors who pay to have their advertisements aired during the show. The **Project on pages 154 and 155** describes the meaning of Nielsen rating points and their relationship to market share.

Objectives

Section 3.1

- **A** To determine whether a given number is a solution of an equation
- **B** To solve an equation of the form $x + a = b$
- **C** To solve an equation of the form $ax = b$
- **D** To solve application problems using the basic percent equation

Section 3.2

- **A** To solve an equation of the form $ax + b = c$
- **B** To solve application problems using formulas

Section 3.3

- **A** To solve an equation of the form $ax + b = cx + d$
- **B** To solve an equation containing parentheses
- **C** To solve application problems using formulas

Section 3.4

- **A** To solve integer problems
- **B** To translate a sentence into an equation and solve

Section 3.5

- **A** To solve perimeter problems
- **B** To solve problems involving angles formed by intersecting lines
- **C** To solve problems involving the angles of a triangle

Section 3.6

- **A** To solve value mixture problems
- **B** To solve percent mixture problems
- **C** To solve uniform motion problems

Need help? For online student resources, such as section quizzes, visit this textbook's website at **math.college.hmco.com/students.**

Prep Test

1. Write $\frac{9}{100}$ as a decimal.

2. Write $\frac{3}{4}$ as a percent.

3. Evaluate $3x^2 - 4x - 1$ when $x = -4$.

4. Simplify: $R - 0.35R$

5. Simplify: $\frac{1}{2}x + \frac{2}{3}x$

6. Simplify: $6x - 3(6 - x)$

7. Simplify: $0.22(3x + 6) + x$

8. Translate into a variable expression: "The difference between 5 and twice a number".

9. A new graphics card for computer games is five times faster than a graphics card made two years ago. Express the speed of the new card in terms of the speed of the old card.

10. A board 5 ft long is cut into two pieces. If x represents the length of the longer piece, write an expression for the shorter piece in terms of x.

Go Figure

How can a donut be cut into 8 equal pieces with three cuts of a knife?

3.1 Introduction to Equations

[illegible]

To determine whether a given number is a solution of an equation

3

Point of Interest

One of the most famous equations ever stated is $E = mc^2$. This equation, stated by Albert Einstein, shows that there is a relationship between mass m and energy E. As a side note, the chemical element einsteinium was named in honor of Einstein.

An **equation** expresses the equality of two mathematical expressions. The expressions can be either numerical or variable expressions.

$$\left.\begin{array}{l} 9 + 3 = 12 \\ 3x - 2 = 10 \\ y^2 + 4 = 2y - 1 \\ z = 2 \end{array}\right\} \text{Equations}$$

The equation at the right is true if the variable is replaced by 5.

$$x + 8 = 13$$

$$5 + 8 = 13 \quad \text{A true equation}$$

The equation is false if the variable is replaced by 7.

$$7 + 8 = 13 \quad \text{A false equation}$$

A **solution** of an equation is a number that, when substituted for the variable, results in a true equation. 5 is a solution of the equation $x + 8 = 13$. 7 is not a solution of the equation $x + 8 = 13$.

TAKE NOTE

The Order of Operations Agreement applies to evaluating $2(-2) + 5$ and $(-2)^2 - 3$.

Is -2 a solution of $2x + 5 = x^2 - 3$?

$$\begin{array}{r|l} \multicolumn{2}{c}{2x + 5 = x^2 - 3} \\ \hline 2(-2) + 5 & (-2)^2 - 3 \\ -4 + 5 & 4 - 3 \\ \multicolumn{2}{c}{1 = 1} \end{array}$$

- Replace x by -2.
- Evaluate the numerical expressions.
- If the results are equal, -2 is a solution of the equation. If the results are not equal, -2 is not a solution of the equation.

Yes, -2 is a solution of the equation.

Example 1

Is -4 a solution of $5x - 2 = 6x + 2$?

Solution

$$\begin{array}{r|l} \multicolumn{2}{c}{5x - 2 = 6x + 2} \\ \hline 5(-4) - 2 & 6(-4) + 2 \\ -20 - 2 & -24 + 2 \\ \multicolumn{2}{c}{-22 = -22} \end{array}$$

Yes, -4 is a solution.

You Try It 1

Is $\frac{1}{4}$ a solution of $5 - 4x = 8x + 2$?

Your solution

Example 2

Is -4 a solution of $4 + 5x = x^2 - 2x$?

Solution

$$\begin{array}{r|l} \multicolumn{2}{c}{4 + 5x = x^2 - 2x} \\ \hline 4 + 5(-4) & (-4)^2 - 2(-4) \\ 4 + (-20) & 16 - (-8) \\ \multicolumn{2}{c}{-16 \neq 24} \end{array}$$

($\neq$ means "is not equal to")

No, -4 is not a solution.

You Try It 2

Is 5 a solution of $10x - x^2 = 3x - 10$?

Your solution

Solutions on p. S4

To solve an equation of the form $x + a = b$

To **solve an equation** means to find a solution of the equation. The simplest equation to solve is an equation of the form *variable* = *constant*, because the constant is the solution.

The solution of the equation $x = 5$ is 5 because $5 = 5$ is a true equation.

The solution of the equation at the right is 7 because $7 + 2 = 9$ is a true equation.

$$x + 2 = 9 \qquad\qquad 7 + 2 = 9$$

Note that if 4 is added to each side of the equation $x + 2 = 9$, the solution is still 7.

$$\begin{aligned} x + 2 &= 9 \\ x + 2 + 4 &= 9 + 4 \\ x + 6 &= 13 \qquad\qquad 7 + 6 = 13 \end{aligned}$$

If -5 is added to each side of the equation $x + 2 = 9$, the solution is still 7.

$$\begin{aligned} x + 2 &= 9 \\ x + 2 + (-5) &= 9 + (-5) \\ x - 3 &= 4 \qquad\qquad 7 - 3 = 4 \end{aligned}$$

Equations that have the same solution are **equivalent equations.** The equations $x + 2 = 9$, $x + 6 = 13$, and $x - 3 = 4$ are equivalent equations; each equation has 7 as its solution. These examples suggest that adding the same number to each side of an equation produces an equivalent equation. This is called the *Addition Property of Equations.*

> **Addition Property of Equations**
>
> The same number can be added to each side of an equation without changing its solution. In symbols, the equation $a = b$ has the same solution as the equation $a + c = b + c$.

In solving an equation, the goal is to rewrite the given equation in the form *variable* = *constant*. The Addition Property of Equations is used to remove a *term* from one side of the equation by adding the opposite of that term to each side of the equation.

Solve: $x - 4 = 2$

$x - 4 = 2$
- The goal is to rewrite the equation as *variable* = *constant*.

$x - 4 + 4 = 2 + 4$
- Add 4 to each side of the equation.

$x + 0 = 6$
- Simplify.

$x = 6$
- The equation is in the form *variable* = *constant*.

Check:
$$\begin{array}{r|l} \multicolumn{2}{c}{x - 4 = 2} \\ \hline 6 - 4 & 2 \\ 2 = & 2 \end{array}$$

A true equation

The solution is 6.

Because subtraction is defined in terms of addition, the Addition Property of Equations also makes it possible to subtract the same number from each side of an equation without changing the solution of the equation.

Solve: $y + \frac{3}{4} = \frac{1}{2}$

$$y + \frac{3}{4} = \frac{1}{2}$$

- The goal is to rewrite the equation in the form *variable* = *constant*.

$$y + \frac{3}{4} - \frac{3}{4} = \frac{1}{2} - \frac{3}{4}$$

- Subtract $\frac{3}{4}$ from each side of the equation.

$$y + 0 = \frac{2}{4} - \frac{3}{4}$$

- Simplify.

$$y = -\frac{1}{4}$$

- The equation is in the form *variable* = *constant*.

The solution is $-\frac{1}{4}$. You should check this solution.

Example 3

Solve: $x + \frac{3}{4} = \frac{1}{3}$

Solution

$$\begin{aligned} x + \frac{3}{4} &= \frac{1}{3} \\ x + \frac{3}{4} - \frac{3}{4} &= \frac{1}{3} - \frac{3}{4} \\ x + 0 &= \frac{4}{12} - \frac{9}{12} \\ x &= -\frac{5}{12} \end{aligned}$$

The solution is $-\frac{5}{12}$.

You Try It 3

Solve: $\frac{5}{6} = y - \frac{3}{8}$

Your solution

Solution on p. S4

To solve an equation of the form $ax = b$

The solution of the equation at the right is 3 because $2 \cdot 3 = 6$ is a true equation.

$$2x = 6 \qquad 2 \cdot 3 = 6$$

Note that if each side of $2x = 6$ is multiplied by 5, the solution is still 3.

$$\begin{aligned} 2x &= 6 \\ 5(2x) &= 5 \cdot 6 \\ 10x &= 30 \end{aligned} \qquad 10 \cdot 3 = 30$$

If each side of $2x = 6$ is multiplied by -4, the solution is still 3.

$$\begin{aligned} 2x &= 6 \\ (-4)(2x) &= (-4)6 \\ -8x &= -24 \end{aligned} \qquad -8 \cdot 3 = -24$$

The equations $2x = 6$, $10x = 30$, and $-8x = -24$ are equivalent equations; each equation has 3 as its solution. These examples suggest that multiplying each side of an equation by the same number produces an equivalent equation.

Multiplication Property of Equations

Each side of an equation can be multiplied by the same *nonzero* number without changing the solution of the equation. In symbols, if $c \neq 0$, then the equation $a = b$ has the same solutions as the equation $ac = bc$.

The Multiplication Property of Equations is used to remove a coefficient by multiplying each side of the equation by the reciprocal of the coefficient.

Solve: $\frac{3}{4}z = 9$

$$\frac{3}{4}z = 9$$

- The goal is to rewrite the equation in the form *variable* = *constant*.

$$\frac{4}{3} \cdot \frac{3}{4}z = \frac{4}{3} \cdot 9$$

- Multiply each side of the equation by $\frac{4}{3}$.

$$1 \cdot z = 12$$

- Simplify.

$$z = 12$$

- The equation is in the form *variable* = *constant*.

The solution is 12. You should check this solution.

Because division is defined in terms of multiplication, each side of an equation can be divided by the same nonzero number without changing the solution of the equation.

TAKE NOTE
Remember to check the solution.

Check:
$$\begin{array}{c|c} 6x & 14 \\ \hline 6\left(\frac{7}{3}\right) & 14 \\ 14 & 14 \end{array}$$

Solve: $6x = 14$

$$6x = 14$$

- The goal is to rewrite the equation in the form *variable* = *constant*.

$$\frac{6x}{6} = \frac{14}{6}$$

- Divide each side of the equation by 6.

$$x = \frac{7}{3}$$

- Simplify. The equation is in the form *variable* = *constant*.

The solution is $\frac{7}{3}$.

When using the Multiplication Property of Equations, multiply each side of the equation by the reciprocal of the coefficient when the coefficient is a fraction. Divide each side of the equation by the coefficient when the coefficient is an integer or a decimal.

Example 4

Solve: $\frac{3x}{4} = -9$

Solution

$$\frac{3x}{4} = -9$$

$$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3}(-9)$$

- $\frac{3x}{4} = \frac{3}{4}x$

$$x = -12$$

The solution is -12.

You Try It 4

Solve: $-\frac{2x}{5} = 6$

Your solution

Example 5 Solve: $5x - 9x = 12$

Solution

$$5x - 9x = 12$$

$$-4x = 12$$

- Combine like terms.

$$\frac{-4x}{-4} = \frac{12}{-4}$$

$$x = -3$$

The solution is -3.

You Try It 5 Solve: $4x - 8x = 16$

Your solution

Solutions on p. S4

[illegible]

To solve application problems using the basic percent equation

An equation that is used frequently in mathematics applications is the basic percent equation.

Basic Percent Equation

Percent · Base = Amount

$$P \cdot B = A$$

In many application problems involving percent, the base follows the word *of*.

20% of what number is 30?

$$P \cdot B = A$$

- Use the basic percent equation.

$$0.20B = 30$$

- $P = 20\% = 0.20$, $A = 30$, and B is unknown.

$$\frac{0.20B}{0.20} = \frac{30}{0.20}$$

$$B = 150$$

- Solve for B.

The number is 150.

TAKE NOTE

We have written $P(80) = 70$ because that is the form of the basic percent equation. We could have written $80P = 70$. The important point is that each side of the equation is divided by 80, the coefficient of P.

70 is what percent of 80?

$$P \cdot B = A$$

- Use the basic percent equation.

$$P(80) = 70$$

- $B = 80$, $A = 70$, and P is unknown.

$$\frac{P(80)}{80} = \frac{70}{80}$$

- Solve for P.

$$P = 0.875$$

$$P = 87.5\%$$

- The question asked for a percent. Convert the decimal to a percent.

70 is 87.5% of 80.

Point of Interest

As noted at the right, 172,841 of the dogs or litters registered were Labrador retrievers. Listed below are the next most popular breeds and their registration numbers.

Golden retrievers:	66,300
German shepherds:	57,600
Dachshunds:	54,773
Beagles:	52,026

During a recent year, nearly 1.2 million dogs or litters were registered with the American Kennel Club. The most popular breed was the Labrador retriever, with 172,841 registered. (Source: American Kennel Club) What percent of the registrations were Labrador retrievers? Round to the nearest tenth of a percent.

Strategy To find the percent, use the basic percent equation.
$B = 1.2$ million $= 1{,}200{,}000$; $A = 172{,}841$; P is unknown.

Solution

$$\begin{aligned} P \cdot B &= A \\ P(1{,}200{,}000) &= 172{,}841 \\ P &\approx 0.144 = 14.4\% \end{aligned}$$

Approximately 14.4% of the registrations were Labrador retrievers.

In most cases, you should write the percent as a decimal before solving the basic percent equation. However, some percents are more easily written as a fraction. For example,

$$33\frac{1}{3}\% = \frac{1}{3} \qquad 66\frac{2}{3}\% = \frac{2}{3} \qquad 16\frac{2}{3}\% = \frac{1}{6} \qquad 83\frac{1}{3}\% = \frac{5}{6}$$

Example 6

12 is $33\frac{1}{3}\%$ of what number?

Solution

$$\begin{aligned} P \cdot B &= A \\ \frac{1}{3}B &= 12 \qquad \bullet\ 33\frac{1}{3}\% = \frac{1}{3} \\ 3 \cdot \frac{1}{3}B &= 3 \cdot 12 \\ B &= 36 \end{aligned}$$

12 is $33\frac{1}{3}\%$ of 36.

You Try It 6

18 is $16\frac{2}{3}\%$ of what number?

Your solution

Example 7

In a recent year, 238 U.S. airports collected \$1.1 billion in passenger taxes. Of this amount, \$88 million was spent on noise reduction. What percent of the passenger taxes collected was spent on noise reduction?

Strategy

To find the percent, solve the basic percent equation using $B = 1.1$ billion $= 1{,}100$ million and $A = 88$ million. The percent is unknown.

Solution

$$\begin{aligned} P \cdot B &= A \\ P(1{,}100) &= 88 \\ \frac{P(1{,}100)}{1{,}100} &= \frac{88}{1{,}100} \\ P &= 0.08 \end{aligned}$$

8% of the passenger taxes collected was spent on noise reduction.

You Try It 7

The total revenue for all football bowl games in 2000 was approximately \$158.3 million. Of that amount, the Big Ten conference received \$22.45 million. What percent of the total revenue did the Big Ten conference receive? Round to the nearest tenth of a percent. (Source: San Diego Union–Tribune, January 18, 2001)

Your strategy

Your solution

Solutions on p. S4

3.1 Exercises

[illegible]

1. What is the difference between an equation and an expression?

2. Explain how to determine whether a given number is a solution of an equation.

3. Is 4 a solution of $2x = 8$?

4. Is 3 a solution of $y + 4 = 7$?

5. Is -1 a solution of $2b - 1 = 3$?

6. Is -2 a solution of $3a - 4 = 10$?

7. Is 1 a solution of $4 - 2m = 3$?

8. Is 2 a solution of $7 - 3n = 2$?

9. Is 5 a solution of $2x + 5 = 3x$?

10. Is 4 a solution of $3y - 4 = 2y$?

11. Is -2 a solution of $3a + 2 = 2 - a$?

12. Is 3 a solution of $z^2 + 1 = 4 + 3z$?

13. Is 2 a solution of $2x^2 - 1 = 4x - 1$?

14. Is -1 a solution of $y^2 - 1 = 4y + 3$?

15. Is -2 a solution of $m^2 - 4 = m + 3$?

16. Is 5 a solution of $x^2 + 2x + 1 = (x + 1)^2$?

17. Is -6 a solution of $(n - 2)^2 = n^2 - 4n + 4$?

18. Is 4 a solution of $x(x + 1) = x^2 + 5$?

19. Is 3 a solution of $2a(a - 1) = 3a + 3$?

20. Is $-\frac{1}{4}$ a solution of $8t + 1 = -1$?

21. Is $\frac{1}{2}$ a solution of $4y + 1 = 3$?

22. Is $\frac{2}{5}$ a solution of $5m + 1 = 10m - 3$?

23. Is $\frac{3}{4}$ a solution of $8x - 1 = 12x + 3$?

[illegible]

24. Can 0 ever be the solution of an equation? If so, give an example of an equation for which 0 is a solution.

25. Without solving the equation $x + \frac{13}{15} = -\frac{21}{43}$, determine whether x is less than or greater than $-\frac{21}{43}$. Explain your answer.

Solve and check.

26. $x + 5 = 7$

27. $y + 3 = 9$

28. $b - 4 = 11$

29. $z - 6 = 10$

30. $2 + a = 8$

31. $5 + x = 12$

32. $m + 9 = 3$

33. $t + 12 = 10$

34. $n - 5 = -2$

35. $x - 6 = -5$

36. $b + 7 = 7$

37. $y - 5 = -5$

38. $a - 3 = -5$ **39.** $x - 6 = -3$ **40.** $z + 9 = 2$ **41.** $n + 11 = 1$

42. $10 + m = 3$ **43.** $8 + x = 5$ **44.** $9 + x = -3$ **45.** $10 + y = -4$

46. $b - 5 = -3$ **47.** $t - 6 = -4$ **48.** $4 + x = 10$ **49.** $9 + a = 20$

50. $2 = x + 7$ **51.** $-8 = n + 1$ **52.** $4 = m - 11$ **53.** $-6 = y - 5$

54. $12 = 3 + w$ **55.** $-9 = 5 + x$ **56.** $4 = -10 + b$ **57.** $-7 = -2 + x$

58. $13 = -6 + a$ **59.** $m + \frac{2}{3} = -\frac{1}{3}$ **60.** $c + \frac{3}{4} = -\frac{1}{4}$ **61.** $x - \frac{1}{2} = \frac{1}{2}$

62. $x - \frac{2}{5} = \frac{3}{5}$ **63.** $\frac{5}{8} + y = \frac{1}{8}$ **64.** $\frac{4}{9} + a = -\frac{2}{9}$

65. $m + \frac{1}{2} = -\frac{1}{4}$ **66.** $b + \frac{1}{6} = -\frac{1}{3}$ **67.** $x + \frac{2}{3} = \frac{3}{4}$

68. $n + \frac{2}{5} = \frac{2}{3}$ **69.** $-\frac{5}{6} = x - \frac{1}{4}$ **70.** $-\frac{1}{4} = c - \frac{2}{3}$

71. $d + 1.3619 = 2.0148$ **72.** $w + 2.932 = 4.801$ **73.** $-0.813 + x = -1.096$

74. $-1.926 + t = -1.042$ **75.** $6.149 = -3.108 + z$ **76.** $5.237 = -2.014 + x$

OBJECTIVE C

77. Without solving the equation $-\frac{15}{41}x = -\frac{23}{25}$, determine whether x is less than or greater than 0. Explain your answer.

78. Explain why multiplying each side of an equation by the reciprocal of the coefficient of the variable is the same as dividing each side of the equation by the coefficient.

Solve and check.

79. $5x = -15$

80. $4y = -28$

81. $3b = 0$

82. $2a = 0$

83. $-3x = 6$

84. $-5m = 20$

85. $-3x = -27$

86. $-\frac{1}{6}n = -30$

87. $20 = \frac{1}{4}c$

88. $18 = 2t$

89. $-32 = 8w$

90. $-56 = 7x$

91. $0 = -5x$

92. $0 = -8a$

93. $-32 = -4y$

94. $-54 = 6c$

95. $49 = -7t$

96. $\frac{x}{3} = 2$

97. $\frac{x}{4} = 3$

98. $-\frac{y}{2} = 5$

99. $-\frac{b}{3} = 6$

100. $\frac{3}{4}y = 9$

101. $\frac{2}{5}x = 6$

102. $-\frac{2}{3}d = 8$

103. $-\frac{3}{5}m = 12$

104. $\frac{2n}{3} = 0$

105. $\frac{5x}{6} = 0$

106. $\frac{-3z}{8} = 9$

107. $\frac{3x}{4} = 2$

108. $\frac{3}{4}c = \frac{3}{5}$

109. $\frac{2}{9} = \frac{2}{3}y$

110. $-\frac{6}{7} = -\frac{3}{4}b$

111. $\frac{1}{5}x = -\frac{1}{10}$

112. $-\frac{2}{3}y = -\frac{8}{9}$

113. $-1 = \frac{2n}{3}$

114. $-\frac{3}{4} = \frac{a}{8}$

115. $-\frac{2}{5}m = -\frac{6}{7}$

116. $5x + 2x = 14$

117. $3n + 2n = 20$

118. $7d - 4d = 9$

119. $10y - 3y = 21$

120. $2x - 5x = 9$

121. $\frac{x}{1.46} = 3.25$

122. $\frac{z}{2.95} = -7.88$

123. $3.47a = 7.1482$

124. $2.31m = 2.4255$

125. $-3.7x = 7.881$

126. $\frac{n}{2.65} = 9.08$

Objective B

127. Without solving an equation, indicate whether 40% of 80 is less than, equal to, or greater than 80% of 40.

128. Without solving an equation, indicate whether $\frac{1}{4}$% of 80 is less than, equal to, or greater than 25% of 80.

129. What is 35% of 80?

130. What percent of 8 is 0.5?

131. Find 1.2% of 60.

132. 8 is what percent of 5?

133. 125% of what is 80?

134. What percent of 20 is 30?

135. 12 is what percent of 50?

136. What percent of 125 is 50?

137. Find 18% of 40.

138. What is 25% of 60?

139. 12% of what is 48?

140. 45% of what is 9?

141. What is $33\frac{1}{3}$% of 27?

142. Find $16\frac{2}{3}$% of 30.

143. What percent of 12 is 3?

144. 10 is what percent of 15?

145. 60% of what is 3?

146. 75% of what is 6?

147. 12 is what percent of 6?

148. 20 is what percent of 16?

149. $5\frac{1}{4}$% of what is 21?

150. $37\frac{1}{2}\%$ of what is 15?

151. Find 15.4% of 50.

152. What is 18.5% of 46?

153. 1 is 0.5% of what?

154. 3 is 1.5% of what?

155. $\frac{3}{4}\%$ of what is 3?

156. $\frac{1}{2}\%$ of what is 3?

157. Find 125% of 16.

158. What is 250% of 12?

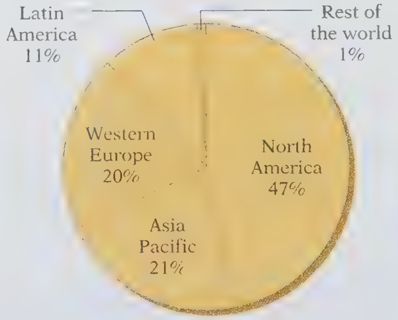
159. 16.43 is what percent of 20.45? Round to the nearest hundredth of a percent.

160. Find 18.37% of 625.43. Round to the nearest hundredth.

161. A university consists of three colleges: business, engineering, and fine arts. There are 2900 students in the business college, 1500 students in the engineering college, and 1000 students in the fine arts college. What percent of the total number of students in the university are in the fine arts college? Round to the nearest percent.

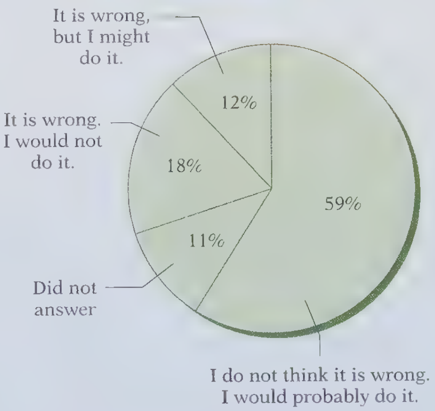
162. Approximately 21% of air is oxygen. Using this estimate, determine how many liters of oxygen there are in a room containing 21,600 L of air.

Regions, Share of Global E-Commerce, 2004



163. The graph at the right, based on data from Forrester Research, shows the percent share of the global e-commerce market for various regions of the world. The total projected revenues for e-commerce business in 2004 are $7500 billion. How many billion dollars more is Western Europe's share than Latin America's share?

164. In January 2001, the Environmental Protection Agency (EPA) required all 54,000 community water systems in the United States to reduce the amount of arsenic in drinking water by 40 parts per billion, from 50 parts per billion to 10 parts per billion. What percent decrease does this represent?

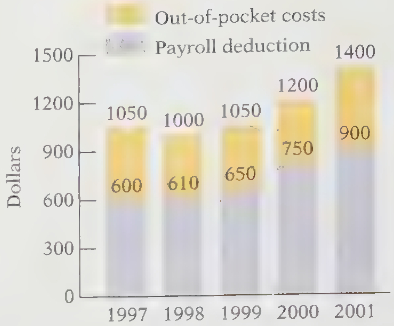


Source: Reprinted with permission from Pete Chenard and *The San Diego Union-Tribune*. As appeared on March 2, 2001.

165. The circle graph at the right shows the responses to a question about downloading music from Napster. How many people were surveyed?

166. To override a presidential veto, at least $66\frac{2}{3}\%$ of the Senate must vote to override the veto. There are 100 senators in the Senate. What is the minimum number of votes needed to override a veto?

167. The graph at the right shows the average health care costs paid by an employee in the United States. What percent of the total average health care cost was out-of-pocket costs for 2001? Round to the nearest tenth of a percent.



Source: Hewitt Associates in the *New York Times*, December 10, 2000

168. In 2000, Ford Motor Company sold about 7.4 million vehicles and had automotive revenues of \$141 billion. This was up from 7.2 million vehicles in 1999 and revenues of \$135 billion. (*Source*: **www.ford.com/2000annualreport**)

a. Which represents the greater percent increase: vehicle sales or revenues?

b. Ford Motor Company set as a goal for 2001 to increase revenues over year-2000 revenues by \$5 billion. Is this a larger or a smaller percent increase than the increase from 1999 revenues to 2000 revenues?

169. Suppose that 125 million people saw a particular 30-second commercial for a soft drink during SuperBowl XXXV. The cost for that commercial was approximately \$2.2 million.

a. If the soft drink manufacturer makes a profit of \$.04 on each can sold, what percent of the people watching the commercial would have to buy one can of the soft drink for the company to recover the cost of the commercial?

b. What percent of the people watching the commercial would have to purchase one 6-pack of the soft drink for the company to recover the cost of the commercial? Round to the nearest tenth of a percent.

APPLYING THE CONCEPTS

170. Solve the equation $ax = b$ for x. Is the solution you have written valid for all real numbers a and b?

171. **a.** Make up an equation of the form $x + a = b$ that has 2 as a solution.
b. Make up an equation of the form $ax = b$ that has -1 as a solution.

172. Solve. **a.** $\dfrac{3}{\frac{1}{x}} = 5$ **b.** $\dfrac{2}{\frac{1}{y}} = -2$ **c.** $\dfrac{3x + 2x}{3} = 2$

173. Write out the steps for solving the equation $\frac{1}{2}x = -3$. Identify each Property of Real Numbers or Property of Equations as you use it.

174. In your own words, state the Addition Property and the Multiplication Property of Equations.

175. If a quantity increases by 100%, how many times its original value is the new value?

3.2 General Equations—Part I

[illegible]

To solve an equation of the form $ax + b = c$

In solving an equation of the form $ax + b = c$, the goal is to rewrite the equation in the form *variable = constant*. This requires the application of both the Addition and the Multiplication Properties of Equations.

Solve: $\frac{3}{4}x - 2 = -11$

The goal is to write the equation in the form *variable = constant*.

$$\frac{3}{4}x - 2 = -11$$

$$\frac{3}{4}x - 2 + 2 = -11 + 2$$

- Add 2 to each side of the equation.

$$\frac{3}{4}x = -9$$

- Simplify.

$$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3}(-9)$$

- Multiply each side of the equation by $\frac{4}{3}$.

$$x = -12$$

- The equation is in the form *variable = constant*.

The solution is −12.

TAKE NOTE

Check: $\frac{3}{4}x - 2 = -11$

$$\begin{array}{c|c} \frac{3}{4}(-12) - 2 & -11 \\ -9 - 2 & -11 \\ \multicolumn{2}{c}{-11 = -11} \end{array}$$

A true equation

Here is an example of solving an equation that contains more than one fraction.

Solve: $\frac{2}{3}x + \frac{1}{2} = \frac{3}{4}$

$$\frac{2}{3}x + \frac{1}{2} = \frac{3}{4}$$

$$\frac{2}{3}x + \frac{1}{2} - \frac{1}{2} = \frac{3}{4} - \frac{1}{2}$$

- Subtract $\frac{1}{2}$ from each side of the equation.

$$\frac{2}{3}x = \frac{1}{4}$$

- Simplify.

$$\frac{3}{2}\left(\frac{2}{3}x\right) = \frac{3}{2}\left(\frac{1}{4}\right)$$

- Multiply each side of the equation by $\frac{3}{2}$, the reciprocal of $\frac{2}{3}$.

$$x = \frac{3}{8}$$

The solution is $\frac{3}{8}$.

It may be easier to solve an equation containing two or more fractions by multiplying each side of the equation by the least common multiple (LCM) of the denominators. For the equation above, the LCM of 3, 2, and 4 is 12. The LCM has the property that 3, 2, and 4 will divide evenly into it. Therefore, if both sides of the equation are multiplied by 12, the denominators will divide evenly into 12. The result is an equation that does not contain any fractions. Multiplying each side of an equation that contains fractions by the LCM of the denominators is called **clearing denominators.** It is an alternative method, as we show in the next example, of solving an equation that contains fractions.

Solve: $\frac{2}{3}x + \frac{1}{2} = \frac{3}{4}$

$$\frac{2}{3}x + \frac{1}{2} = \frac{3}{4}$$

$$12\left(\frac{2}{3}x + \frac{1}{2}\right) = 12\left(\frac{3}{4}\right)$$

- Multiply each side of the equation by 12, the LCM of 3, 2, and 4.

$$12\left(\frac{2}{3}x\right) + 12\left(\frac{1}{2}\right) = 12\left(\frac{3}{4}\right)$$

- Use the Distributive Property.

$$8x + 6 = 9$$

- Simplify.

$$8x + 6 - 6 = 9 - 6$$

- Subtract 6 from each side of the equation.

$$8x = 3$$

$$\frac{8x}{8} = \frac{3}{8}$$

- Divide each side of the equation by 8.

$$x = \frac{3}{8}$$

The solution is $\frac{3}{8}$.

TAKE NOTE

Observe that after we multiply by the LCM and simplify, the equation no longer contains fractions.

Note that both methods give exactly the same solution. You may use either method to solve an equation containing fractions.

Example 1 Solve: $3x - 7 = -5$

Solution

$$\begin{aligned} 3x - 7 &= -5 \\ 3x - 7 + 7 &= -5 + 7 \\ 3x &= 2 \\ \frac{3x}{3} &= \frac{2}{3} \\ x &= \frac{2}{3} \end{aligned}$$

The solution is $\frac{2}{3}$.

You Try It 1 Solve: $5x + 7 = 10$

Your solution

Example 2 Solve: $5 = 9 - 2x$

Solution

$$\begin{aligned} 5 &= 9 - 2x \\ 5 - 9 &= 9 - 9 - 2x \\ -4 &= -2x \\ \frac{-4}{-2} &= \frac{-2x}{-2} \\ 2 &= x \end{aligned}$$

The solution is 2.

You Try It 2 Solve: $2 = 11 + 3x$

Your solution

Solutions on pp.S4–S5

Example 3

Solve: $\frac{2}{3} - \frac{x}{2} = \frac{3}{4}$

Solution

$$\frac{2}{3} - \frac{x}{2} = \frac{3}{4}$$
$$\frac{2}{3} - \frac{2}{3} - \frac{x}{2} = \frac{3}{4} - \frac{2}{3}$$
$$-\frac{x}{2} = \frac{1}{12}$$
$$-2\left(-\frac{x}{2}\right) = -2\left(\frac{1}{12}\right)$$
$$x = \frac{1}{6}$$

The solution is $\frac{1}{6}$.

You Try It 3

Solve: $\frac{5}{8} - \frac{2x}{3} = \frac{5}{4}$

Your solution

Example 4

Solve $\frac{4}{5}x - \frac{1}{2} = \frac{3}{4}$ by first clearing denominators.

Solution The LCM of 5, 2, and 4 is 20.

$$\frac{4}{5}x - \frac{1}{2} = \frac{3}{4}$$
$$20\left(\frac{4}{5}x - \frac{1}{2}\right) = 20\left(\frac{3}{4}\right)$$
$$20\left(\frac{4}{5}x\right) - 20\left(\frac{1}{2}\right) = 20\left(\frac{3}{4}\right)$$
$$16x - 10 = 15$$
$$16x - 10 + 10 = 15 + 10$$
$$16x = 25$$
$$\frac{16x}{16} = \frac{25}{16}$$
$$x = \frac{25}{16}$$

The solution is $\frac{25}{16}$.

You Try It 4

Solve $\frac{2}{3}x + 3 = \frac{7}{2}$ by first clearing denominators.

Your solution

Solutions on p.S5

Example 5

Solve: $2x + 4 - 5x = 10$

Solution

$$\begin{aligned} 2x + 4 - 5x &= 10 \\ -3x + 4 &= 10 \\ -3x + 4 - 4 &= 10 - 4 \\ -3x &= 6 \\ \frac{-3x}{-3} &= \frac{6}{-3} \\ x &= -2 \end{aligned}$$

- Combine like terms.

The solution is −2.

You Try It 5

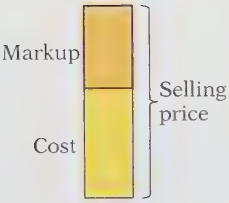
Solve: $x - 5 + 4x = 25$

Your solution

Solution on p. S5

Objective B To solve application problems using formulas

In this objective we will be solving application problems using formulas. Two of the formulas we will use are related to markup and discount.



Cost is the price a business pays for a product. **Selling price** is the price for which a business sells a product to a customer. The difference between selling price and cost is called **markup.** Markup is added to the cost to cover the expenses of operating a business. The diagram at the left illustrates these terms. The total length is the selling price. One part of the diagram is the cost, and the other part is the markup.

When the markup is expressed as a percent of the retailer's cost, it is called the **markup rate.**

The basic markup equations used by a business are

$$\begin{array}{ccccccc} \text{Selling price} & = & \text{cost} & + & \text{markup} \\ S & = & C & + & M \end{array} \qquad \begin{array}{ccccc} \text{Markup} & = & \text{markup rate} & \cdot & \text{cost} \\ M & = & r & \cdot & C \end{array}$$

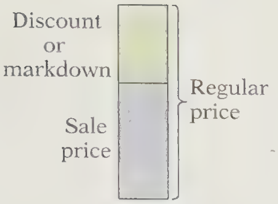
Substituting $r \cdot C$ for M in the first equation results in $S = C + (r \cdot C)$, or $S = C + rC$.

➡ The manager of a clothing store buys a jacket for \$80 and sells the jacket for \$116. Find the markup rate.

$$\begin{aligned} S &= C + rC \\ 116 &= 80 + 80r \\ 36 &= 80r \\ \frac{36}{80} &= \frac{80r}{80} \\ 0.45 &= r \end{aligned}$$

- Use the equation $S = C + rC$.
- Given: $C = \$80$ and $S = \$116$
- Subtract 80 from each side of the equation.
- Divide both sides of the equation by 80.

The markup rate is 45%.



A retailer may reduce the regular price of a product because the goods are damaged, odd sizes, or discontinued items. The **discount,** or **markdown,** is the amount by which a retailer reduces the regular price of a product. The percent discount is called the **discount rate** and is usually expressed as a percent of the original selling price (the regular price).

The basic discount equations used by a business are

$$\text{Sale price} = \text{regular price} - \text{discount} \qquad \text{Discount} = \text{discount rate} \cdot \text{regular price}$$

$$S = R - D \qquad D = r \cdot R$$

Substituting $r \cdot R$ for D in the first equation results in $S = R - (r \cdot R)$, or $S = R - rR$.

A portable computer that regularly sells for \$1850 is on sale for \$1480. Find the discount rate.

$$S = R - rR$$
- Use the equation $S = R - rR$.

$$1480 = 1850 - 1850r$$
- Given: $S =$ \$1480 and $R =$ \$1850

$$-370 = -1850r$$
- Subtract 1850 from each side of the equation.

$$\frac{-370}{-1850} = \frac{-1850r}{-1850}$$
- Divide each side of the equation by −1850.

$$0.2 = r$$

The discount rate on the portable computer is 20%.

Example 6

A markup rate of 40% was used on a refrigerator that has a selling price of \$749. Find the cost of the refrigerator. Use the formula $S = C + rC$.

Strategy

Given: $S =$ \$749
$r = 40\% = 0.40$
Unknown: C

Solution

$$S = C + rC$$
$$749 = C + 0.40C$$
- $C + 0.40C = 1C + 0.40C$

$$749 = 1.40C$$
- Combine like terms.

$$\frac{749}{1.40} = \frac{1.40C}{1.40}$$
$$535 = C$$

The cost of the refrigerator is \$535.

You Try It 6

A markup rate of 45% was used on an outboard motor that has a selling price of \$986. Find the cost of the outboard motor. Use the formula $S = C + rC$.

Your strategy

Your solution

Solution on p. S5

Example 7

A necklace that is marked down 35% has a sale price of \$292.50. Find the regular price of the necklace. Use the formula $S = R - rR$.

Strategy

Given: $S = 292.50$

$r = 35\% = 0.35$

Unknown: R

Solution

$$\begin{aligned}
S &= R - rR \\
292.50 &= R - 0.35R & &\bullet\ R - 0.35R = 1R - 0.35R \\
292.50 &= 0.65R & &\bullet\ \text{Combine like terms.} \\
\frac{292.50}{0.65} &= \frac{0.65R}{0.65} \\
450 &= R
\end{aligned}$$

The regular price of the necklace is \$450.

You Try It 7

A garage door opener, marked down 25%, is on sale for \$159. Find the regular price of the garage door opener. Use the formula $S = R - rR$.

Your strategy

Your solution

Example 8

To determine the total cost of production, an economist uses the equation $T = U \cdot N + F$, where T is the total cost, U is the unit cost, N is the number of units made, and F is the fixed cost. Use this equation to find the number of units made during a month when the total cost was \$9000, the unit cost was \$25, and the fixed cost was \$3000.

Strategy

Given: $T = \$9000$

$U = \$25$

$F = \$3000$

Unknown: N

Solution

$$\begin{aligned}
T &= U \cdot N + F \\
9000 &= 25N + 3000 \\
6000 &= 25N \\
\frac{6000}{25} &= \frac{25N}{25} \\
240 &= N
\end{aligned}$$

There were 240 units made.

You Try It 8

The pressure at a certain depth in the ocean can be approximated by the equation $P = 15 + \frac{1}{2}D$, where P is the pressure in pounds per square inch and D is the depth in feet. Use this equation to find the depth when the pressure is 45 pounds per square inch.

Your strategy

Your solution

Solutions on p. S5

3.2 Exercises

Solve and check.

1. $3x + 1 = 10$
2. $4y + 3 = 11$
3. $2a - 5 = 7$
4. $5m - 6 = 9$

5. $5 = 4x + 9$
6. $2 = 5b + 12$
7. $2x - 5 = -11$
8. $3n - 7 = -19$

9. $4 - 3w = -2$
10. $5 - 6x = -13$
11. $8 - 3t = 2$
12. $12 - 5x = 7$

13. $4a - 20 = 0$
14. $3y - 9 = 0$
15. $6 + 2b = 0$
16. $10 + 5m = 0$

17. $-2x + 5 = -7$
18. $-5d + 3 = -12$
19. $-12x + 30 = -6$
20. $-13 = -11y + 9$

21. $2 = 7 - 5a$
22. $3 = 11 - 4n$
23. $-35 = -6b + 1$
24. $-8x + 3 = -29$

25. $-3m - 21 = 0$
26. $-5x - 30 = 0$
27. $-4y + 15 = 15$
28. $-3x + 19 = 19$

29. $9 - 4x = 6$
30. $3t - 2 = 0$
31. $9x - 4 = 0$
32. $7 - 8z = 0$

33. $1 - 3x = 0$
34. $9d + 10 = 7$
35. $12w + 11 = 5$
36. $6y - 5 = -7$

37. $8b - 3 = -9$
38. $5 - 6m = 2$
39. $7 - 9a = 4$
40. $9 = -12c + 5$

41. $10 = -18x + 7$

42. $2y + \frac{1}{3} = \frac{7}{3}$

43. $4a + \frac{3}{4} = \frac{19}{4}$

44. $2n - \frac{3}{4} = \frac{13}{4}$

45. $3x - \frac{5}{6} = \frac{13}{6}$

46. $5y + \frac{3}{7} = \frac{3}{7}$

47. $9x + \frac{4}{5} = \frac{4}{5}$

48. $8 = 7d - 1$

49. $8 = 10x - 5$

50. $4 = 7 - 2w$

51. $7 = 9 - 5a$

52. $8t + 13 = 3$

53. $12x + 19 = 3$

54. $-6y + 5 = 13$

55. $-4x + 3 = 9$

56. $\frac{1}{2}a - 3 = 1$

57. $\frac{1}{3}m - 1 = 5$

58. $\frac{2}{5}y + 4 = 6$

59. $\frac{3}{4}n + 7 = 13$

60. $-\frac{2}{3}x + 1 = 7$

61. $-\frac{3}{8}b + 4 = 10$

62. $\frac{x}{4} - 6 = 1$

63. $\frac{y}{5} - 2 = 3$

64. $\frac{2x}{3} - 1 = 5$

65. $\frac{2}{3}x - \frac{5}{6} = -\frac{1}{3}$

66. $\frac{5}{4}x + \frac{2}{3} = \frac{1}{4}$

67. $\frac{1}{2} - \frac{2}{3}x = \frac{1}{4}$

68. $\frac{3}{4} - \frac{3}{5}x = \frac{19}{20}$

69. $\frac{3}{2} = \frac{5}{6} + \frac{3x}{8}$

70. $-\frac{1}{4} = \frac{5}{12} + \frac{5x}{6}$

71. $\frac{11}{27} = \frac{4}{9} - \frac{2x}{3}$

72. $\frac{37}{24} = \frac{7}{8} - \frac{5x}{6}$

73. $7 = \frac{2x}{5} + 4$

74. $5 - \frac{4c}{7} = 8$

75. $7 - \frac{5}{9}y = 9$

76. $6a + 3 + 2a = 11$

77. $5y + 9 + 2y = 23$

78. $7x - 4 - 2x = 6$

79. $11z - 3 - 7z = 9$

80. $2x - 6x + 1 = 9$

81. Solve $3x + 4y = 13$ when $y = -2$.

82. Solve $2x - 3y = 8$ when $y = 0$.

83. Solve $-4x + 3y = 9$ when $x = 0$.

84. Solve $5x - 2y = -3$ when $x = -3$.

85. If $2x - 3 = 7$, evaluate $3x + 4$.

86. If $3x + 5 = -4$, evaluate $2x - 5$.

87. If $4 - 5x = -1$, evaluate $x^2 - 3x + 1$.

88. If $2 - 3x = 11$, evaluate $x^2 + 2x - 3$.

89. If $5x + 3 - 2x = 12$, evaluate $4 - 5x$.

90. If $2x - 4 - 7x = 16$, evaluate $x^2 + 1$.

[illegible] *Application Problems*

Solve. Use the markup equation $S = C + rC$, where S is selling price, C is cost, and r is the markup rate.

91. A watch costing $98 is sold for $156.80. Find the markup rate on the watch.

92. A set of golf clubs costing $360 is sold for $630. Find the markup rate on the set of golf clubs.

93. A markup rate of 40% was used on a basketball with a selling price of $82.60. Find the cost of the basketball.

94. A portable tape player with a selling price of $57 has a markup rate of 50%. Find the cost of the tape player.

95. A freezer costing $360 is sold for $520. Find the markup rate. Round to the nearest tenth of a percent.

96. A sofa costing $320 is sold for $479. Find the markup rate. Round to the nearest tenth of a percent.

97. A digitally recorded compact disk has a selling price of $11.90. The markup rate is 40%. Find the cost of the CD.

98. A markup rate of 25% is used on a computer that has a selling price of $2187.50. Find the cost of the computer.

Solve. Use the discount equation $S = R - rR$, where S is the sale price, R is the regular price, and r is the discount rate.

99. An oak bedroom set with a regular price of \$1295 is on sale for \$995. Find the discount rate. Round to the nearest tenth of a percent.

100. A stereo system with a regular price of \$495 is on sale for \$395. Find the markdown rate. Round to the nearest tenth of a percent.

101. A mechanic's tool set is on sale for \$180 after a markdown of 40% off the regular price. Find the regular price.

102. A battery with a discount price of \$65 is on sale for 22% off the regular price. Find the regular price. Round to the nearest cent.

103. A compact disk player with a regular price of \$325 is on sale for \$201.50. Find the markdown rate.

104. A luggage set with a regular price of \$178 is on sale for \$103.24. Find the discount rate.

105. A telescope is on sale for \$165 after a markdown of 40% off the regular price. Find the regular price.

106. An exercise bike is on sale for \$390, having been marked down 25% off the regular price. Find the regular price.

The distance s, in feet, that an object will fall in t seconds is given by $s = 16t^2 + vt$, where v is the initial velocity of the object in feet per second.

107. Find the initial velocity of an object that falls 80 ft in 2 s.

108. Find the initial velocity of an object that falls 144 ft in 3 s.

A company uses the equation $V = C - 6000t$ to determine the depreciated value V, after t years, of a milling machine that originally cost C dollars. Equations like this are used in accounting for straight-line depreciation.

109. A milling machine originally cost \$50,000. In how many years will the depreciated value of the machine be \$38,000?

110. A milling machine originally cost \$78,000. In how many years will the depreciated value of the machine be \$48,000?

Anthropologists approximate the height of a primate by the size of its humerus (the bone from the elbow to the shoulder) using the equation $H = 1.2L + 27.8$, where L is the length of the humerus and H is the height, in inches, of the primate.

111. An anthropologist estimates the height of a primate to be 66 in. What is the approximate length of the humerus of this primate? Round to the nearest tenth of an inch.

112. An anthropologist estimates the height of a primate to be 62 in. What is the approximate length of the humerus of this primate?

Black ice is an ice covering on roads that is especially difficult to see and therefore extremely dangerous for motorists. The distance that a car traveling 30 mph will slide after its brakes are applied is related to the outside temperature by the formula $C = \frac{1}{4}D - 45$, where C is the Celsius temperature and D is the distance in feet that the car will slide.

113. Determine the distance a car will slide on black ice when the outside temperature is -3°C.

114. Determine the distance a car will slide on black ice when the outside temperature is -11°C.

A telephone company estimates that the number N of phone calls per day between two cities of populations P_1 and P_2 that are d miles apart is given by the equation $N = \frac{2.51P_1P_2}{d^2}$.

115. Estimate the population (P_1) of a city, given that the population of a second city (P_2) is 48,000, the number of phone calls per day between the two cities is 1,100,000, and the distance between the cities is 75 mi. Round to the nearest thousand.

116. Estimate the population (P_1) of a city given that the population of a second city (P_2) is 125,000, the number of phone calls per day between the two cities is 2,500,000, and the distance between the cities is 50 mi. Round to the nearest thousand.

The world record time for a 1-mile race can be approximated by the equation $t = 17.08 - 0.0067y$, where t is the time in minutes and y is the year of the race.

117. Approximate the year in which the first "4-minute mile" was run. (The actual year was 1954.) Round to the nearest whole number.

118. In 1985, the world record for a 1-mile race was 3.77 min. For what year does the equation predict this record time? Round to the nearest whole number.

APPLYING THE CONCEPTS

119. A pair of shoes that now sells for \$63 has been marked up 40%. Find the markup on the pair of shoes.

120. The sale price of a typewriter is 25% off the regular price. The discount is \$70. Find the sale price.

121. The sale price of a television was \$180. Find the regular price if the sale price was computed by taking one-third off the regular price, followed by an additional 15% discount on the reduced price.

122. A customer buys four tires, three at the regular price and one for 20% off the regular price. The four tires cost \$323. What was the regular price of a tire?

123. Solve: $x \div 15 = 25$ remainder 10

124. Does the sentence, "Solve $3x - 4(x - 1)$" make sense? Why or why not?

125. Explain what is wrong with the following demonstration, which suggests that $2 = 3$.

$$2x + 5 = 3x + 5$$

$$2x + 5 - 5 = 3x + 5 - 5$$

• Subtract 5 from each side of the equation.

$$2x = 3x$$

$$2 = 3$$

• Divide each side of the equation by x.

126. The following problem does not contain enough information for us to find only one solution. Supply some additional information so that the problem has exactly one solution. Then write and solve an equation.

The sum of two numbers is 15. Find the numbers.

127. The following problem does not contain enough information. What additional information is needed to answer the question?

How many hours does it take to fly from Los Angeles to New York?

3.3

General Equations—Part II

To solve an equation of the form $ax + b = cx + d$

In solving an equation of the form $ax + b = cx + d$, the goal is to rewrite the equation in the form *variable = constant*. Begin by rewriting the equation so that there is only one variable term in the equation. Then rewrite the equation so that there is only one constant term.

Solve: $2x + 3 = 5x - 9$

$$2x + 3 = 5x - 9$$

$$2x - 5x + 3 = 5x - 5x - 9$$
- Subtract $5x$ from each side of the equation.

$$-3x + 3 = -9$$
- Simplify. There is only one variable term.

$$-3x + 3 - 3 = -9 - 3$$
- Subtract 3 from each side of the equation.

$$-3x = -12$$
- Simplify. There is only one constant term.

$$\frac{-3x}{-3} = \frac{-12}{-3}$$
- Divide each side of the equation by -3.

$$x = 4$$
- The equation is in the form *variable = constant*.

The solution is 4. You should verify this by checking this solution.

Example 1 Solve: $4x - 5 = 8x - 7$

Solution

$$4x - 5 = 8x - 7$$
$$4x - 8x - 5 = 8x - 8x - 7$$
$$-4x - 5 = -7$$
$$-4x - 5 + 5 = -7 + 5$$
$$-4x = -2$$
$$\frac{-4x}{-4} = \frac{-2}{-4}$$
$$x = \frac{1}{2}$$

The solution is $\frac{1}{2}$.

You Try It 1 Solve: $5x + 4 = 6 + 10x$

Your solution

Solution on p. S5

Example 2 Solve: $3x + 4 - 5x = 2 - 4x$

Solution

$$\begin{aligned} 3x + 4 - 5x &= 2 - 4x \\ -2x + 4 &= 2 - 4x \\ -2x + 4x + 4 &= 2 - 4x + 4x \\ 2x + 4 &= 2 \\ 2x + 4 - 4 &= 2 - 4 \\ 2x &= -2 \\ \frac{2x}{2} &= \frac{-2}{2} \\ x &= -1 \end{aligned}$$

The solution is -1.

You Try It 2 Solve: $5x - 10 - 3x = 6 - 4x$

Your solution

Solution on pp.S5–S6

Objective B To solve an equation containing parentheses 3

When an equation contains parentheses, one of the steps in solving the equation requires the use of the Distributive Property. The Distributive Property is used to remove parentheses from a variable expression.

⇨ Solve: $4 + 5(2x - 3) = 3(4x - 1)$

$$\begin{aligned} 4 + 5(2x - 3) &= 3(4x - 1) \\ 4 + 10x - 15 &= 12x - 3 && \text{• Use the Distributive Property. Then simplify.} \\ 10x - 11 &= 12x - 3 \\ 10x - 12x - 11 &= 12x - 12x - 3 && \text{• Subtract } 12x \text{ from each side of the equation.} \\ -2x - 11 &= -3 && \text{• Simplify.} \\ -2x - 11 + 11 &= -3 + 11 && \text{• Add 11 to each side of the equation.} \\ -2x &= 8 && \text{• Simplify.} \\ \frac{-2x}{-2} &= \frac{8}{-2} && \text{• Divide each side of the equation by } -2. \\ x &= -4 && \text{• The equation is in the form } \textit{variable} = \textit{constant}. \end{aligned}$$

The solution is -4. You should verify this by checking this solution.

Some application problems result in an equation with parentheses and decimal numbers.

Solve: $16 + 0.55x = 0.75(x + 20)$

$$\begin{aligned}
16 + 0.55x &= 0.75(x + 20) \\
16 + 0.55x &= 0.75x + 15 \\
16 + 0.55x - 0.75x &= 0.75x - 0.75x + 15 \\
16 - 0.20x &= 15 \\
16 - 16 - 0.20x &= 15 - 16 \\
-0.20x &= -1 \\
\frac{-0.20x}{-0.20} &= \frac{-1}{-0.20} \\
x &= 5
\end{aligned}$$

- Use the Distributive Property.
- Subtract $0.75x$ from each side of the equation.
- Simplify.
- Subtract 16 from each side of the equation.
- Simplify.
- Divide each side of the equation by -0.20.
- The equation is in the form *variable = constant*.

The solution is 5.

Example 3

Solve: $3x - 4(2 - x) = 3(x - 2) - 4$

Solution

$$\begin{aligned}
3x - 4(2 - x) &= 3(x - 2) - 4 \\
3x - 8 + 4x &= 3x - 6 - 4 \\
7x - 8 &= 3x - 10 \\
7x - 3x - 8 &= 3x - 3x - 10 \\
4x - 8 &= -10 \\
4x - 8 + 8 &= -10 + 8 \\
4x &= -2 \\
\frac{4x}{4} &= \frac{-2}{4} \\
x &= -\frac{1}{2}
\end{aligned}$$

The solution is $-\frac{1}{2}$.

You Try It 3

Solve: $5x - 4(3 - 2x) = 2(3x - 2) + 6$

Your solution

Example 4

Solve: $3[2 - 4(2x - 1)] = 4x - 10$

Solution

$$\begin{aligned}
3[2 - 4(2x - 1)] &= 4x - 10 \\
3[2 - 8x + 4] &= 4x - 10 \\
3[6 - 8x] &= 4x - 10 \\
18 - 24x &= 4x - 10 \\
18 - 24x - 4x &= 4x - 4x - 10 \\
18 - 28x &= -10 \\
18 - 18 - 28x &= -10 - 18 \\
-28x &= -28 \\
\frac{-28x}{-28} &= \frac{-28}{-28} \\
x &= 1
\end{aligned}$$

The solution is 1.

You Try It 4

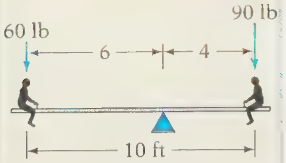
Solve: $-2[3x - 5(2x - 3)] = 3x - 8$

Your solution

Solutions on p.S6

To solve application problems using formulas

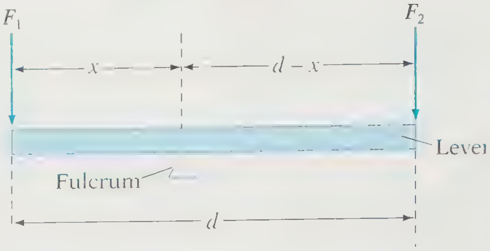
TAKE NOTE



This system balances because

$$
\begin{aligned}
F_1x &= F_2(d-x)\\
60(6) &= 90(10-6)\\
60(6) &= 90(4)\\
360 &= 360
\end{aligned}
$$

A lever system is shown at the right. It consists of a lever, or bar; a fulcrum; and two forces, F_1 and F_2. The distance d represents the length of the lever, x represents the distance from F_1 to the fulcrum, and $d - x$ represents the distance from F_2 to the fulcrum.



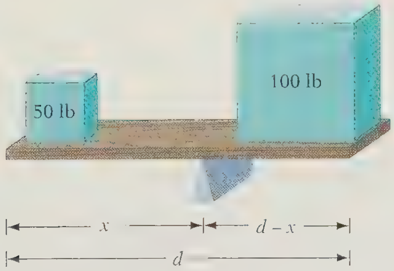
A principle of physics states that when the lever system balances, $F_1x = F_2(d - x)$.

Example 5

A lever is 15 ft long. A force of 50 lb is applied to one end of the lever, and a force of 100 lb is applied to the other end. Where is the fulcrum located when the system balances?

Strategy

Make a drawing.



Given: $F_1 = 50$

$F_2 = 100$

$d = 15$

Unknown: x

Solution

$$
\begin{aligned}
F_1x &= F_2(d-x)\\
50x &= 100(15-x)\\
50x &= 1500-100x\\
50x+100x &= 1500-100x+100x\\
150x &= 1500\\
\frac{150x}{150} &= \frac{1500}{150}\\
x &= 10
\end{aligned}
$$

The fulcrum is 10 ft from the 50-pound force.

You Try It 5

A lever is 25 ft long. A force of 45 lb is applied to one end of the lever, and a force of 80 lb is applied to the other end. Where is the location of the fulcrum when the system balances?

Your strategy

Your solution

Solution on p. S6

3.3 Exercises

[illegible]

Solve and check.

1. $8x + 5 = 4x + 13$

2. $6y + 2 = y + 17$

3. $5x - 4 = 2x + 5$

4. $13b - 1 = 4b - 19$

5. $15x - 2 = 4x - 13$

6. $7a - 5 = 2a - 20$

7. $3x + 1 = 11 - 2x$

8. $n - 2 = 6 - 3n$

9. $2x - 3 = -11 - 2x$

10. $4y - 2 = -16 - 3y$

11. $2b + 3 = 5b + 12$

12. $m + 4 = 3m + 8$

13. $4y - 8 = y - 8$

14. $5a + 7 = 2a + 7$

15. $6 - 5x = 8 - 3x$

16. $10 - 4n = 16 - n$

17. $5 + 7x = 11 + 9x$

18. $3 - 2y = 15 + 4y$

19. $2x - 4 = 6x$

20. $2b - 10 = 7b$

21. $8m = 3m + 20$

22. $9y = 5y + 16$

23. $8b + 5 = 5b + 7$

24. $6y - 1 = 2y + 2$

25. $7x - 8 = x - 3$

26. $2y - 7 = -1 - 2y$

27. $2m - 1 = -6m + 5$

28. If $5x = 3x - 8$, evaluate $4x + 2$.

29. If $7x + 3 = 5x - 7$, evaluate $3x - 2$.

30. If $2 - 6a = 5 - 3a$, evaluate $4a^2 - 2a + 1$.

31. If $1 - 5c = 4 - 4c$, evaluate $3c^2 - 4c + 2$.

32. If $2y + 3 = 5 - 4y$, evaluate $6y - 7$.

33. If $3z + 1 = 1 - 5z$, evaluate $3z^2 - 7z + 8$.

Solve and check.

34. $5x + 2(x + 1) = 23$

35. $6y + 2(2y + 3) = 16$

36. $9n - 3(2n - 1) = 15$

37. $12x - 2(4x - 6) = 28$

38. $7a - (3a - 4) = 12$

39. $9m - 4(2m - 3) = 11$

40. $5(3 - 2y) + 4y = 3$

41. $4(1 - 3x) + 7x = 9$

42. $5y - 3 = 7 + 4(y - 2)$

43. $0.22(x + 6) = 0.2x + 1.8$

44. $0.05(4 - x) + 0.1x = 0.32$

45. $0.3x + 0.3(x + 10) = 300$

46. $2a - 5 = 4(3a + 1) - 2$

47. $5 - (9 - 6x) = 2x - 2$

48. $7 - (5 - 8x) = 4x + 3$

49. $3[2 - 4(y - 1)] = 3(2y + 8)$

50. $5[2 - (2x - 4)] = 2(5 - 3x)$

51. $3a + 2[2 + 3(a - 1)] = 2(3a + 4)$

52. $5 + 3[1 + 2(2x - 3)] = 6(x + 5)$

53. $-2[4 - (3b + 2)] = 5 - 2(3b + 6)$

54. $-4[x - 2(2x - 3)] + 1 = 2x - 3$

55. If $4 - 3a = 7 - 2(2a + 5)$, evaluate $a^2 + 7a$.

56. If $9 - 5x = 12 - (6x + 7)$, evaluate $x^2 - 3x - 2$.

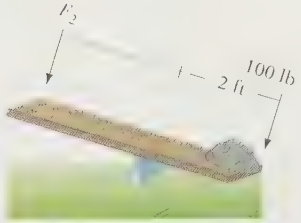
57. If $2z - 5 = 3(4z + 5)$, evaluate $\frac{z^2}{z - 2}$.

58. If $3n - 7 = 5(2n + 7)$, evaluate $\frac{n^2}{2n - 6}$.

Application Problems

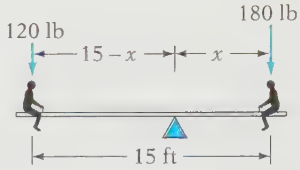
Solve. Use the lever system equation $F_1x = F_2(d - x)$.

59. A lever 10 ft long is used to move a 100-pound rock. The fulcrum is placed 2 ft from the rock. What force must be applied to the other end of the lever to move the rock?



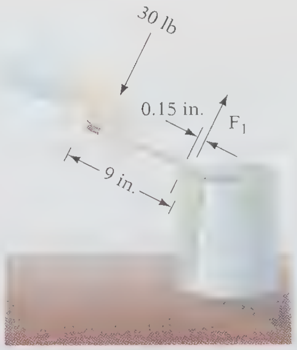
60. An adult and a child are on a seesaw 14 ft long. The adult weighs 175 lb and the child weighs 70 lb. How many feet from the child must the fulcrum be placed so that the seesaw balances?

61. Two people are sitting 15 ft apart on a seesaw. One person weighs 180 lb. The second person weighs 120 lb. How far from the 180-pound person should the fulcrum be placed so that the seesaw balances?



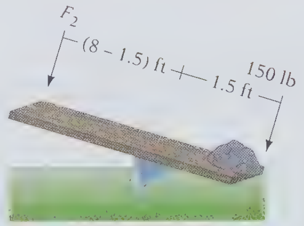
62. Two children are sitting on a seesaw that is 12 ft long. One child weighs 60 lb. The other child weighs 90 lb. How far from the 90-pound child should the fulcrum be placed so that the seesaw balances?

63. In preparation for a stunt, two acrobats are standing on a plank 18 ft long. One acrobat weighs 128 lb and the second acrobat weighs 160 lb. How far from the 128-pound acrobat must the fulcrum be placed so that the acrobats are balanced on the plank?



64. A screwdriver 9 in. long is used as a lever to open a can of paint. The tip of the screwdriver is placed under the lip of the can with the fulcrum 0.15 in. from the lip. A force of 30 lb is applied to the other end of the screwdriver. Find the force on the lip of the can.

65. A metal bar 8 ft long is used to move a 150-pound rock. The fulcrum is placed 1.5 ft from the rock. What minimum force must be applied to the other end of the bar to move the rock? Round to the nearest tenth.



To determine the breakeven point, or the number of units that must be sold so that no profit or loss occurs, an economist uses the formula $Px = Cx + F$, where P is the selling price per unit, x is the number of units that must be sold to break even, C is the cost to make each unit, and F is the fixed cost.

66. A business analyst has determined that the selling price per unit for a laser printer is \$1600. The cost to make the laser printer is \$950, and the fixed cost is \$211,250. Find the breakeven point.

67. A business analyst has determined that the selling price per unit for a gas barbecue is \$325. The cost to make one gas barbecue is \$175, and the fixed cost is \$39,000. Find the breakeven point.

68. A manufacturer of thermostats determines that the cost per unit for a programmable thermostat is \$38 and that the fixed cost is \$24,400. The selling price for the thermostat is \$99. Find the breakeven point.

69. A manufacturing engineer determines that the cost per unit for a desk lamp is \$12 and that the fixed cost is \$19,240. The selling price for the desk lamp is \$49. Find the breakeven point.

70. A manufacturing engineer determines the cost to make one compact disc to be \$3.35 and the fixed cost to be \$6180. The selling price for each compact disc is \$8.50. Find the number of compact discs that must be sold to break even.

71. To manufacture a softball bat requires two steps. The first step is to cut a rough shape. The second step is to sand the bat to its final form. The cost to rough-shape a bat is \$.45, and the cost to sand a bat to final form is \$1.05. The total fixed cost for the two steps is \$16,500. How many softball bats must be sold at a price of \$7.00 to break even?

APPLYING THE CONCEPTS

72. Write an equation of the form $ax + b = cx + d$ that has 4 as the solution.

Solve. If the equation has no solution, write "no solution."

73. $3(2x - 1) - (6x - 4) = -9$

74. $7(3x + 6) - 4(3 + 5x) = 13 + x$

75. $\frac{1}{5}(25 - 10a) + 4 = \frac{1}{3}(12a - 15) + 14$

76. $5[m + 2(3 - m)] = 3[2(4 - m) - 5]$

77. The equation $x = x + 1$ has no solution, whereas the solution of the equation $2x + 3 = 3$ is zero. Is there a difference between no solution and a solution of zero? Explain your answer.

78. Archimedes supposedly said, "Give me a long enough lever and I can move the world." Explain what Archimedes meant by that statement.

3.4 Translating Sentences into Equations

To solve integer problems

An equation states that two mathematical expressions are equal. Therefore, to **translate** a sentence into an equation requires recognition of the words or phrases that mean "equals." Some of these phrases are listed below.

$$\left.\begin{array}{l}\text{equals}\\ \text{is}\\ \text{is equal to}\\ \text{amounts to}\\ \text{represents}\end{array}\right\}\ \text{translate to } =$$

Once the sentence is translated into an equation, the equation can be solved by rewriting the equation in the form *variable* = *constant*.

TAKE NOTE
You can check the solution to a translation problem.

Check:

$$\begin{array}{r|l} \multicolumn{2}{l}{\underline{\text{5 less than 18 is 13}}} \\ 18 - 5 & 13 \\ 13 = & 13 \end{array}$$

Translate "five less than a number is thirteen" into an equation and solve.

The unknown number: n

- Assign a variable to the unknown number.

Five less than a number	is	thirteen

- Find two verbal expressions for the same value.

$$n - 5 \quad = \quad 13$$

- Write a mathematical expression for each verbal expression. Write the equals sign.

$$n - 5 + 5 = 13 + 5$$

- Solve the equation.

$$n = 18$$

The number is 18.

Recall that the integers are the numbers $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$. An **even integer** is an integer that is divisible by 2. Examples of even integers are -8, 0, and 22. An **odd integer** is an integer that is not divisible by 2. Examples of odd integers are -17, 1, and 39.

Consecutive integers are integers that follow one another in order. Examples of consecutive integers are shown at the right. (Assume that the variable n represents an integer.)

11, 12, 13
$-8, -7, -6$
$n, n + 1, n + 2$

Examples of **consecutive even integers** are shown at the right. (Assume that the variable n represents an even integer.)

24, 26, 28
$-10, -8, -6$
$n, n + 2, n + 4$

TAKE NOTE
Both consecutive even and consecutive odd integers are represented using $n, n + 2, n + 4, \ldots$.

Examples of **consecutive odd integers** are shown at the right. (Assume that the variable n represents an odd integer.)

19, 21, 23
$-1, 1, 3$
$n, n + 2, n + 4$

The sum of three consecutive odd integers is forty-five. Find the integers.

Strategy

- First odd integer: n
  Second odd integer: $n + 2$
  Third odd integer: $n + 4$
- The sum of the three odd integers is 45.

- Represent three consecutive odd integers.

Solution

$$n + (n + 2) + (n + 4) = 45$$

- Write an equation.

$$3n + 6 = 45$$

- Solve the equation.

$$3n = 39$$

$$n = 13$$

- The first odd integer is 13.

$$n + 2 = 13 + 2 = 15$$

- Find the second odd integer.

$$n + 4 = 13 + 4 = 17$$

- Find the third odd integer.

The three consecutive odd integers are 13, 15, and 17.

Example 1

The sum of two numbers is sixteen. The difference between four times the smaller number and two is two more than twice the larger number. Find the two numbers.

Solution

The smaller number: n
The larger number: $16 - n$

The difference between four times the smaller and two	is	two more than twice the larger

$$\begin{aligned}
4n - 2 &= 2(16 - n) + 2 \\
4n - 2 &= 32 - 2n + 2 \\
4n - 2 &= 34 - 2n \\
4n + 2n - 2 &= 34 - 2n + 2n \\
6n - 2 &= 34 \\
6n - 2 + 2 &= 34 + 2 \\
6n &= 36 \\
\frac{6n}{6} &= \frac{36}{6} \\
n &= 6
\end{aligned}$$

$$16 - n = 16 - 6 = 10$$

The smaller number is 6.
The larger number is 10.

You Try It 1

The sum of two numbers is twelve. The total of three times the smaller number and six amounts to seven less than the product of four and the larger number. Find the two numbers.

Your solution

Solution on p. S6

Example 2

Find three consecutive even integers such that three times the second equals four more than the sum of the first and third.

Strategy

- First even integer: n
  Second even integer: $n + 2$
  Third even integer: $n + 4$
- Three times the second equals four more than the sum of the first and third.

Solution

$$\begin{aligned}3(n + 2) &= n + (n + 4) + 4\\ 3n + 6 &= 2n + 8\\ 3n - 2n + 6 &= 2n - 2n + 8\\ n + 6 &= 8\\ n &= 2\\ n + 2 = 2 + 2 &= 4\\ n + 4 = 2 + 4 &= 6\end{aligned}$$

The three integers are 2, 4, and 6.

You Try It 2

Find three consecutive integers whose sum is negative six.

Your strategy

Your solution

Solution on p. S6

OBJECTIVE B To translate a sentence into an equation and solve

3

Example 3

A wallpaper hanger charges a fee of \$25 plus \$12 for each roll of wallpaper used in a room. If the total charge for hanging wallpaper is \$97, how many rolls of wallpaper were used?

Strategy

To find the number of rolls of wallpaper used, write and solve an equation using n to represent the number of rolls of wallpaper used.

Solution

$$\boxed{\text{\$25 plus \$12 for each roll of wallpaper}}\ \text{is}\ \boxed{\text{\$97}}$$

$$\begin{aligned}25 + 12n &= 97\\ 12n &= 72\\ \frac{12n}{12} &= \frac{72}{12}\\ n &= 6\end{aligned}$$

6 rolls of wallpaper were used.

You Try It 3

The fee charged by a ticketing agency for a concert is \$3.50 plus \$17.50 for each ticket purchased. If your total charge for tickets is \$161, how many tickets are you purchasing?

Your strategy

Your solution

Solution on p. S7

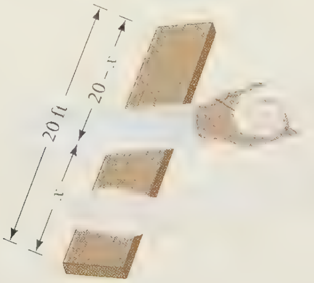
Example 4

A board 20 ft long is cut into two pieces. Five times the length of the shorter piece is 2 ft more than twice the length of the longer piece. Find the length of each piece.

Strategy

Let x represent the length of the shorter piece. Then $20 - x$ represents the length of the longer piece.

Make a drawing.



To find the lengths, write and solve an equation using x to represent the length of the shorter piece and $20 - x$ to represent the length of the longer piece.

Solution

$$\boxed{\text{Five times the length of the shorter piece}} \text{ is } \boxed{\text{2 ft more than twice the length of the longer}}$$

$$\begin{aligned}
5x &= 2(20 - x) + 2 \\
5x &= 40 - 2x + 2 \\
5x &= 42 - 2x \\
5x + 2x &= 42 - 2x + 2x \\
7x &= 42 \\
\frac{7x}{7} &= \frac{42}{7} \\
x &= 6
\end{aligned}$$

$20 - x = 20 - 6 = 14$

The length of the shorter piece is 6 ft.
The length of the longer piece is 14 ft.

You Try It 4

A wire 22 in. long is cut into two pieces. The length of the longer piece is 4 in. more than twice the length of the shorter piece. Find the length of each piece.

Your strategy

Your solution

Solution on p. S7

3.4 Exercises

[illegible]

In Exercises 1 to 19, translate into an equation and solve.

1. The difference between a number and fifteen is seven. Find the number.

2. The sum of five and a number is three. Find the number.

3. The product of seven and a number is negative twenty-one. Find the number.

4. The quotient of a number and four is two. Find the number.

5. The difference between nine and a number is seven. Find the number.

6. Three-fifths of a number is negative thirty. Find the number.

7. The difference between five and twice a number is one. Find the number.

8. Four more than three times a number is thirteen. Find the number.

9. The sum of twice a number and five is fifteen. Find the number.

10. The difference between nine times a number and six is twelve. Find the number.

11. Six less than four times a number is twenty-two. Find the number.

12. Four times the sum of twice a number and three is twelve. Find the number.

13. Three times the difference between four times a number and seven is fifteen. Find the number.

14. Twelve is six times the difference between a number and three. Find the number.

15. Twice the difference between a number and twenty-five is three times the number. Find the number.

16. The sum of two numbers is twenty. Three times the smaller is equal to two times the larger. Find the two numbers.

17. The sum of two numbers is fifteen. One less than three times the smaller is equal to the larger. Find the two numbers.

18. The sum of two numbers is fourteen. The difference between two times the smaller and the larger is one. Find the two numbers.

19. The sum of two numbers is eighteen. The total of three times the smaller and twice the larger is forty-four. Find the two numbers.

20. The sum of three consecutive odd integers is fifty-one. Find the integers.

21. Find three consecutive even integers whose sum is negative eighteen.

22. Find three consecutive odd integers such that three times the middle integer is one more than the sum of the first and third integers.

23. Twice the smallest of three consecutive odd integers is seven more than the largest. Find the integers.

24. Find two consecutive even integers such that three times the first equals twice the second.

25. Find two consecutive even integers such that four times the first is three times the second.

26. Seven times the first of two consecutive odd integers is five times the second. Find the integers.

27. Find three consecutive even integers such that three times the middle integer is four more than the sum of the first and third integers.

[illegible]

Application Problems

28. The processor speed of a personal computer is 80 megahertz (mHz). This is one-third of the processor speed of a newer model personal computer. Find the processor speed of the newer personal computer.

29. The storage capacity of a hard-disk drive is 0.5 gigabyte. This is one-fourth of the storage capacity of a second hard-disk drive. Find the storage capacity of the second hard-disk drive.

30. A union charges monthly dues of $4.00 plus $.15 for each hour worked during the month. A union member's dues for March were $29.20. How many hours did the union member work during the month of March?

31. A technical information hotline charges a customer $9.00 plus $.50 per minute to answer questions about software. How many minutes did a customer who received a bill for $14.50 use this service?

32. The total cost to paint the inside of a house was $1346. This cost included $125 for materials and $33 per hour for labor. How many hours of labor were required to paint the inside of the house?

33. The cellular phone service for a business executive is $35 per month plus $.40 per minute of phone use. In a month when the executive's cellular phone bill was $99.80, how many minutes did the executive use the phone?

34. A computer screen consists of tiny dots of light called pixels. In a certain graphics mode, there are 640 horizontal pixels. This is 40 more than three times the number of vertical pixels. Find the number of vertical pixels.

35. The cost of electricity in a certain city is $.08 for each of the first 300 kWh (kilowatt-hours) and $.13 for each kilowatt-hour over 300 kWh. Find the number of kilowatt-hours used by a family with a $51.95 electric bill.

36. A 12-foot board is cut into two pieces. Twice the length of the shorter piece is 3 ft less than the length of the longer piece. Find the length of each piece.

37. A 14-yard fishing line is cut into two pieces. Three times the length of the longer piece is four times the length of the shorter piece. Find the length of each piece.

38. Seven thousand dollars is divided into two scholarships. Twice the amount of the smaller scholarship is \$1000 less than the larger scholarship. What is the amount of the larger scholarship?

39. An investment of \$10,000 is divided into two accounts, one for stocks and one for mutual funds. The value of the stock account is \$2000 less than twice the value of the mutual fund account. Find the amount in each account.

APPLYING THE CONCEPTS

40. Make up two word problems; one that requires solving the equation $6x = 123$ and one that requires solving the equation $8x + 100 = 300$ to find the answer to the problem.

41. A formula is an equation that relates variables in a known way. Find two examples of formulas that are used in your college major. Explain what each of the variables represents.

An equation that is never true is called a **contradiction.** For example, $x = x + 1$ is a contradiction; there is no value of x that will make the equation true. An equation that is true for all real numbers is called an **identity.** The equation $x + x = 2x$ is an identity; this equation is true for any real number. A **conditional equation** is one that is true for some real numbers and false for some real numbers. The equation $2x = 4$ is a conditional equation; this equation is true when x is 2 and false for any other real number. Determine whether each equation below is a contradiction, an identity, or a conditional equation. If it is a conditional equation, find the solution.

42. $6x + 2 = 5 + 3(2x - 1)$

43. $3 - 2(4x + 1) = 5 + 8(1 - x)$

44. $3t - 5(t + 1) = 2(2 - t) - 9$

45. $6 + 4(2y + 1) = 5 - 8y$

46. $3v - 2 = 5v - 2(2 + v)$

47. $9z = 15z$

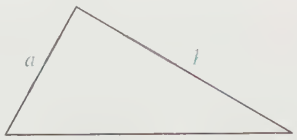
48. It is always important to check the answer to an application problem to be sure the answer makes sense. Consider the following problem. A 4-quart mixture of fruit juices is made from apple juice and cranberry juice. There are 6 more quarts of apple juice than of cranberry juice. Write and solve an equation for the number of quarts of each juice used. Does the answer to this question make sense? Explain.

3.5 Geometry Problems

To solve perimeter problems 3

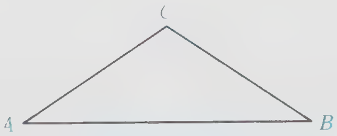
The **perimeter** of a plane geometric figure is a measure of the distance around the figure. Perimeter is used in buying fencing for a lawn or determining how much baseboard is needed for a room.

The perimeter of a triangle is the sum of the lengths of the three sides. Therefore, if a, b, and c represent the lengths of the sides of a triangle, the perimeter, P, of the triangle is given by $P = a + b + c$.



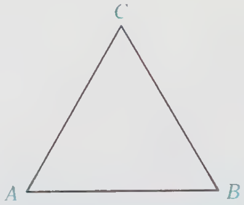
$P = a + b + c$

Two special types of triangles are shown below. An **isosceles triangle** has two sides of equal length. The three sides of an **equilateral triangle** are of equal length.



Isosceles triangle

$AC = BC$



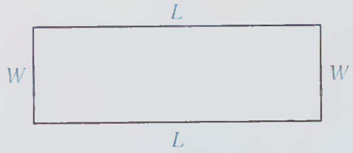
Equilateral triangle

$AB = AC = BC$

Point of Interest

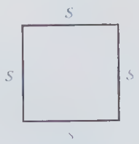
Leonardo da Vinci painted the Mona Lisa on a rectangular canvas whose height was approximately 1.6 times its width. Rectangles with these proportions, called golden rectangles, were used extensively in Renaissance art.

The perimeter of a rectangle is the sum of the lengths of the four sides. Let L represent the length and W the width of a rectangle. Then the perimeter, P, of the rectangle is given by $P = L + W + L + W$. After we combine like terms, the formula is $P = 2L + 2W$.



$P = 2L + 2W$

A square is a rectangle in which each side has the same length. Let s represent the length of each side of a square. Then the perimeter, P, of the square is given by $P = s + s + s + s$. After we combine like terms, the formula is $P = 4s$.

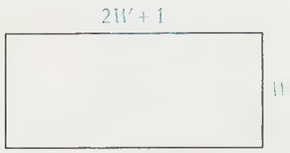


$P = 4s$

Formulas for Perimeters of Geometric Figures

Perimeter of a triangle	$P = a + b + c$
Perimeter of a rectangle	$P = 2L + 2W$
Perimeter of a square	$P = 4s$

The perimeter of a rectangle is 26 ft. The length of the rectangle is 1 ft more than twice the width. Find the width and length of the rectangle.



Strategy

- Width: W
  Length: $2W + 1$
- Use the formula for the perimeter of a rectangle.

- Let a variable represent the width.
- Represent the length in terms of that variable.

Solution

$P = 2L + 2W$

$26 = 2(2W + 1) + 2W$ • $P = 26$. Substitute $2W + 1$ for L.

$26 = 4W + 2 + 2W$ • Use the Distributive Property.

$26 = 6W + 2$ • Combine like terms.

$24 = 6W$ • Subtract 2 from each side of the equation.

$4 = W$ • Divide each side of the equation by 6.

$L = 2W + 1$ • Find the length of the rectangle by substituting 4 for W in $2W + 1$.

$= 2(4) + 1 = 8 + 1 = 9$

The width is 4 ft. The length is 9 ft.

Example 1

The perimeter of an isosceles triangle is 25 ft. The length of the third side of the triangle is 2 ft less than the length of one of the equal sides. Find the measures of each of the three sides of the triangle.

Strategy

- Length of each equal side: x
  Length of the third side: $x - 2$
- Use the equation for the perimeter of a triangle.

Solution

$P = a + b + c$

$25 = x + x + (x - 2)$

$25 = 3x - 2$

$27 = 3x$

$9 = x$

$x - 2 = 9 - 2 = 7$

Each of the equal sides measures 9 ft.
The third side measures 7 ft.

You Try It 1

A carpenter is designing a square patio with a perimeter of 52 ft. What is the length of each side?

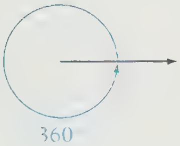
Your strategy

Your solution

Solution on p. S7

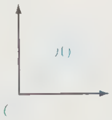
To solve problems involving angles formed by intersecting lines

A unit used to measure angles is the **degree.** The symbol for degree is °. ∠ is the symbol for angle.



One complete revolution is 360°. It is probable that the Babylonians chose 360° for a circle because they knew that there were 365 days in one year, and 360 is the closest number to 365 that is divisible by many numbers.

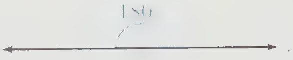
A 90° angle is called a **right angle.** The symbol ⊾ represents a right angle. Angle C (∠C) is a right angle.



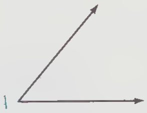
Point of Interest

The word *degree* first appeared in Chaucer's *Canterbury Tales*, which was written in 1386.

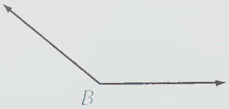
A 180° angle is called a **straight angle.** The angle at the right is a straight angle.



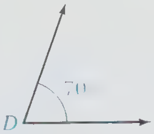
An **acute angle** is an angle whose measure is between 0° and 90°. ∠A at the right is an acute angle.

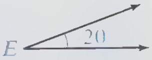


An **obtuse angle** is an angle whose measure is between 90° and 180°. ∠B at the right is an obtuse angle.



Complementary angles are two angles whose measures have the sum 90°.

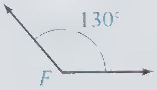


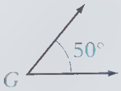


$\angle D + \angle E = 70° + 20° = 90°$

∠D and ∠E are complementary angles.

Supplementary angles are two angles whose measures have the sum 180°.

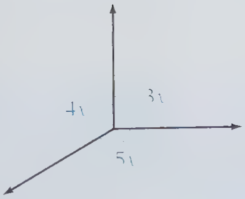




$\angle F + \angle G = 130° + 50° = 180°$

∠F and ∠G are supplementary angles.

Given the diagram at the left, find x.



Strategy

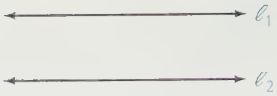
The sum of the measures of the three angles is 360°. To find x, write an equation and solve for x.

Solution

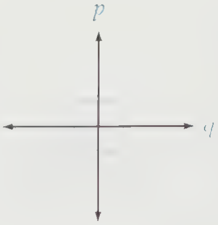
$$\begin{aligned} 3x + 4x + 5x &= 360° \\ 12x &= 360° \\ x &= 30° \end{aligned}$$

The measure of x is 30°.

Parallel lines never meet. The distance between them is always the same. The symbol $\parallel$ means "is parallel to." In the figure at the right, $\ell_1 \parallel \ell_2$.

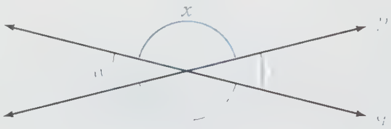


Perpendicular lines are intersecting lines that form right angles. The symbol $\perp$ means "is perpendicular to." In the figure at the right, $p \perp q$.



Four angles are formed by the intersection of two lines. If the two lines are perpendicular, then each of the four angles is a right angle. If the two lines are not perpendicular, then two of the angles formed are acute angles and two of the angles are obtuse angles. The two acute angles are always opposite each other, and the two obtuse angles are always opposite each other.

In the figure at the right, $\angle w$ and $\angle y$ are acute angles, and $\angle x$ and $\angle z$ are obtuse angles.



Two angles that are on opposite sides of the intersection of two lines are called **vertical angles.** Vertical angles have the same measure. $\angle w$ and $\angle y$ are vertical angles. $\angle x$ and $\angle z$ are vertical angles.

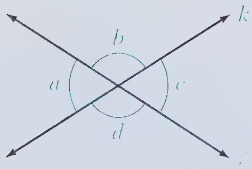
Vertical angles have the same measure.

$$\angle w = \angle y$$
$$\angle x = \angle z$$

Two angles that share a common side are called **adjacent angles.** For the figure shown above, $\angle x$ and $\angle y$ are adjacent angles, as are $\angle y$ and $\angle z$, $\angle z$ and $\angle w$, and $\angle w$ and $\angle x$. Adjacent angles of intersecting lines are supplementary angles.

Adjacent angles of intersecting lines are supplementary angles.

$$\angle x + \angle y = 180^\circ$$
$$\angle y + \angle z = 180^\circ$$
$$\angle z + \angle w = 180^\circ$$
$$\angle w + \angle x = 180^\circ$$



⇒ In the diagram at the left, $\angle b = 115^\circ$. Find the measures of angles a, c, and d.

$$\angle a + \angle b = 180^\circ$$
$$\angle a + 115^\circ = 180^\circ$$
$$\angle a = 65^\circ$$

- $\angle a$ is supplementary to $\angle b$ because $\angle a$ and $\angle b$ are adjacent angles of intersecting lines.

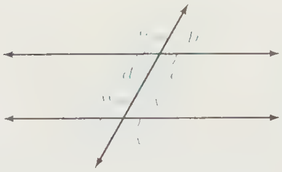
$$\angle c = 65^\circ$$

- $\angle c = \angle a$ because $\angle c$ and $\angle a$ are vertical angles.

$$\angle d = 115^\circ$$

- $\angle d = \angle b$ because $\angle d$ and $\angle b$ are vertical angles.

A line that intersects two other lines at different points is called a **transversal.** If the lines cut by a transversal *t* are parallel lines and the transversal is not perpendicular to the parallel lines, then all four acute angles have the same measure and all four obtuse angles have the same measure. In the figure at the right,



[illegible]

[illegible]

Alternate interior angles are two angles that are on opposite sides of the transversal and between the lines. In the figure above, $\angle c$ and $\angle w$ are alternate interior angles, and $\angle d$ and $\angle x$ are alternate interior angles. Alternate interior angles have the same measure.

Alternate interior angles have the same measure.

$$\angle c = \angle w$$
$$\angle d = \angle x$$

Alternate exterior angles are two angles that are on opposite sides of the transversal and outside the parallel lines. In the figure above, $\angle a$ and $\angle y$ are alternate exterior angles, and $\angle b$ and $\angle z$ are alternate exterior angles. Alternate exterior angles have the same measure.

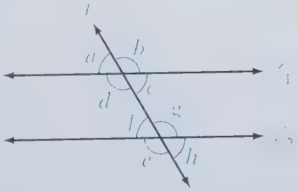
Alternate exterior angles have the same measure.

$$\angle a = \angle y$$
$$\angle b = \angle z$$

Corresponding angles are two angles that are on the same side of the transversal and are both acute angles or are both obtuse angles. In the figure above, the following pairs of angles are corresponding angles: $\angle a$ and $\angle w$, $\angle d$ and $\angle z$, $\angle b$ and $\angle x$, and $\angle c$ and $\angle y$. Corresponding angles have the same measure.

Corresponding angles have the same measure.

$$\angle a = \angle w$$
$$\angle d = \angle z$$
$$\angle b = \angle x$$
$$\angle c = \angle y$$



⇨ In the diagram at the left, $\ell_1 \parallel \ell_2$ and $\angle f = 58°$. Find the measures of $\angle a$, $\angle c$, and $\angle d$.

$\angle a = \angle f = 58°$ • $\angle a$ and $\angle f$ are corresponding angles.

$\angle c = \angle f = 58°$ • $\angle c$ and $\angle f$ are alternate interior angles.

$\angle d + \angle a = 180°$ • $\angle d$ is supplementary to $\angle a$.
$\angle d + 58° = 180°$
$\angle d = 122°$

Example 2

Find the complement of a 39° angle.

Strategy

To find the complement, let x represent the complement of a 39° angle. Use the fact that complementary angles are two angles whose sum is 90° to write an equation. Solve for x.

Solution

$$x + 39^\circ = 90^\circ$$
$$x = 51^\circ$$

The complement of a 39° angle is a 51° angle.

You Try It 2

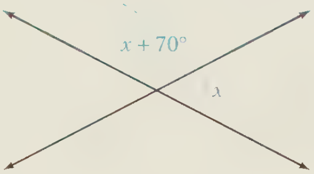
Find the supplement of a 107° angle.

Your strategy

Your solution

Example 3

Find x.



Strategy

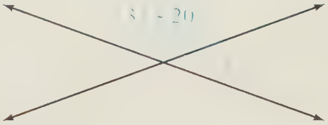
The angles labeled are adjacent angles of intersecting lines and are therefore supplementary angles. To find x, write an equation and solve for x.

Solution

$$x + (x + 70^\circ) = 180^\circ$$
$$2x + 70^\circ = 180^\circ$$
$$2x = 110^\circ$$
$$x = 55^\circ$$

You Try It 3

Find x.

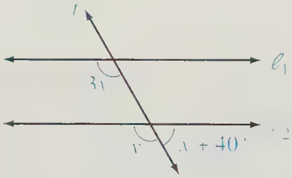


Your strategy

Your solution

Example 4

Given $\ell_1 \parallel \ell_2$, find x.



Strategy

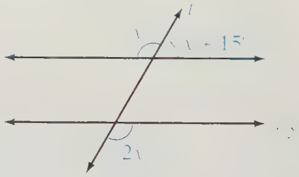
$3x = y$ because corresponding angles have the same measure. $y + (x + 40^\circ) = 180^\circ$ because adjacent angles of intersecting lines are supplementary angles. Substitute $3x$ for y and solve for x.

Solution

$$y + (x + 40^\circ) = 180^\circ$$
$$3x + (x + 40^\circ) = 180^\circ$$
$$4x + 40^\circ = 180^\circ$$
$$4x = 140^\circ$$
$$x = 35^\circ$$

You Try It 4

Given $\ell_1 \parallel \ell_2$, find x.



Your strategy

Your solution

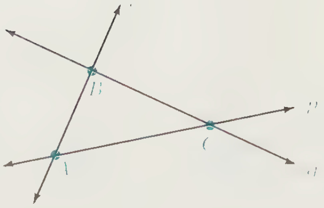
Solutions on p.S7

Objective C

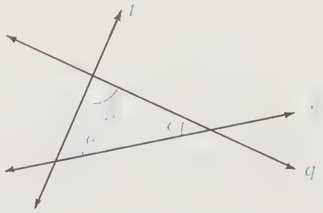
To solve problems involving the angles of a triangle

3

If the lines cut by a transversal are not parallel lines, then the three lines will intersect at three points. In the figure at the right, the transversal t intersects lines p and q. The three lines intersect at points A, B, and C. The geometric figure formed by AB, BC, and AC is a **triangle.**



The angles within the region enclosed by the triangle are called **interior angles.** In the figure at the right, angles a, b, and c are interior angles. The sum of the measures of the interior angles is 180°.

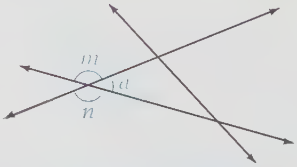


$\angle a + \angle b + \angle c = 180$

The Sum of the Measures of the Interior Angles of a Triangle

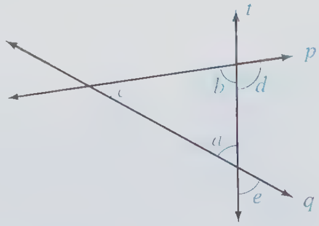
The sum of the measures of the interior angles of a triangle is 180°.

An angle adjacent to an interior angle is an **exterior angle.** In the figure at the right, angles m and n are exterior angles for angle a. The sum of the measures of an interior and an exterior angle is 180°.



$\angle a + \angle m = 180°$
$\angle a + \angle n = 180°$

➡ Given that $\angle c = 40°$ and $\angle e = 60°$, find the measure of $\angle d$.



$\angle a = \angle e = 60°$

- $\angle a$ and $\angle e$ are vertical angles.

$$\begin{aligned}\angle c + \angle a + \angle b &= 180° \\ 40° + 60° + \angle b &= 180° \\ 100° + \angle b &= 180° \\ \angle b &= 80°\end{aligned}$$

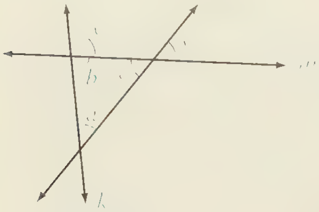
- The sum of the interior angles is 180°.

$$\begin{aligned}\angle b + \angle d &= 180° \\ 80° + \angle d &= 180° \\ \angle d &= 100°\end{aligned}$$

- $\angle b$ and $\angle d$ are supplementary angles.

Example 5

Given that $\angle a = 45°$ and $\angle x = 100°$, find the measures of angles b, c, and y.



Strategy

- To find the measure of $\angle b$, use the fact that $\angle b$ and $\angle x$ are supplementary angles.
- To find the measure of $\angle c$, use the fact that the sum of the measures of the interior angles of a triangle is 180°.
- To find the measure of $\angle y$, use the fact that $\angle c$ and $\angle y$ are vertical angles.

Solution

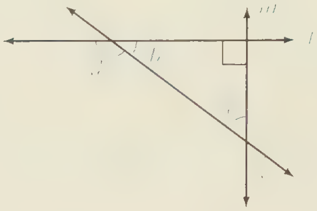
$$\begin{aligned} \angle b + \angle x &= 180° \\ \angle b + 100° &= 180° \\ \angle b &= 80° \end{aligned}$$

$$\begin{aligned} \angle a + \angle b + \angle c &= 180° \\ 45° + 80° + \angle c &= 180° \\ 125° + \angle c &= 180° \\ \angle c &= 55° \end{aligned}$$

$$\angle y = \angle c = 55°$$

You Try It 5

Given that $\angle y = 55°$, find the measures of angles a, b, and d.



Your strategy

Your solution

Example 6

Two angles of a triangle measure 43° and 86°. Find the measure of the third angle.

Strategy

To find the measure of the third angle, use the fact that the sum of the measures of the interior angles of a triangle is 180°. Write an equation using x to represent the measure of the third angle. Solve the equation for x.

Solution

$$\begin{aligned} x + 43° + 86° &= 180° \\ x + 129° &= 180° \\ x &= 51° \end{aligned}$$

The measure of the third angle is 51°.

You Try It 6

One angle in a triangle is a right angle, and one angle measures 27°. Find the measure of the third angle.

Your strategy

Your solution

Solutions on pp. S7–S8

3.5 Exercises

[illegible]

1. What is the difference between an isosceles triangle and an equilateral triangle?

2. Is there enough information given in the problem "Find the length and width of a rectangle whose perimeter is 40 cm." to solve it? Suppose the problem is changed to "Find the length and width of a square whose perimeter is 40 cm." Is there enough information to solve this problem?

3. In an isosceles triangle, the length of the third side is 50% of the length of one of the equal sides. Find the length of each side when the perimeter is 125 ft.

4. In an isosceles triangle, the length of one of the equal sides is 3 times the length of the third side. The perimeter is 21 m. Find the length of each side.

5. The perimeter of a rectangle is 42 m. The length of the rectangle is 3 m less than twice the width. Find the length and width of the rectangle.

6. The width of a rectangle is 25% of the length. The perimeter is 250 cm. Find the length and width of the rectangle.

7. The perimeter of a rectangle is 120 ft. The length of the rectangle is twice the width. Find the length and width of the rectangle.

8. The perimeter of a rectangle is 50 m. The width of the rectangle is 5 m less than the length. Find the length and width of the rectangle.

9. The perimeter of a triangle is 110 cm. One side is twice the second side. The third side is 30 cm more than the second side. Find the length of each side.

10. The perimeter of a triangle is 33 ft. One side of the triangle is 1 ft longer than the second side. The third side is 2 ft longer than the second side. Find the length of each side.

11. The width of the rectangular foundation of a building is 30% of the length. The perimeter of the foundation is 338 ft. Find the length and width of the foundation.

12. The perimeter of a rectangular playground is 440 ft. If the width is 100 ft, what is the length of the playground?

13. A rectangular vegetable garden has a perimeter of 64 ft. The length of the garden is 20 ft. What is the width of the garden?

14. Each of two sides of a triangular banner measures 18 in. If the perimeter of the banner is 46 in., what is the length of the third side of the banner?

15. The perimeter of a square picture frame is 48 in. Find the length of each side of the frame.

16. A square rug has a perimeter of 32 ft. Find the length of each side of the rug.

[illegible]

17. Place the words *supplementary* and *complementary* in the correct positions in the following sentence. ____________ angles are to right angles as ____________ angles are to straight angles.

18. Place the phrases *on the same* and *on different* in the correct positions in the following sentence. Corresponding angles are ____________ side(s) of the transversal; alternate interior angles are ____________ side(s) of the transversal.

19. Find the complement of a 28° angle.

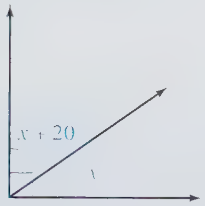
20. Find the complement of a 46° angle.

21. Find the supplement of a 73° angle.

22. Find the supplement of a 119° angle.

Find the measure of $\angle x$.

23.



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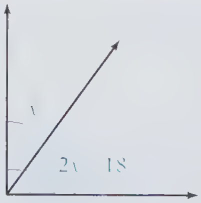

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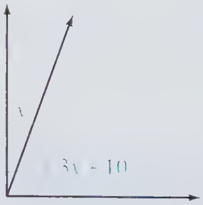
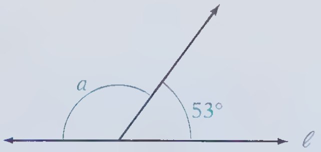


Find the measure of $\angle a$.

26.



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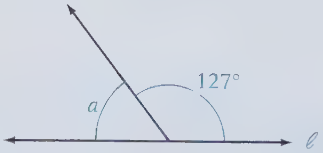
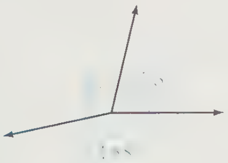


Find the measure of ∠a.

28.



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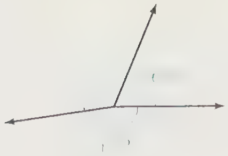
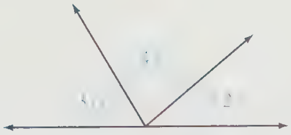


Find x.

30.



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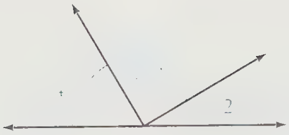

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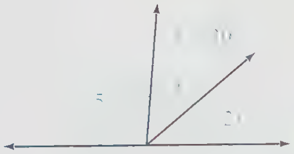

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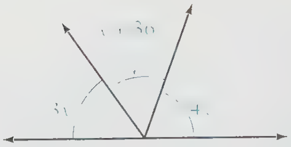

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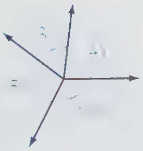

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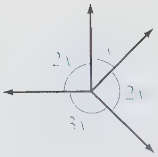
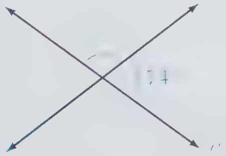


Find the measure of ∠x.

36.



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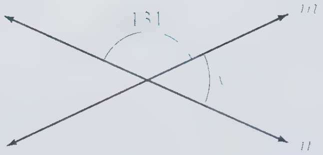
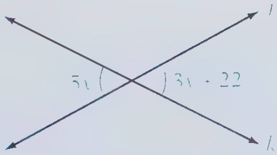


Find x.

38.



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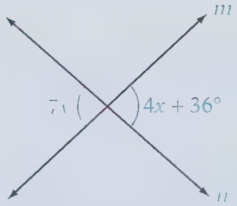
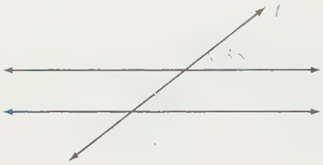


Given that $\ell_1 \parallel \ell_2$, find the measures of angles a and b.

40.



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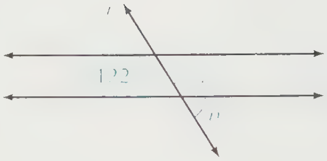

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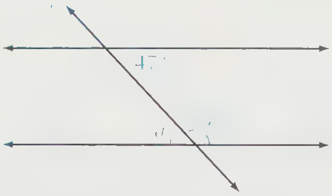

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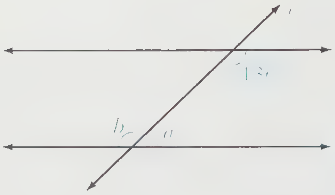
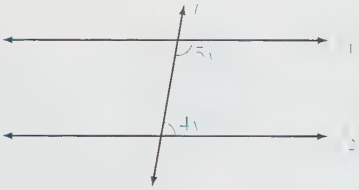


Given that $\ell_1 \parallel \ell_2$, find x.

44.



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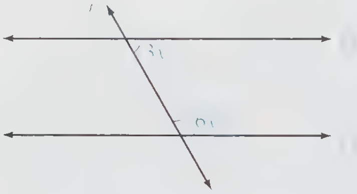

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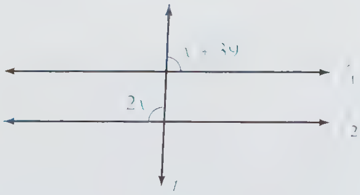

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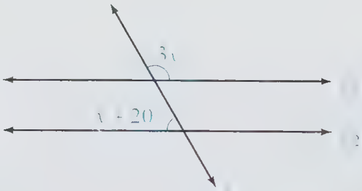
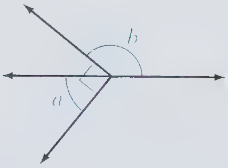
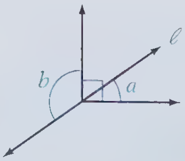


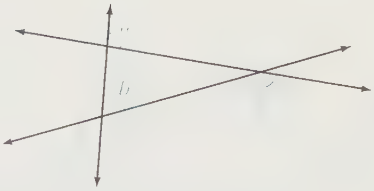
48. Given that $\angle a = 51°$, find the measure of $\angle b$.



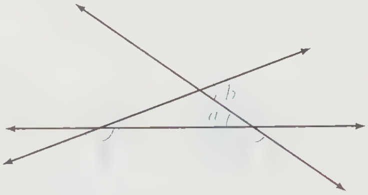
49. Given that $\angle a = 38°$, find the measure of $\angle b$.



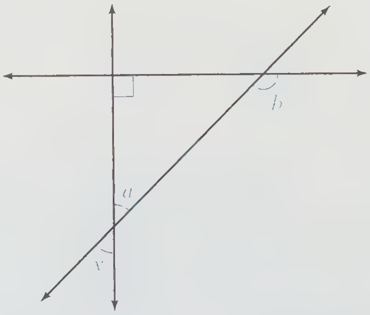
50. Given that $\angle a = 95°$ and $\angle b = 70°$, find the measures of angles x and y.



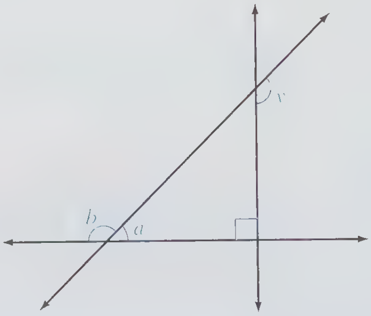
51. Given that $\angle a = 35°$ and $\angle b = 55°$, find the measures of angles x and y.



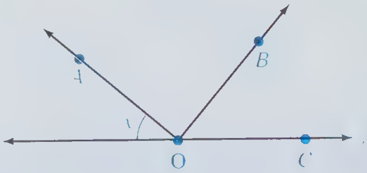
52. Given that $\angle y = 45°$, find the measures of angles a and b.



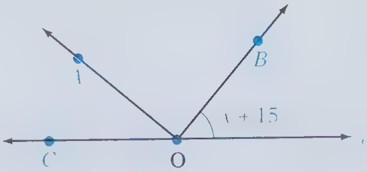
53. Given that $\angle y = 130°$, find the measures of angles a and b.



54. Given that $AO \perp OB$, express in terms of x the number of degrees in $\angle BOC$.



55. Given that $AO \perp OB$, express in terms of x the number of degrees in $\angle AOC$.



56. One angle in a triangle is a right angle, and one angle measures 30°. What is the measure of the third angle?

57. A triangle has a 45° angle and a right angle. Find the measure of the third angle.

58. Two angles of a triangle measure 42° and 103°. Find the measure of the third angle.

59. Two angles of a triangle measure 62° and 45°. Find the measure of the third angle.

60. A triangle has a 13° angle and a 65° angle. What is the measure of the third angle?

61. A triangle has a 105° angle and a 32° angle. What is the measure of the third angle?

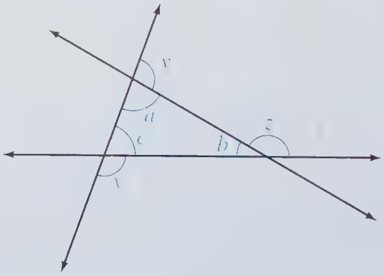
APPLYING THE CONCEPTS

62. A rectangle and an equilateral triangle have the same perimeter. The length of the rectangle is three times the width. Each side of the triangle is 8 cm. Find the length and width of the rectangle.

63. The length of a rectangle is 1 cm more than twice the width. If the length of the rectangle is decreased by 2 cm and the width is decreased by 1 cm, the perimeter is 20 cm. Find the length and width of the original rectangle.

64. For the figure at the right, find the sum of the measures of angles x, y, and z.

65. For the figure at the right, explain why $\angle a + \angle b = \angle x$. Write a rule that describes the relationship between an exterior angle of a triangle and the opposite interior angles. Use the rule to write an equation involving angles a, c, and z.



66. The length of a rectangle is $14x$. The perimeter is $50x$. Find the width of the rectangle in terms of the variable x.

3.6 Mixture and Uniform Motion Problems

Objective A

To solve value mixture problems

3

A value mixture problem involves combining two ingredients that have different prices into a single blend. For example, a coffee merchant may blend two types of coffee into a single blend, or a candy manufacturer may combine two types of candy to sell as a variety pack.

TAKE NOTE
The equation $AC = V$ is used to find the value of an ingredient. For example, the value of 4 lb of cashews costing \$6 per pound is

$$AC = V$$
$$4 \cdot 6 = V$$
$$\$24 = V$$

The solution of a value mixture problem is based on the equation $AC = V$, where A is the amount of an ingredient, C is the cost per unit of the ingredient, and V is the value of the ingredient.

A coffee merchant wants to make 6 lb of a blend of coffee costing \$5 per pound. The blend is made using a \$6-per-pound grade and a \$3-per-pound grade of coffee. How many pounds of each of these grades should be used?

Strategy for Solving a Value Mixture Problem

1. For each ingredient in the mixture, write a numerical or variable expression for the amount of the ingredient used, the unit cost of the ingredient, and the value of the amount used. For the blend, write a numerical or variable expression for the amount, the unit cost of the blend, and the value of the amount. The results can be recorded in a table.

The sum of the amounts is 6 lb.

Amount of \$6 coffee: x
Amount of \$3 coffee: $6 - x$

TAKE NOTE
Use the information given in the problem to fill in the amount and unit cost columns of the table. Fill in the value column by multiplying the two expressions you wrote in each row. Use the expressions in the last column to write the equation.

	Amount, A	·	*Unit Cost, C*	=	*Value, V*
\$6 grade	x	·	6	=	$6x$
\$3 grade	$6 - x$	·	3	=	$3(6 - x)$
\$5 blend	6	·	5	=	$5(6)$

2. Determine how the values of the ingredients are related. Use the fact that the sum of the values of all the ingredients is equal to the value of the blend.

The sum of the values of the \$6 grade and the \$3 grade is equal to the value of the \$5 blend.

$$6x + 3(6 - x) = 5(6)$$
$$6x + 18 - 3x = 30$$
$$3x + 18 = 30$$
$$3x = 12$$
$$x = 4$$

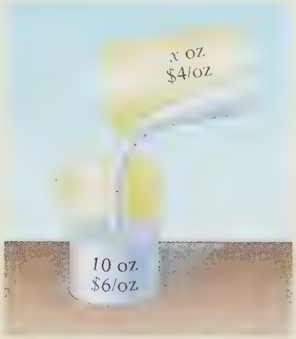
$$6 - x = 6 - 4 = 2$$

The merchant must use 4 lb of the \$6 coffee and 2 lb of the \$3 coffee.

Example 1

How many ounces of a silver alloy that costs \$4 an ounce must be mixed with 10 oz of an alloy that costs \$6 an ounce to make a mixture that costs \$4.32 an ounce?

Strategy



- Ounces of \$4 alloy: x

	Amount	*Cost*	*Value*
\$4 alloy	x	4	$4x$
\$6 alloy	10	6	$6(10)$
\$4.32 mixture	$10 + x$	4.32	$4.32(10 + x)$

- The sum of the values before mixing equals the value after mixing.

Solution

$$\begin{aligned} 4x + 6(10) &= 4.32(10 + x) \\ 4x + 60 &= 43.2 + 4.32x \\ -0.32x + 60 &= 43.2 \\ -0.32x &= -16.8 \\ x &= 52.5 \end{aligned}$$

52.5 oz of the \$4 silver alloy must be used.

You Try It 1

A gardener has 20 lb of a lawn fertilizer that costs \$.80 per pound. How many pounds of a fertilizer that costs \$.55 per pound should be mixed with this 20 lb of lawn fertilizer to produce a mixture that costs \$.75 per pound?

Your strategy

Your solution

Solution on p. S8

Objective C To solve percent mixture problems

The amount of a substance in a solution can be given as a percent of the total solution. For example, in a 5% salt water solution, 5% of the total solution is salt. The remaining 95% is water.

Solving a percent mixture problem can be done using the equation $Ar = Q$, where A is the amount of a solution, r is the percent concentration of a substance in the solution, and Q is the quantity of the substance in the solution.

For example, a 500-milliliter bottle is filled with a 4% solution of hydrogen peroxide.

$$\begin{aligned} Ar &= Q \\ 500(0.04) &= Q \\ 20 &= Q \end{aligned}$$

The bottle contains 20 ml of hydrogen peroxide.

How many gallons of a 20% salt solution must be mixed with 6 gal of a 30% salt solution to make a 22% salt solution?

Strategy for Solving a Percent Mixture Problem

1. For each solution, write a numerical or variable expression for the amount of solution, the percent concentration, and the quantity of the substance in the solution. The results can be recorded in a table.

The unknown quantity of 20% solution: x

TAKE NOTE
Use the information given in the problem to fill in the amount and percent columns of the table. Fill in the quantity column by multiplying the two expressions you wrote in each row. Use the expressions in the last column to write the equation.

	Amount of Solution, A	·	*Percent Concentration, r*	=	*Quantity of Substance, Q*
20% solution	x	·	0.20	=	$0.20x$
30% solution	6	·	0.30	=	$0.30(6)$
22% solution	$x + 6$	·	0.22	=	$0.22(x + 6)$

2. Determine how the quantities of the substances in the solutions are related. Use the fact that the sum of the quantities of the substances being mixed is equal to the quantity of the substance after mixing.

The sum of the quantities of the substances in the 20% solution and the 30% solution is equal to the quantity of the substance in the 22% solution.

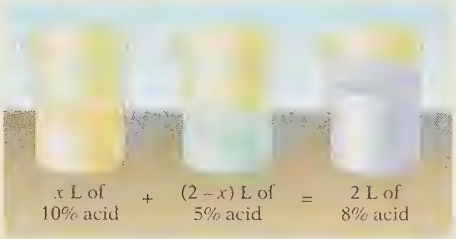
$$\begin{aligned} 0.20x + 0.30(6) &= 0.22(x + 6) \\ 0.20x + 1.80 &= 0.22x + 1.32 \\ -0.02x + 1.80 &= 1.32 \\ -0.02x &= -0.48 \\ x &= 24 \end{aligned}$$

24 gal of the 20% solution are required.

Example 2

A chemist wishes to make 2 L of an 8% acid solution by mixing a 10% acid solution and a 5% acid solution. How many liters of each solution should the chemist use?

Strategy



- Liters of 10% solution: x
  Liters of 5% solution: $2 - x$

	Amount	*Percent*	*Quantity*
10% solution	x	0.10	$0.10x$
5% solution	$2 - x$	0.05	$0.05(2 - x)$
8% solution	2	0.08	$0.08(2)$

- The sum of the quantities before mixing is equal to the quantity after mixing.

Solution

$$\begin{aligned}
0.10x + 0.05(2 - x) &= 0.08(2)\\
0.10x + 0.10 - 0.05x &= 0.16\\
0.05x + 0.10 &= 0.16\\
0.05x &= 0.06\\
x &= 1.2
\end{aligned}$$

$$2 - x = 2 - 1.2 = 0.8$$

The chemist needs 1.2 L of the 10% solution and 0.8 L of the 5% solution.

You Try It 2

A pharmacist dilutes 5 L of a 12% solution with a 6% solution. How many liters of the 6% solution are added to make an 8% solution?

Your strategy

Your solution

Solution on p. S8

To solve uniform motion problems

A train that travels constantly in a straight line at 50 mph is in *uniform motion.* **Uniform motion** means that the speed or direction of an object does not change.

The solution of a uniform motion problem is based on the equation $rt = d$, where r is the rate of travel, t is the time spent traveling, and d is the distance traveled.

For example, a car travels 50 mph for 3 h.

$$\begin{aligned} rt &= d \\ 50 \cdot 3 &= d \\ 150 &= d \end{aligned}$$

The car travels a distance of 150 mph.

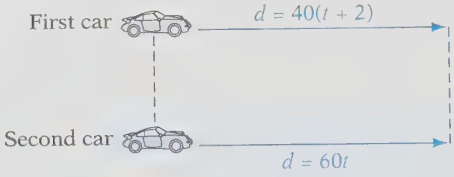
A car leaves a town traveling at 40 mph. Two hours later, a second car leaves the same town, on the same road, traveling at 60 mph. In how many hours will the second car pass the first car?

Strategy for Solving a Uniform Motion Problem

1. For each object, write a numerical or variable expression for the rate, time, and distance. The results can be recorded in a table.

The first car traveled 2 h longer than the second car.

Unknown time for the second car: t
Time for the first car: $t + 2$



	Rate, r	·	Time, t	=	Distance, d
First car	40	·	$t + 2$	=	$40(t + 2)$
Second car	60	·	t	=	$60t$

TAKE NOTE
Use the information given in the problem to fill in the rate and time columns of the table. Find the expression in the distance column by multiplying the two expressions you wrote in each row.

2. Determine how the distances traveled by the two objects are related. For example, the total distance traveled by both objects may be known, or it may be known that the two objects traveled the same distance.

The two cars travel the same distance.

$$\begin{aligned} 40(t + 2) &= 60t \\ 40t + 80 &= 60t \\ 80 &= 20t \\ 4 &= t \end{aligned}$$

The second car will pass the first car in 4 h.

Example 3

Two cars, one traveling 10 mph faster than the other, start at the same time from the same point and travel in opposite directions. In 3 h they are 300 mi apart. Find the rate of each car.

Strategy

- Rate of 1st car: r
  Rate of 2nd car: $r + 10$

	Rate	*Time*	*Distance*
1st car	r	3	$3r$
2nd car	$r + 10$	3	$3(r + 10)$

- The total distance traveled by the two cars is 300 mi.

Solution

$$
\begin{aligned}
3r + 3(r + 10) &= 300 \\
3r + 3r + 30 &= 300 \\
6r + 30 &= 300 \\
6r &= 270 \\
r &= 45
\end{aligned}
$$

$$r + 10 = 45 + 10 = 55$$

The first car is traveling 45 mph.
The second car is traveling 55 mph.

You Try It 3

Two trains, one traveling at twice the speed of the other, start at the same time on parallel tracks from stations that are 288 mi apart and travel toward each other. In 3 h, the trains pass each other. Find the rate of each train.

Your strategy

Your solution

Example 4

How far can the members of a bicycling club ride out into the country at a speed of 12 mph and return over the same road at 8 mph if they travel a total of 10 h?

Strategy

- Time spent riding out: t
  Time spent riding back: $10 - t$

	Rate	*Time*	*Distance*
Out	12	t	$12t$
Back	8	$10 - t$	$8(10 - t)$

- The distance out equals the distance back.

Solution

$$
\begin{aligned}
12t &= 8(10 - t) \\
12t &= 80 - 8t \\
20t &= 80 \\
t &= 4 \quad \text{(The time is 4 h.)}
\end{aligned}
$$

The distance out $= 12t = 12(4) = 48$ mi.
The club can ride 48 mi into the country.

You Try It 4

A pilot flew out to a parcel of land and back in 5 h. The rate out was 150 mph, and the rate returning was 100 mph. How far away was the parcel of land?

Your strategy

Your solution

Solutions on p. S8

3.6 Exercises

Application Problems

1. An herbalist has 30 oz of herbs costing $2 per ounce. How many ounces of herbs costing $1 per ounce should be mixed with the 30 oz to produce a mixture costing $1.60 per ounce?

2. The manager of a farmer's market has 500 lb of grain that costs $1.20 per pound. How many pounds of meal costing $.80 per pound should be mixed with the 500 lb of grain to produce a mixture that costs $1.05 per pound?

3. Find the cost per pound of a meatloaf mixture made from 3 lb of ground beef costing $1.99 per pound and 1 lb of ground turkey costing $1.39 per pound.

4. Find the cost per ounce of a sunscreen made from 100 oz of a lotion that costs $2.50 per ounce and 50 oz of a lotion that costs $4.00 per ounce.

5. A snack food is made by mixing 5 lb of popcorn that costs $.80 per pound with caramel that costs $2.40 per pound. How much caramel is needed to make a mixture that costs $1.40 per pound?

6. A wild birdseed mix is made by combining 100 lb of millet seed costing $.60 per pound with sunflower seeds costing $1.10 per pound. How many pounds of sunflower seeds are needed to make a mixture that costs $.70 per pound?

7. Ten cups of a restaurant's house Italian dressing is made by blending olive oil costing $1.50 per cup with vinegar that costs $.25 per cup. How many cups of each are used if the cost of the blend is $.50 per cup?

8. A high-protein diet supplement that costs $6.75 per pound is mixed with a vitamin supplement that costs $3.25 per pound. How many pounds of each should be used to make 5 lb of a mixture that costs $4.65 per pound?

9. Find the cost per ounce of a mixture of 200 oz of a cologne that costs $5.50 per ounce and 500 oz of a cologne that costs $2.00 per ounce.

10. Find the cost per pound of a trail mix made from 40 lb of raisins that cost $4.40 per pound and 100 lb of granola that costs $2.30 per pound.

11. A 20-ounce alloy of platinum that costs $220 per ounce is mixed with an alloy that costs $400 per ounce. How many ounces of the $400 alloy should be used to make an alloy that costs $300 per ounce?

12. How many liters of a blue dye that costs $1.60 per liter must be mixed with 18 L of anil that costs $2.50 per liter to make a mixture that costs $1.90 per liter?

13. The manager of a specialty food store combined almonds that cost $4.50 per pound with walnuts that cost $2.50 per pound. How many pounds of each were used to make a 100-pound mixture that costs $3.24 per pound?

14. A goldsmith combined an alloy that cost $4.30 per ounce with an alloy that cost $1.80 per ounce. How many ounces of each were used to make a mixture of 200 oz costing $2.50 per ounce?

15. Adult tickets for a play cost $6.00 and children's tickets cost $2.50. For one performance, 370 tickets were sold. Receipts for the performance were $1723. Find the number of adult tickets sold.

16. Tickets for a piano concert sold for $4.50 for each adult. Student tickets sold for $2.00 each. The total receipts for 1720 tickets were $5980. Find the number of adult tickets sold.

17. Find the cost per pound of sugar-coated breakfast cereal made from 40 lb of sugar that costs $1.00 per pound and 120 lb of corn flakes that cost $.60 per pound.

18. Find the cost per pound of a coffee mixture made from 8 lb of coffee that costs $9.20 per pound and 12 lb of coffee that costs $5.50 per pound.

Objective 3 Application Problems

19. Forty ounces of a 30% gold alloy are mixed with 60 oz of a 20% gold alloy. Find the percent concentration of the resulting gold alloy.

20. One hundred ounces of juice that is 50% tomato juice is added to 200 oz of a vegetable juice that is 25% tomato juice. What is the percent concentration of tomato juice in the resulting mixture?

21. How many gallons of a 15% acid solution must be mixed with 5 gal of a 20% acid solution to make a 16% acid solution?

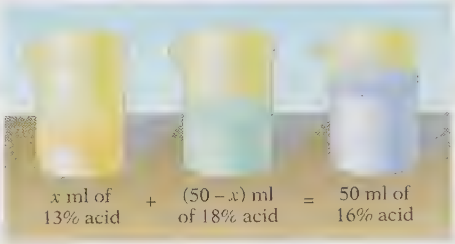
22. How many pounds of a chicken feed that is 50% corn must be mixed with 400 lb of a feed that is 80% corn to make a chicken feed that is 75% corn?

23. A rug is made by weaving 20 lb of yarn that is 50% wool with a yarn that is 25% wool. How many pounds of the yarn that is 25% wool are used if the finished rug is 35% wool?

24. Five gallons of a light green latex paint that is 20% yellow paint is combined with a darker green latex paint that is 40% yellow paint. How many gallons of the darker green paint must be used to create a green paint that is 25% yellow paint?

25. How many gallons of a plant food that is 9% nitrogen must be combined with another plant food that is 25% nitrogen to make 10 gal of a solution that is 15% nitrogen?

26. A chemist wants to make 50 ml of a 16% acid solution by mixing a 13% acid solution and an 18% acid solution. How many milliliters of each solution should the chemist use?



27. Five grams of sugar are added to a 45-gram serving of a breakfast cereal that is 10% sugar. What is the percent concentration of sugar in the resulting mixture?

28. A goldsmith mixes 8 oz of a 30% gold alloy with 12 oz of a 25% gold alloy. What is the percent concentration of the resulting alloy?

29. How many pounds of coffee that is 40% java beans must be mixed with 80 lb of coffee that is 30% java beans to make a coffee blend that is 32% java beans?

30. The manager of a garden shop mixes grass seed that is 60% rye grass with 70 lb of grass seed that is 80% rye grass to make a mixture that is 74% rye grass. How much of the 60% rye grass is used?

31. A hair dye is made by blending a 7% hydrogen peroxide solution and a 4% hydrogen peroxide solution. How many milliliters of each are used to make a 300-milliliter solution that is 5% hydrogen peroxide?

32. A tea that is 20% jasmine is blended with a tea that is 15% jasmine. How many pounds of each tea are used to make 5 lb of tea that is 18% jasmine?

33. How many ounces of pure chocolate must be added to 150 oz of chocolate topping that is 50% chocolate to make a topping that is 75% chocolate?

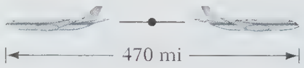
34. How many ounces of pure bran flakes must be added to 50 oz of cereal that is 40% bran flakes to produce a mixture that is 50% bran flakes?

35. Thirty ounces of pure silver are added to 50 oz of a silver alloy that is 20% silver. What is the percent concentration of the resulting alloy?

36. A clothing manufacturer has some pure silk thread and some thread that is 85% silk. How many kilograms of each must be woven together to make 75 kg of cloth that is 96% silk?

Application Problems

37. Two small planes start from the same point and fly in opposite directions. The first plane is flying 25 mph slower than the second plane. In 2 h, the planes are 470 mi apart. Find the rate of each plane.



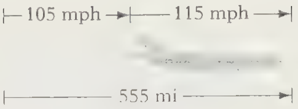
38. Two cyclists start from the same point and ride in opposite directions. One cyclist rides twice as fast as the other. In 3 h, they are 81 mi apart. Find the rate of each cyclist.

39. Two planes leave an airport at 8 A.M., one flying north at 480 km/h and the other flying south at 520 km/h. At what time will they be 3000 km apart?

40. A long-distance runner started on a course running at an average speed of 6 mph. One-half hour later, a second runner began the same course at an average speed of 7 mph. How long after the second runner started will the second runner overtake the first runner?

41. A motorboat leaves a harbor and travels at an average speed of 9 mph toward a small island. Two hours later a cabin cruiser leaves the same harbor and travels at an average speed of 18 mph toward the same island. In how many hours after the cabin cruiser leaves the harbor will it be alongside the motorboat?

42. A 555-mile, 5-hour plane trip was flown at two speeds. For the first part of the trip, the average speed was 105 mph. For the remainder of the trip, the average speed was 115 mph. For how long did the plane fly at each speed?



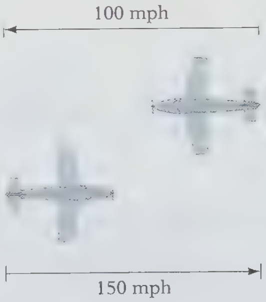
43. An executive drove from home at an average speed of 30 mph to an airport where a helicopter was waiting. The executive boarded the helicopter and flew to the corporate offices at an average speed of 60 mph. The entire distance was 150 mi. The entire trip took 3 h. Find the distance from the airport to the corporate offices.

44. After a sailboat had been on the water for 3 h, a change in the wind direction reduced the average speed of the boat by 5 mph. The entire distance sailed was 57 mi. The total time spent sailing was 6 h. How far did the sailboat travel in the first 3 h?

45. A car and a bus set out at 3 P.M. from the same point headed in the same direction. The average speed of the car is twice the average speed of the bus. In 2 h the car is 68 mi ahead of the bus. Find the rate of the car.

46. A passenger train leaves a train depot 2 h after a freight train leaves the same depot. The freight train is traveling 20 mph slower than the passenger train. Find the rate of each train if the passenger train overtakes the freight train in 3 h.

47. As part of flight training, a student pilot was required to fly to an airport and then return. The average speed on the way to the airport was 100 mph, and the average speed returning was 150 mph. Find the distance between the two airports if the total flying time was 5 h.



48. A ship traveling east at 25 mph is 10 mi from a harbor when another ship leaves the harbor traveling east at 35 mph. How long does it take the second ship to catch up to the first ship?

49. At 10 A.M. a plane leaves Boston, Massachusetts, for Seattle, Washington, a distance of 3000 mi. One hour later a plane leaves Seattle for Boston. Both planes are traveling at a speed of 500 mph. How many hours after the plane leaves Seattle will the planes pass each other?

50. At noon a train leaves Washington, D.C., headed for Charleston, South Carolina, a distance of 500 mi. The train travels at a speed of 60 mph. At 1 P.M. a second train leaves Charleston headed for Washington, D.C., traveling at 50 mph. How long after the train leaves Charleston will the two trains pass each other?

51. Two cyclists start at the same time from opposite ends of a course that is 51 mi long. One cyclist is riding at a rate of 16 mph, and the second cyclist is riding at a rate of 18 mph. How long after they begin will they meet?

52. A bus traveled on a level road for 2 h at an average speed that was 20 mph faster than its average speed on a winding road. The time spent on the winding road was 3 h. Find the average speed on the winding road if the total trip was 210 mi.

53. A bus traveling at a rate of 60 mph overtakes a car traveling at a rate of 45 mph. If the car had a 1-hour head start, how far from the starting point does the bus overtake the car?

54. A car traveling at 48 mph overtakes a cyclist who, riding at 12 mph, had a 3-hour head start. How far from the starting point does the car overtake the cyclist?

APPLYING THE CONCEPTS

55. How many grams of pure water must be added to 50 g of pure acid to make a solution that is 40% acid?

56. How many ounces of water must be evaporated from 50 oz of a 12% salt solution to produce a 15% salt solution?

57. A radiator contains 15 gal of a 20% antifreeze solution. How many gallons must be drained from the radiator and replaced by pure antifreeze so that the radiator will contain 15 gal of a 40% antifreeze solution?

58. At 10 A.M., two campers left their campsite by canoe and paddled downstream at an average speed of 12 mph. They then turned around and paddled back upstream at an average rate of 4 mph. The total trip took 1 h. At what time did the campers turn around downstream?

59. A bicyclist rides for 2 h at a speed of 10 mph and then returns at a speed of 20 mph. Find the cyclist's average speed for the trip.

60. A car travels a 1-mile track at an average speed of 30 mph. At what average speed must the car travel the next mile so that the average speed for the 2 mi is 60 mph?

Focus on Problem Solving

Trial-and-Error Approach to Problem Solving

The questions below require an answer of always true, sometimes true, or never true. These problems are best solved by the trial-and-error method. The trial-and-error method of arriving at a solution to a problem involves repeated tests or experiments.

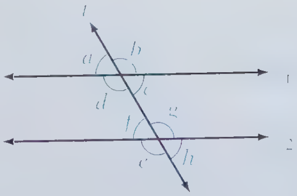
For example, consider the statement

> Both sides of an equation can be divided by the same number without changing the solution of the equation.

The solution of the equation $6x = 18$ is 3. If we divide both sides of the equation by 2, the result is $3x = 9$ and the solution is still 3. So the answer "never true" has been eliminated. We still need to determine whether there is a case for which the statement is not true. Is there a number that we could divide both sides of the equation by and the result would be an equation for which the solution is not 3? If we divide both sides of the equation by 0, the result is $\frac{6x}{0} = \frac{18}{0}$; the solution of this equation is not 3 because the expressions on either side of the equals sign are undefined. Thus the statement is true for some numbers and not true for 0. The statement is sometimes true.

Determine whether the statement is always true, sometimes true, or never true.

1. Both sides of an equation can be multiplied by the same number without changing the solution of the equation.
2. For an equation of the form $ax = b$, $a \neq 0$, multiplying both sides of the equation by the reciprocal of a will result in an equation of the form $x = constant$.
3. The Multiplication Property of Equations is used to remove a term from one side of an equation.
4. Adding -3 to each side of an equation yields the same result as subtracting 3 from each side of the equation.
5. An equation contains an equals sign.
6. The same variable term can be added to both sides of an equation without changing the solution of the equation.
7. An equation of the form $ax + b = c$ cannot be solved if a is a negative number.
8. The solution of the equation $\frac{x}{0} = 0$ is 0.



9. For the diagram at the left, $\angle b = \angle d = \angle e = \angle g$.
10. In solving an equation of the form $ax + b = cx + d$, subtracting cx from each side of the equation results in an equation with only one variable term in it.

11. If a rope 8 m long is cut into two pieces and one of the pieces has length x meters, then the length of the other piece can be represented as $(x - 8)$ meters.

12. An even integer is a multiple of 2.

13. If the first of three consecutive odd integers is n, then the second and third consecutive odd integers are represented by $n + 1$ and $n + 3$.

14. Suppose we are mixing two salt solutions. Then the variable Q in the percent mixture equation $Q = Ar$ represents the amount of salt in a solution.

15. If 100 oz of a silver alloy is 25% silver, then the alloy contains 25 oz of silver.

16. If we combine an alloy that costs \$8 an ounce with an alloy that costs \$5 an ounce, the cost of the resulting mixture will be greater than \$8 an ounce.

17. If we combine a 9% acid solution with a solution that is 4% acid, the resulting solution will be less than 4% acid.

18. If the speed of one train is 20 mph slower than that of a second train, then the speeds of the two trains can be represented as r and $20 - r$.

Projects and Group Activities

Nielsen Ratings

Point of Interest

The 12 top-ranked programs in prime time for the week of April 8 to April 15, 2001, ranked by Nielsen Media Research, were

- *Survivor II*
- *CSI*
- *Who Wants to Be a Millionaire—Tuesday*
- *Everybody Loves Raymond*
- *CSI—Saturday*
- *Becker*
- *Law and Order*
- *Who Wants to Be a Millionaire—Sunday*
- *Who Wants to Be a Millionaire—Thursday*
- *Judging Amy*
- *60 Minutes*
- *NYPD Blue*

Nielsen Media Research surveys television viewers to determine the numbers of people watching particular shows. There are an estimated 102.2 million U.S. households with televisions. Each **rating point** represents 1% of that number, or 1,022,000 households. Therefore, for instance, if *60 Minutes* received a rating of 9.6, then 9.6%, or $(0.096)(102{,}200{,}000) = 9{,}811{,}200$ households, watched that program.

A rating point does not mean that 1,022,000 people are watching a program. A rating point refers to the number of TV sets tuned to that program; there may be more than one person watching a television set in the household.

Nielsen Media Research also describes a program's share of the market. **Share** is the percent of television sets in use that are tuned to a program. Suppose the same week that *60 Minutes* received 9.6 rating points, the show received a share of 19%. This would mean that 19% of all households with a television *turned on* were tuned to *60 Minutes,* whereas 9.6% of all households with a television were tuned to the program.

1. If *Law and Order* received a Nielsen rating of 10.8 and a share of 19, how many TV households watched the program that week? How many TV households were watching television during that hour? Round to the nearest hundred thousand.

2. Suppose *Survivor II* received a rating of 16.2 and a share of 28. How many TV households watched the program that week? How many TV households were watching television during that hour? Round to the nearest hundred-thousand.

3. Suppose *Who Wants to Be a Millionaire—Tuesday* received a rating of 13.6 during a week in which 34,750,000 people were watching the show. Find the average number of people per TV household who watched the program. Round to the nearest tenth.

The cost to advertise during a program is related to its Nielsen rating. The sponsor (the company paying for the advertisement) pays a certain number of dollars for each rating point a show receives.

4. Suppose a television network charges $35,000 per rating point for a 30-second commercial on a daytime talk show. Determine the cost for three 30-second commercials if the Nielsen rating of the show is 11.5.

Nielsen Media Research also tracks the exposure of advertisements. For example, it might be reported that commercials for McDonald's had 500,000,000 household exposures during a week when its advertisement was aired 90 times.

5. Information regarding household exposure of advertisements can be found in *USA Today* each Monday. For a recent week, find the information for the top four advertised brands. For each brand, calculate the average household exposure for each time the ad was aired.

Nielsen Media Research has a web site on the Internet. You can locate the site by using a search engine. The site does not list rating points and market share, but these statistics can be found on other web sites by using a search engine.

6. Find the top two prime-time television shows for last week. Calculate the number of TV households that watched each program. Compare these figures with the top two sports programs for last week.

Chapter Summary

Key Words

An *equation* expresses the equality of two mathematical expressions. [p.87]

A *solution* of an equation is a number that, when substituted for the variable, results in a true equation. [p.87]

To *solve an equation* means to find a solution of the equation. The goal is to rewrite the equation in the form *variable* = *constant*. [p.88]

Cost is the price that a business pays for a product. *Selling price* is the price for which a business sells a product to a customer. *Markup* is the difference between selling price and cost. When the markup is expressed as a percent of the retailer's cost, it is called the *markup rate*. [p.102]

Discount, or *markdown*, is the amount by which a retailer reduces the regular price of a product. The percent discount is called the *discount rate*. [p.103]

To *translate* a sentence into an equation requires recognition of the words or phrases that mean "equals." Some of these phrases are *equals, is, is equal to, amounts to,* and *represents*. [p.119]

The *perimeter* of a geometric figure is a measure of the distance around the figure. [p.127]

An *isosceles triangle* has two sides of equal measure. The three sides of an *equilateral triangle* are of equal measure. [p.127]

An angle is measured in *degrees.* A 90° angle is a *right angle.* A 180° angle is a *straight angle.* One complete revolution is 360°. An *acute angle* is an angle whose measure is between 0° and 90°. An *obtuse angle* is an angle whose measure is between 90° and 180°. [p.129]

Complementary angles are two angles whose measures have the sum 90°. [p.129]

Supplementary angles are two angles whose measures have the sum 180°. [p. 129]

Parallel lines never meet; the distance between them is always the same. [p.130]

Perpendicular lines are intersecting lines that form right angles. [p.130]

Two angles that are on opposite sides of the intersection of two lines are *vertical angles;* vertical angles have the same measure. Two angles that share a common side are *adjacent angles;* adjacent angles of intersecting lines are supplementary angles. [p.130]

A line that intersects two other lines at two different points is a *transversal.* If the lines cut by a transversal are parallel lines, equal angles are formed: *alternate interior angles, alternate exterior angles,* and *corresponding angles.* [p.131]

Uniform motion means that an object, traveling at a constant speed, moves in a straight line. [p.145]

Essential Rules

Addition Property of Equations [p.88]

The same number can be added to each side of an equation without changing the solution of the equation.

If $a = b$, then
$a + c = b + c$.

Multiplication Property of Equations [p.89]

Each side of an equation can be multiplied by the same nonzero number without changing the solution of the equation.

If $a = b$ and $c \neq 0$,
then $ac = bc$.

Basic Percent Equation [p.91]

Percent · Base = Amount

$P \cdot B = A$

Consecutive Integers [p.119] $n, n + 1, n + 2, \ldots$

Consecutive Even Integers [p.119] $n, n + 2, n + 4, \ldots$

Consecutive Odd Integers [p.119] $n, n + 2, n + 4, \ldots$

Formulas for Perimeter [p.127]

Triangle $P = a + b + c$
Rectangle $P = 2L + 2W$
Square $P = 4s$

Sum of the Angles of a Triangle [p.133]

The sum of the measures of the interior angles of a triangle is 180°.
The sum of an interior and corresponding exterior angle is 180°.

$\angle a + \angle b + \angle c = 180°$

Value Mixture Equation [p.141]

Amount · Unit Cost = Value

$AC = V$

Percent Mixture Equation [p.143]

Amount · Percent Concentration = Quantity

$Ar = Q$

Uniform Motion Equation [p.145]

Rate · Time = Distance

$rt = d$

Chapter Review

1. Solve: $x + 3 = 24$

2. Solve: $x + 5(3x - 20) = 10(x - 4)$

3. Solve: $5x - 6 = 29$

4. Is 3 a solution of $5x - 2 = 4x + 5$?

5. Solve: $\frac{3}{5}a = 12$

6. Solve: $6x + 3(2x - 1) = -27$

7. 30 is what percent of 12?

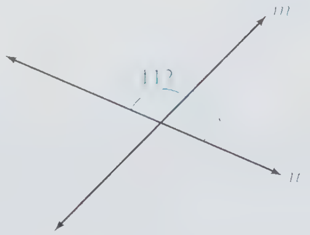
8. Solve: $5x + 3 = 10x - 17$

9. Solve: $7 - [4 + 2(x - 3)] = 11(x + 2)$

10. Solve: $-6x + 16 = -2x$

11. A furniture store uses a markup rate of 60%. The store sells a solid oak curio cabinet for \$1074. Find the cost of the curio cabinet. Use the formula $S = C + rC$, where S is the selling price, C is the cost, and r is the markup rate.

12. Find the measure of $\angle x$.

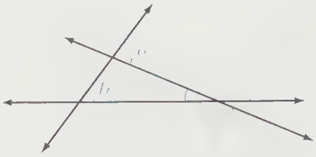


13. A lever is 12 ft long. At a distance of 2 ft from the fulcrum, a force of 120 lb is applied. How large a force must be applied to the other end so that the system will balance? Use the lever system equation $F_1x = F_2(d - x)$.

14. A bus traveled on a level road for 2 h at an average speed of 20 mph faster than it traveled on a winding road. The time spent on the winding road was 3 h. Find the average speed on the winding road if the total trip was 200 mi.

15. A ceiling fan, which regularly sells for \$60, is on sale for \$40. Find the discount rate. Use the formula $S = R - rR$, where S is the sale price, R is the regular price, and r is the discount rate.

16. Given that $\angle a = 74°$ and $\angle b = 52°$, find the measures of angles x and y.



17. A health food store combined cranberry juice that cost \$1.79 per quart with apple juice that cost \$1.19 per quart. How many quarts of each were used to make 10 qt of cranapple juice costing \$1.61 per quart?

18. Four times the second of three consecutive integers equals the sum of the first and third integers. Find the integers.

19. One angle of a triangle is 15° more than the measure of the second angle. The third angle is 15° less than the measure of the second angle. Find the measure of each angle.

20. Translate "four less than the product of five and a number is sixteen" into an equation and solve.

21. The Empire State Building is 1472 ft tall. This is 654 ft less than twice the height of the Eiffel Tower. Find the height of the Eiffel Tower.

22. The length of a rectangle is four times the width of the rectangle. The perimeter is 200 ft. Find the length and width of the rectangle.

23. A jet plane traveling at 600 mph overtakes a propeller-driven plane that had a 2-hour head start. The propeller-driven plane is traveling at 200 mph. How far from the starting point does the jet overtake the propeller-driven plane?

24. The sum of two numbers is twenty-one. Three times the smaller number is two less than twice the larger number. Find the two numbers.

25. A dairy owner mixed 5 gal of cream containing 30% butterfat with 8 gal of milk containing 4% butterfat. What is the percent of butterfat in the resulting mixture?

Chapter Test

1. Solve: $3x - 2 = 5x + 8$

2. Solve: $x - 3 = -8$

3. Solve: $3x - 5 = -14$

4. Solve: $4 - 2(3 - 2x) = 2(5 - x)$

5. Is -2 a solution of $x^2 - 3x = 2x - 6$?

6. Solve: $7 - 4x = -13$

7. What is 0.5% of 8?

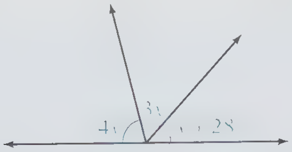
8. Solve: $5x - 2(4x - 3) = 6x + 9$

9. Solve: $5x + 3 - 7x = 2x - 5$

10. Solve: $\frac{3}{4}x = -9$

11. A baker wants to make a 15-pound blend of flour that costs $.60 per pound. The blend is made using a rye flour that costs $.70 per pound and a wheat flour that costs $.40 per pound. How many pounds of each flour should be used?

12. Find x.



13. A television that regularly sells for $450 is on sale for $360. Find the discount rate. Use the formula $S = R - rR$, where S is the sale price, R is the regular price, and r is the discount rate.

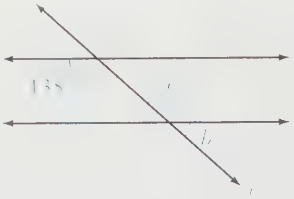
14. A financial manager has determined that the cost per unit for a calculator is $15 and that the fixed cost per month is $2000. Find the number of calculators produced during a month in which the total cost was $5000. Use the equation $T = U \cdot N + F$, where T is the total cost, U is the cost per unit, N is the number of units produced, and F is the fixed cost.

15. In an isosceles triangle, two angles are equal. The third angle of the triangle is 30° less than one of the equal angles. Find the measure of one of the equal angles.

16. Find three consecutive even integers whose sum is 36.

17. How many gallons of water must be mixed with 5 gal of a 20% salt solution to make a 16% salt solution?

18. Given that $\ell_1 \parallel \ell_2$, find the measures of angles a and b.



19. Translate "The difference between three times a number and fifteen is twenty-seven" into an equation and solve.

20. A cross-country skier leaves a camp to explore a wilderness area. Two hours later a friend leaves the camp in a snowmobile, traveling 4 mph faster than the skier. This friend meets the skier 1 h later. Find the rate of the snowmobile.

21. A company makes 140 televisions per day. Three times the number of black-and-white TVs made equals 20 less than the number of color TVs made. Find the number of color TVs made each day.

22. The sum of two numbers is eighteen. The difference between four times the smaller number and seven is equal to the sum of two times the larger number and five. Find the two numbers.

23. As part of flight training, a student pilot was required to fly to an airport and then return. The average speed to the airport was 90 mph, and the average speed returning was 120 mph. Find the distance between the two airports if the total flying time was 7 h.

24. The perimeter of a triangle is 23 ft. One side is twice the second side. The third side is 3 ft more than the second side. Find the measure of each side.

25. A chemist mixes 100 g of water at 80°C with 50 g of water at 20°C. To find the final temperature of the water after mixing, use the equation $m_1(T_1 - T) = m_2(T - T_2)$, where m_1 is the quantity of water at the hotter temperature, T_1 is the temperature of the hotter water, m_2 is the quantity of water at the cooler temperature, T_2 is the temperature of the cooler water, and T is the final temperature of the water after mixing.

Cumulative Review

1. Subtract: $-6 - (-20) - 8$

2. Multiply: $(-2)(-6)(-4)$

3. Subtract: $-\frac{5}{6} - \left(-\frac{7}{16}\right)$

4. Divide: $-2\frac{1}{3} \div 1\frac{1}{6}$

5. Simplify: $-4^2 \cdot \left(-\frac{3}{2}\right)^3$

6. Simplify: $25 - 3\frac{(5-2)^2}{2^3+1} - (-2)$

7. Evaluate $3(a - c) - 2ab$ when $a = 2$, $b = 3$, and $c = -4$.

8. Simplify: $3x - 8x + (-12x)$

9. Simplify: $2a - (-3b) - 7a - 5b$

10. Simplify: $(16x)\left(\frac{1}{8}\right)$

11. Simplify: $-4(-9y)$

12. Simplify: $-2(-x^2 - 3x + 2)$

13. Simplify: $-2(x - 3) + 2(4 - x)$

14. Simplify: $-3[2x - 4(x - 3)] + 2$

15. Is -3 a solution of $x^2 + 6x + 9 = x + 3$?

16. Is $\frac{1}{2}$ a solution of $3 - 8x = 12x - 2$?

17. Find 32% of 60.

18. Solve: $\frac{3}{5}x = -15$

19. Solve: $7x - 8 = -29$

20. Solve: $13 - 9x = -14$

21. Solve: $8x - 3(4x - 5) = -2x - 11$

22. Solve: $6 - 2(5x - 8) = 3x - 4$

23. Solve: $5x - 8 = 12x + 13$

24. Solve: $11 - 4x = 2x + 8$

25. A chemist mixes 300 g of water at 75°C with 100 g of water at 15°C. To find the final temperature of the water after mixing, use the equation $m_1(T_1 - T) = m_2(T - T_2)$, where m_1 is the quantity of water at the hotter temperature, T_1 is the temperature of the hotter water, m_2 is the quantity of water at the cooler temperature, T_2 is the temperature of the cooler water, and T is the final temperature of the water after mixing.

26. Translate "The difference between twelve and the product of five and a number is negative eighteen" into an equation and solve.

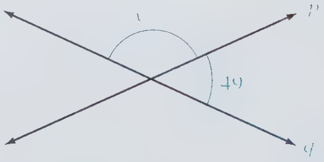
27. The area of a cement foundation of a house is 2000 ft^2. This is 200 ft^2 more than three times the area of the garage. Find the area of the garage.

28. How many pounds of an oat flour that costs \$.80 per pound must be mixed with 40 lb of a wheat flour that costs \$.50 per pound to make a blend that costs \$.60 per pound?

29. How many grams of pure gold must be added to 100 g of a 20% gold alloy to make an alloy that is 36% gold?

30. The perimeter of a rectangular office is 44 ft. The length of the office is 2 ft more than the width. Find the dimensions of the office.

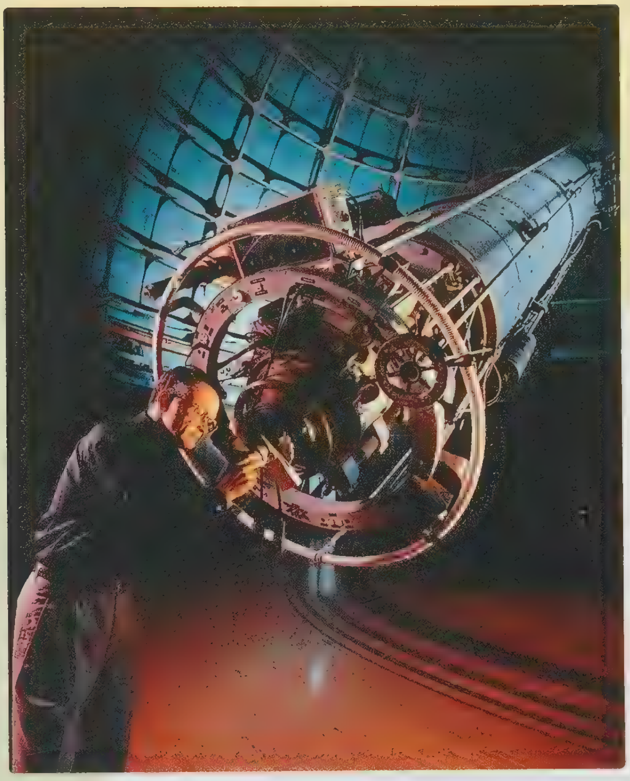
31. Find the measure of $\angle x$.



32. In an equilateral triangle, all three angles are equal. Find the measure of one of the angles of an equilateral triangle.

33. A sprinter ran to the end of a track at an average rate of 8 m/s and then jogged back to the starting point at an average rate of 3 m/s. The sprinter took 55 s to run to the end of the track and jog back. Find the length of the track.

Chapter 4 Polynomials



Telescopes allow scientists to look at objects that are extremely large but far away. Astronomers, like the one shown here, use scientific notation to describe distances in space. Scientific notation replaces very large and very small numbers with more concise expressions, making these numbers easier to read and write. **Exercises 115 to 122 on pages 189 and 190** provide examples of situations in which scientific notation is used.

Objectives

Section 4.1

- **A** To add polynomials
- **B** To subtract polynomials

Section 4.2

- **A** To multiply monomials
- **B** To simplify powers of monomials

Section 4.3

- **A** To multiply a polynomial by a monomial
- **B** To multiply two polynomials
- **C** To multiply two binomials
- **D** To multiply binomials that have special products
- **E** To solve application problems

Section 4.4

- **A** To divide monomials
- **B** To write a number in scientific notation

Section 4.5

- **A** To divide a polynomial by a monomial
- **B** To divide polynomials

Need help? For online student resources, such as section quizzes, visit this textbook's website at **math.college.hmco.com/students.**

Prep Test

1. Subtract: $-2 - (-3)$

2. Multiply: $-3(6)$

3. Simplify: $-\dfrac{24}{-36}$

4. Evaluate $3n^4$ when $n = -2$.

5. If $\dfrac{a}{b}$ is a fraction is simplest form, what number is not a possible value of b?

6. Are $2x^2$ and $2x$ like terms?

7. Simplify: $3x^2 - 4x + 1 + 2x^2 - 5x - 7$

8. Simplify: $-4y + 4y$

9. Simplify: $-3(2x - 8)$

10. Simplify: $3xy - 4y - 2(5xy - 7y)$

Go Figure

If $x + y$, xy, and $\dfrac{x}{y}$ all equal the same number, find the values of x and y.

4.1 Addition and Subtraction of Polynomials

[illegible]

To add polynomials 4

TAKE NOTE

The expression $3\sqrt{x}$ is not a monomial because $\sqrt{x}$ cannot be written as a product of variables. The expression $\frac{2x}{y}$ is not a monomial because it is a *quotient* of variables.

A monomial is a number, a variable, or a product of numbers and variables. For instance,

7	b	$\frac{2}{3}a$	$12xy^2$
A number	A variable	A product of a number and a variable	A product of a number and variables

A **polynomial** is a variable expression in which the terms are monomials.

A polynomial of *one* term is a **monomial.**
A polynomial of *two* terms is a **binomial.**
A polynomial of *three* terms is a **trinomial.**

$-7x^2$ is a monomial.
$4x + 2$ is a binomial.
$7x^2 + 5x - 7$ is a trinomial.

The terms of a polynomial in one variable are usually arranged so that the exponents of the variable decrease from left to right. This is called **descending order.**

$5x^3 - 4x^2 + 6x - 1$

$7z^4 + 4z^2 + z - 6$

$2y^4 + y^3 - 2y^2 + 4y - 1$

The **degree** of a polynomial in one variable is the value of the largest exponent on a variable. The degree of $4x^3 - 5x^2 + 7x - 8$ is 3; the degree of $2y^4 + y^2 - 1$ is 4. The degree of a nonzero constant is zero. For instance, the degree of 7 is zero. The number zero has no degree.

Polynomials can be added, using either a horizontal or a vertical format, by combining like terms.

Simplify $(3x^3 - 7x + 2) + (7x^2 + 2x - 7)$. Use a horizontal format.

$$(3x^3 - 7x + 2) + (7x^2 + 2x - 7)$$
$$= 3x^3 + 7x^2 + (-7x + 2x) + (2 - 7)$$

- Use the Commutative and Associative Properties of Addition to rearrange and group like terms.

$$= 3x^3 + 7x^2 - 5x - 5$$

- Then combine like terms.

Simplify $(-4x^2 + 6x - 9) + (12 - 8x + 2x^3)$. Use a vertical format.

$$\begin{array}{rrrr} & -4x^2 & +\,6x & -\;9 \\ 2x^3 & & -\,8x & +\,12 \\ \hline 2x^3 & -\,4x^2 & -\,2x & +\;3 \end{array}$$

- Arrange the terms of each polynomial in descending order with like terms in the same column.
- Combine the terms in each column.

Example 1
Use a horizontal format to simplify $(8x^2 - 4x - 9) + (2x^2 + 9x - 9)$.

Solution
$$(8x^2 - 4x - 9) + (2x^2 + 9x - 9)$$
$$= (8x^2 + 2x^2) + (-4x + 9x) + (-9 - 9)$$
$$= 10x^2 + 5x - 18$$

You Try It 1
Use a horizontal format to simplify $(-4x^3 + 2x^2 - 8) + (4x^3 + 6x^2 - 7x + 5)$.

Your solution

Solution on p. S9

Example 2
Use a vertical format to simplify
$(-5x^3 + 4x^2 - 7x + 9) + (2x^3 + 5x - 11)$.

Solution

$$\begin{array}{rrrr} -5x^3 & + 4x^2 & - 7x & + 9 \\ 2x^3 & & + 5x & - 11 \\ \hline -3x^3 & + 4x^2 & - 2x & - 2 \end{array}$$

You Try It 2
Use a vertical format to simplify
$(6x^3 + 2x + 8) + (-9x^3 + 2x^2 - 12x - 8)$.

Your solution

Solution on p. S9

Objective B

To subtract polynomials

4

The **opposite** of the polynomial $(3x^2 - 7x + 8)$ is $-(3x^2 - 7x + 8)$.

To simplify the opposite of a polynomial, change the sign of each term inside the parentheses.

$$-(3x^2 - 7x + 8) = -3x^2 + 7x - 8$$

TAKE NOTE
This is the same definition used for subtraction of integers: Subtraction is addition of the opposite.

Polynomials can be subtracted using either a horizontal or a vertical format. To subtract, add the opposite of the second polynomial to the first.

⇨ Simplify $(4y^2 - 6y + 7) - (2y^3 - 5y - 4)$. Use a horizontal format.

$$\begin{aligned} &(4y^2 - 6y + 7) - (2y^3 - 5y - 4) \\ &\quad = (4y^2 - 6y + 7) + (-2y^3 + 5y + 4) \\ &\quad = -2y^3 + 4y^2 + (-6y + 5y) + (7 + 4) \\ &\quad = -2y^3 + 4y^2 - y + 11 \end{aligned}$$

- Add the opposite of the second polynomial to the first.
- Combine like terms.

⇨ Simplify $(9 + 4y + 3y^3) - (2y^2 + 4y - 21)$. Use a vertical format.

The opposite of $2y^2 + 4y - 21$ is $-2y^2 - 4y + 21$.

$$\begin{array}{rrrr} 3y^3 & & + 4y & + 9 \\ & -2y^2 & - 4y & + 21 \\ \hline 3y^3 & - 2y^2 & & + 30 \end{array}$$

- Arrange the terms of each polynomial in descending order with like terms in the same column.
- Note that $4y - 4y = 0$, but 0 is not written.

Example 3
Use a horizontal format to simplify
$(7c^2 - 9c - 12) - (9c^2 + 5c - 8)$.

Solution

$$\begin{aligned} &(7c^2 - 9c - 12) - (9c^2 + 5c - 8) \\ &\quad = (7c^2 - 9c - 12) + (-9c^2 - 5c + 8) \\ &\quad = -2c^2 - 14c - 4 \end{aligned}$$

You Try It 3
Use a horizontal format to simplify
$(-4w^3 + 8w - 8) - (3w^3 - 4w^2 - 2w - 1)$.

Your solution

Example 4
Use a vertical format to simplify
$(3k^2 - 4k + 1) - (k^3 + 3k^2 - 6k - 8)$.

Solution

$$\begin{array}{rrrr} & 3k^2 & - 4k & + 1 \\ -k^3 & - 3k^2 & + 6k & + 8 \\ \hline -k^3 & & + 2k & + 9 \end{array}$$

- Add the opposite of $(k^3 + 3k^2 - 6k - 8)$ to the first polynomial.

You Try It 4
Use a vertical format to simplify
$(13y^3 - 6y - 7) - (4y^2 - 6y - 9)$.

Your solution

Solutions on p. S9

4.1 Exercises

[illegible]

State whether the expression is a monomial.

1. 17

2. $3x^4$

3. $\frac{17}{\sqrt{x}}$

4. xyz

5. $\frac{2}{3}y$

6. $\frac{xy}{z}$

7. $\sqrt{5}\,x$

8. πx

State whether the expression is a monomial, a binomial, a trinomial, or none of these.

9. $3x + 5$

10. $2y - 3\sqrt{y}$

11. $9x^2 - x - 1$

12. $x^2 + y^2$

13. $\frac{2}{x} - 3$

14. $\frac{ab}{4}$

15. $6x^2 + 7x$

16. $12a^4 - 3a + 2$

Simplify. Use a vertical format.

17. $(x^2 + 7x) + (-3x^2 - 4x)$

18. $(3y^2 - 2y) + (5y^2 + 6y)$

19. $(y^2 + 4y) + (-4y - 8)$

20. $(3x^2 + 9x) + (6x - 24)$

21. $(2x^2 + 6x + 12) + (3x^2 + x + 8)$

22. $(x^2 + x + 5) + (3x^2 - 10x + 4)$

23. $(-7x + x^3 + 4) + (2x^2 + x - 10)$

24. $(y^2 + 3y^3 + 1) + (-4y^3 - 6y - 3)$

25. $(2a^3 - 7a + 1) + (1 - 4a - 3a^2)$

26. $(5r^3 - 6r^2 + 3r) + (-3 - 2r + r^2)$

Simplify. Use a horizontal format.

27. $(4x^2 + 2x) + (x^2 + 6x)$

28. $(-3y^2 + y) + (4y^2 + 6y)$

29. $(4x^2 - 5xy) + (3x^2 + 6xy - 4y^2)$

30. $(2x^2 - 4y^2) + (6x^2 - 2xy + 4y^2)$

31. $(2a^2 - 7a + 10) + (a^2 + 4a + 7)$

32. $(-6x^2 + 7x + 3) + (3x^2 + x + 3)$

33. $(7x + 5x^3 - 7) + (10x^2 - 8x + 3)$

34. $(4y + 3y^3 + 9) + (2y^2 + 4y - 21)$

35. $(7 - 5r + 2r^2) + (3r^3 - 6r)$

36. $(14 + 4y + 3y^3) + (-4y^2 + 21)$

[illegible]

Simplify. Use a vertical format.

37. $(x^2 - 6x) - (x^2 - 10x)$

38. $(y^2 + 4y) - (y^2 + 10y)$

39. $(2y^2 - 4y) - (-y^2 + 2)$

40. $(-3a^2 - 2a) - (4a^2 - 4)$

41. $(x^2 - 2x + 1) - (x^2 + 5x + 8)$

42. $(3x^2 + 2x - 2) - (5x^2 - 5x + 6)$

43. $(4x^3 + 5x + 2) - (1 + 2x - 3x^2)$

44. $(5y^2 - y + 2) - (-3 + 3y - 2y^3)$

45. $(-2y + 6y^2 + 2y^3) - (4 + y^2 + y^3)$

46. $(4 - x - 2x^2) - (-2 + 3x - x^3)$

Simplify. Use a horizontal format.

47. $(y^2 - 10xy) - (2y^2 + 3xy)$

48. $(x^2 - 3xy) - (-2x^2 + xy)$

49. $(3x^2 + x - 3) - (4x + x^2 - 2)$

50. $(5y^2 - 2y + 1) - (-y - 2 - 3y^2)$

51. $(-2x^3 + x - 1) - (-x^2 + x - 3)$

52. $(2x^2 + 5x - 3) - (3x^3 + 2x - 5)$

53. $(1 - 2a + 4a^3) - (a^3 - 2a + 3)$

54. $(7 - 8b + b^2) - (4b^3 - 7b - 8)$

55. $(-1 - y + 4y^3) - (3 - 3y - 2y^2)$

56. $(-3 - 2x + 3x^2) - (4 - 2x^2 + 2x^3)$

APPLYING THE CONCEPTS

57. What polynomial must be added to $3x^2 - 6x + 9$ so that the sum is $4x^2 + 3x - 2$?

58. What polynomial must be subtracted from $2x^2 - x - 2$ so that the difference is $5x^2 + 3x + 1$?

59. In your own words, explain the terms *monomial, binomial, trinomial,* and *polynomial.* Give an example of each.

60. Is it possible to subtract two polynomials, each of degree 3, and have the difference be a polynomial of degree 2? If so, give an example. If not, explain why not.

61. Is it possible to add two polynomials, each of degree 3, and have the sum be a polynomial of degree 2? If so, give an example. If not, explain why not.

4.2 Multiplication of Monomials

Objective A To multiply monomials 4

Recall that in an exponential expression such as x^6, x is the base and 6 is the exponent. The exponent indicates the number of times the base occurs as a factor.

The product of exponential expressions with the *same* base can be simplified by writing each expression in factored form and writing the result with an exponent.

$$x^3 \cdot x^2 = \underbrace{\overbrace{(x \cdot x \cdot x)}^{3\text{ factors}} \cdot \overbrace{(x \cdot x)}^{2\text{ factors}}}_{5\text{ factors}}$$

$$= x^5$$

Note that adding the exponents results in the same product.

$$x^3 \cdot x^2 = x^{3+2} = x^5$$

Rule for Multiplying Exponential Expressions

If m and n are positive integers, then $x^m \cdot x^n = x^{m+n}$.

Simplify: $y^4 \cdot y \cdot y^3$

$$y^4 \cdot y \cdot y^3 = y^{4+1+3}$$
$$= y^8$$

- The bases are the same. Add the exponents. Recall that $y = y^1$.

Simplify: $(-3a^4b^3)(2ab^4)$

$$(-3a^4b^3)(2ab^4) = (-3 \cdot 2)(a^4 \cdot a)(b^3 \cdot b^4)$$

- Use the Commutative and Associative Properties of Multiplication to rearrange and group factors.

$$= -6(a^{4+1})(b^{3+4})$$

- Multiply variables with the same base by adding their exponents.

$$= -6a^5b^7$$

- Simplify.

TAKE NOTE

The Rule for Multiplying Exponential Expressions requires the bases to be the same. The expression a^5b^7 cannot be simplified.

Example 1 Simplify: $(-5ab^3)(4a^5)$

Solution

$$(-5ab^3)(4a^5) = (-5 \cdot 4)(a \cdot a^5)b^3$$
$$= -20a^6b^3$$

You Try It 1 Simplify: $(8m^3n)(-3n^5)$

Your solution

Example 2 Simplify: $(6x^3y^2)(4x^4y^5)$

Solution

$$(6x^3y^2)(4x^4y^5) = (6 \cdot 4)(x^3 \cdot x^4)(y^2 \cdot y^5)$$
$$= 24x^7y^7$$

You Try It 2 Simplify: $(12p^4q^3)(-3p^5q^2)$

Your solution

Solutions on p. S9

To simplify powers of monomials

The power of a monomial can be simplified by writing the power in factored form and then using the Rule for Multiplying Exponential Expressions.

$$(x^4)^3 = x^4 \cdot x^4 \cdot x^4 \qquad (a^2b^3)^2 = (a^2b^3)(a^2b^3)$$

- Write in factored form.

$$= x^{4+4+4} \qquad = a^{2+2}b^{3+3}$$

- Use the Rule for Multiplying Exponential Expressions.

$$= x^{12} \qquad = a^4b^6$$

Note that multiplying each exponent inside the parentheses by the exponent outside the parentheses results in the same product.

$$(x^4)^3 = x^{4\cdot 3} = x^{12} \qquad (a^2b^3)^2 = a^{2\cdot 2}b^{3\cdot 2} = a^4b^6$$

- Multiply each exponent inside the parentheses by the exponent outside the parentheses.

Point of Interest

One of the first symbolic representations of powers was given by Diophantus (c. 250 A.D.) in his book *Arithmetica*. He used Δ^Y for x^2 and κ^Y for x^3. The symbol Δ^Y was the first two letters of the Greek word *dunamis*, which means "power"; κ^Y was from the Greek word *kubos*, which means "cube." He also combined these symbols to denote higher powers. For instance, $\Delta\kappa^Y$ was the symbol for x^5.

Rule for Simplifying the Power of an Exponential Expression

If m and n are positive integers, then $(x^m)^n = x^{mn}$.

Rule for Simplifying Powers of Products

If m, n, and p are positive integers, then $(x^my^n)^p = x^{mp}y^{np}$.

Simplify: $(5x^2y^3)^3$

$$(5x^2y^3)^3 = 5^{1\cdot 3}x^{2\cdot 3}y^{3\cdot 3}$$

- Multiply each exponent inside the parentheses by the exponent outside the parentheses. Note that $5 = 5^1$.

$$= 5^3x^6y^9$$

$$= 125x^6y^9$$

- Evaluate 5^3.

Example 3 Simplify: $(-2p^3r)^4$

Solution

$$(-2p^3r)^4 = (-2)^{1\cdot 4}p^{3\cdot 4}r^{1\cdot 4}$$
$$= (-2)^4p^{12}r^4 = 16p^{12}r^4$$

You Try It 3 Simplify: $(-3a^4bc^2)^3$

Your solution

Example 4 Simplify: $(2a^2b)(2a^3b^2)^3$

Solution

$$(2a^2b)(2a^3b^2)^3$$
$$= (2a^2b)(2^{1\cdot 3}a^{3\cdot 3}b^{2\cdot 3})$$
$$= (2a^2b)(2^3a^9b^6)$$
$$= (2a^2b)(8a^9b^6) = 16a^{11}b^7$$

You Try It 4 Simplify: $(-xy^4)(-2x^3y^2)^2$

Your solution

Solutions on p. S9

4.2 Exercises

Objective A

1. Explain how to multiply two monomials. Provide an example.

2. Explain how to simplify the power of a monomial. Provide an example.

Simplify.

3. $(6x^2)(5x)$

4. $(-4y^3)(2y)$

5. $(7c^2)(-6c^4)$

6. $(-8z^5)(5z^8)$

7. $(-3a^3)(-3a^4)$

8. $(-5a^6)(-2a^5)$

9. $(x^2)(xy^4)$

10. $(x^2y^4)(xy^7)$

11. $(-2x^4)(5x^5y)$

12. $(-3a^3)(2a^2b^4)$

13. $(-4x^2y^4)(-3x^5y^4)$

14. $(-6a^2b^4)(-4ab^3)$

15. $(2xy)(-3x^2y^4)$

16. $(-3a^2b)(-2ab^3)$

17. $(x^2yz)(x^2y^4)$

18. $(-ab^2c)(a^2b^5)$

19. $(-a^2b^3)(-ab^2c^4)$

20. $(-x^2y^3z)(-x^3y^4)$

21. $(-5a^2b^2)(6a^3b^6)$

22. $(7xy^4)(-2xy^3)$

23. $(-6a^3)(-a^2b)$

24. $(-2a^2b^3)(-4ab^2)$

25. $(-5y^4z)(-8y^6z^5)$

26. $(3x^2y)(-4xy^2)$

27. $(x^2y)(yz)(xyz)$

28. $(xy^2z)(x^2y)(z^2y^2)$

29. $(3ab^2)(-2abc)(4ac^2)$

30. $(-2x^3y^2)(-3x^2z^2)(-5y^3z^3)$

31. $(4x^4z)(-yz^3)(-2x^3z^2)$

32. $(-a^3b^4)(-3a^4c^2)(4b^3c^4)$

33. $(-2x^2y^3)(3xy)(-5x^3y^4)$

34. $(4a^2b)(-3a^3b^4)(a^5b^2)$

35. $(3a^2b)(-6bc)(2ac^2)$

Objective B

Simplify.

36. $(z^4)^3$

37. $(x^3)^5$

38. $(y^4)^2$

39. $(x^7)^2$

40. $(-y^5)^3$

41. $(-x^2)^4$

42. $(-x^2)^3$

43. $(-y^3)^4$

44. $(-3y)^3$

45. $(-2x^2)^3$

46. $(a^3b^4)^3$ **47.** $(x^2y^3)^2$ **48.** $(2x^3y^4)^5$ **49.** $(3x^2y)^2$ **50.** $(-2ab^3)^4$

51. $(-3x^3y^2)^5$ **52.** $(3b^2)(2a^3)^4$ **53.** $(-2x)(2x^3)^2$ **54.** $(2y)(-3y^4)^3$

55. $(3x^2y)(2x^2y^2)^3$ **56.** $(a^3b)^2(ab)^3$ **57.** $(ab^2)^2(ab)^2$ **58.** $(-x^2y^3)^2(-2x^3y)^3$

59. $(-2x)^3(-2x^3y)^3$ **60.** $(-3y)(-4x^2y^3)^3$ **61.** $(-2x)(-3xy^2)^2$ **62.** $(-3y)(-2x^2y)^3$

63. $(ab^2)(-2a^2b)^3$ **64.** $(a^2b^2)(-3ab^4)^2$ **65.** $(-2a^3)(3a^2b)^3$ **66.** $(-3b^2)(2ab^2)^3$

APPLYING THE CONCEPTS

Simplify.

67. $3x^2 + (3x)^2$ **68.** $4x^2 - (4x)^2$ **69.** $2x^6y^2 + (3x^2y)^2$ **70.** $(x^2y^2)^3 + (x^3y^3)^2$

71. $(2a^3b^2)^3 - 8a^9b^6$ **72.** $4y^2z^4 - (2yz^2)^2$ **73.** $(x^2y^4)^2 + (2xy^2)^4$ **74.** $(3a^3)^2 - 4a^6 + (2a^2)^3$

For Exercises 75 to 78, answer true or false. If the answer is false, correct the right-hand side of the equation.

75. $(-a)^5 = -a^5$ **76.** $(-b)^8 = b^8$ **77.** $(x^2)^5 = x^{2+5} = x^7$ **78.** $x^3 + x^3 = 2x^{3+3} = 2x^6$

79. Evaluate $(2^3)^2$ and $2^{(3^2)}$. Are the results the same? If not, which expression has the larger value?

80. If n is a positive integer and $x^n = y^n$, does $x = y$? Explain your answer.

81. The distance a rock will fall in t seconds is $16t^2$ (neglecting air resistance). Find other examples of quantities that can be expressed in terms of an exponential expression, and explain where the expression is used.

4.3 Multiplication of Polynomials

To multiply a polynomial by a monomial

To multiply a polynomial by a monomial, use the Distributive Property and the Rule for Multiplying Exponential Expressions.

Simplify: $-3a(4a^2 - 5a + 6)$

$$\begin{aligned} -3a(4a^2 - 5a + 6) &= -3a(4a^2) - (-3a)(5a) + (-3a)(6) \\ &= -12a^3 + 15a^2 - 18a \end{aligned}$$

- Use the Distributive Property.

Example 1
Simplify: $(5x + 4)(-2x)$

Solution
$(5x + 4)(-2x) = -10x^2 - 8x$

You Try It 1
Simplify: $(-2y + 3)(-4y)$

Your solution

Example 2
Simplify: $2a^2b(4a^2 - 2ab + b^2)$

Solution
$2a^2b(4a^2 - 2ab + b^2)$
$= 8a^4b - 4a^3b^2 + 2a^2b^3$

You Try It 2
Simplify: $-a^2(3a^2 + 2a - 7)$

Your solution

Solutions on p. S9

To multiply two polynomials

Multiplication of two polynomials requires the repeated application of the Distributive Property.

$$\begin{aligned} (y - 2)(y^2 + 3y + 1) &= (y - 2)(y^2) + (y - 2)(3y) + (y - 2)(1) \\ &= y^3 - 2y^2 + 3y^2 - 6y + y - 2 \\ &= y^3 + y^2 - 5y - 2 \end{aligned}$$

A convenient method of multiplying two polynomials is to use a vertical format similar to that used for multiplication of whole numbers.

$$\begin{array}{rrrrl} & y^2 & + 3y & + 1 & \\ & & y & - 2 & \\ \hline & -2y^2 & - 6y & - 2 & = -2(y^2 + 3y + 1) \\ y^3 & + 3y^2 & + y & & = y(y^2 + 3y + 1) \\ \hline y^3 & + y^2 & - 5y & - 2 & \end{array}$$

- Multiply by -2.
- Multiply by y.
- Add the terms in each column.

Simplify: $(2a^3 + a - 3)(a + 5)$

$$\begin{array}{rrrrr}
 & 2a^3 & & +\ a & -\ 3 \\
 & & & a & +\ 5 \\
\hline
 & 10a^3 & & +\ 5a & -\ 15 \\
2a^4 & & +\ a^2 & -\ 3a & \\
\hline
2a^4 & +\ 10a^3 & +\ a^2 & +\ 2a & -\ 15
\end{array}$$

- Note that spaces are provided in each product so that like terms are in the same column.
- Add the terms in each column.

Example 3
Simplify: $(2b^3 - b + 1)(2b + 3)$

Solution

$$\begin{array}{rrrrr}
 & 2b^3 & & -\ b & +\ 1 \\
 & & & 2b & +\ 3 \\
\hline
 & 6b^3 & & -\ 3b & +\ 3 \\
4b^4\ + & & -\ 2b^2 & +\ 2b & \\
\hline
4b^4 & +\ 6b^3 & -\ 2b^2 & -\ b & +\ 3
\end{array}$$

You Try It 3
Simplify: $(2y^3 + 2y^2 - 3)(3y - 1)$

Your solution

Solution on p. S9

Objective C To multiply two binomials

It is frequently necessary to find the product of two binomials. The product can be found using a method called **FOIL**, which is based on the Distributive Property. The letters of FOIL stand for **F**irst, **O**uter, **I**nner, and **L**ast.

TAKE NOTE
FOIL is not really a different way of multiplying. It is based on the Distributive Property.

$$\begin{aligned}
&(2x + 3)(x + 5) \\
&= 2x(x + 5) + 3(x + 5) \\
&\qquad\ \ \text{F} \qquad \text{O} \qquad \text{I} \qquad \text{L} \\
&= 2x^2 + 10x + 3x + 15 \\
&= 2x^2 + 13x + 15
\end{aligned}$$

Simplify: $(2x + 3)(x + 5)$

Multiply the First terms. $(2x + 3)(x + 5)$ $2x \cdot x = 2x^2$

Multiply the Outer terms. $(2x + 3)(x + 5)$ $2x \cdot 5 = 10x$

Multiply the Inner terms. $(2x + 3)(x + 5)$ $3 \cdot x = 3x$

Multiply the Last terms. $(2x + 3)(x + 5)$ $3 \cdot 5 = 15$

Add the products. $(2x + 3)(x + 5)$ $\overset{\text{F}}{=\ 2x^2} + \overset{\text{O}}{10x} + \overset{\text{I}}{3x} + \overset{\text{L}}{15}$

Combine like terms. $= 2x^2 + 13x + 15$

Simplify: $(4x - 3)(3x - 2)$

$$\begin{aligned}
(4x - 3)(3x - 2) &= 4x(3x) + 4x(-2) + (-3)(3x) + (-3)(-2) \\
&= 12x^2 - 8x - 9x + 6 \\
&= 12x^2 - 17x + 6
\end{aligned}$$

Simplify: $(3x - 2y)(x + 4y)$

$$\begin{aligned}
(3x - 2y)(x + 4y) &= 3x(x) + 3x(4y) + (-2y)(x) + (-2y)(4y) \\
&= 3x^2 + 12xy - 2xy - 8y^2 \\
&= 3x^2 + 10xy - 8y^2
\end{aligned}$$

Example 4
Simplify: $(2a - 1)(3a - 2)$

Solution

$$\begin{aligned}(2a - 1)(3a - 2) &= 6a^2 - 4a - 3a + 2 \\ &= 6a^2 - 7a + 2\end{aligned}$$

You Try It 4
Simplify: $(4y - 5)(2y - 3)$

Your solution

Example 5
Simplify: $(3x - 2)(4x + 3)$

Solution

$$\begin{aligned}(3x - 2)(4x + 3) &= 12x^2 + 9x - 8x - 6 \\ &= 12x^2 + x - 6\end{aligned}$$

You Try It 5
Simplify: $(3b + 2)(3b - 5)$

Your solution

Solutions on p. S9

Objective D To multiply binomials that have special products

Using FOIL, it is possible to find a pattern for the product of the sum and difference of two terms and for the square of a binomial.

The Sum and Difference of Two Terms

$$\begin{aligned}(a + b)(a - b) &= a^2 - ab + ab - b^2 \\ &= \underset{\uparrow}{a^2} - \underset{\uparrow}{b^2}\end{aligned}$$

Square of the first term
Square of the second term

The Square of a Binomial

$$\begin{aligned}(a + b)^2 = (a + b)(a + b) &= a^2 + ab + ab + b^2 \\ &= \underset{\uparrow}{a^2} + \underset{\uparrow}{2ab} + \underset{\uparrow}{b^2}\end{aligned}$$

Square of the first term
Twice the product of the two terms
Square of the last term

Simplify: $(2x + 3)(2x - 3)$

$$\begin{aligned}(2x + 3)(2x - 3) &= (2x)^2 - 3^2 \\ &= 4x^2 - 9\end{aligned}$$

- This is the sum and difference of two terms.

Simplify: $(3x - 2)^2$

$$\begin{aligned}(3x - 2)^2 &= (3x)^2 + 2(3x)(-2) + (-2)^2 \\ &= 9x^2 - 12x + 4\end{aligned}$$

- This is the square of a binomial.

Example 6

Simplify: $(4z - 2w)(4z + 2w)$

Solution

$(4z - 2w)(4z + 2w) = 16z^2 - 4w^2$

You Try It 6

Simplify: $(2a + 5c)(2a - 5c)$

Your solution

Example 7

Simplify: $(2r - 3s)^2$

Solution

$(2r - 3s)^2 = 4r^2 - 12rs + 9s^2$

You Try It 7

Simplify: $(3x + 2y)^2$

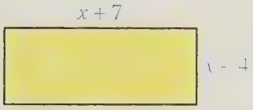
Your solution

Solutions on p.S9

Objective C To solve application problems 4

Example 8

The length of a rectangle is $(x + 7)$ m. The width is $(x - 4)$ m. Find the area of the rectangle in terms of the variable x.



Strategy

To find the area, replace the variables L and W in the equation $A = L \cdot W$ by the given values and solve for A.

Solution

$A = L \cdot W$

$A = (x + 7)(x - 4)$

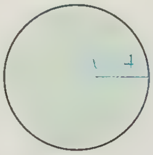
$A = x^2 - 4x + 7x - 28$

$A = x^2 + 3x - 28$

The area is $(x^2 + 3x - 28)$ m^2.

You Try It 8

The radius of a circle is $(x - 4)$ ft. Use the equation $A = \pi r^2$, where r is the radius, to find the area of the circle in terms of x. Leave the answer in terms of π.



Your strategy

Your solution

Solution on p. S9

4.3 Exercises

[illegible]

Replace the question marks to make a true statement.

1. $3(4x - 5) = (?)(4x) - (?)(5) = ?$

2. $2x^2(3x + 7) = (?)(3x) + (?)(7) = ?$

Simplify.

3. $x(x - 2)$

4. $y(3 - y)$

5. $-x(x + 7)$

6. $-y(7 - y)$

7. $3a^2(a - 2)$

8. $4b^2(b + 8)$

9. $-5x^2(x^2 - x)$

10. $-6y^2(y + 2y^2)$

11. $-x^3(3x^2 - 7)$

12. $-y^4(2y^2 - y^6)$

13. $2x(6x^2 - 3x)$

14. $3y(4y - y^2)$

15. $(2x - 4)3x$

16. $(3y - 2)y$

17. $(3x + 4)x$

18. $(2x + 1)2x$

19. $-xy(x^2 - y^2)$

20. $-x^2y(2xy - y^2)$

21. $x(2x^3 - 3x + 2)$

22. $y(-3y^2 - 2y + 6)$

23. $-a(-2a^2 - 3a - 2)$

24. $-b(5b^2 + 7b - 35)$

25. $x^2(3x^4 - 3x^2 - 2)$

26. $y^3(-4y^3 - 6y + 7)$

27. $2y^2(-3y^2 - 6y + 7)$

28. $4x^2(3x^2 - 2x + 6)$

29. $(a^2 + 3a - 4)(-2a)$

30. $(b^3 - 2b + 2)(-5b)$

31. $-3y^2(-2y^2 + y - 2)$

32. $-5x^2(3x^2 - 3x - 7)$

33. $xy(x^2 - 3xy + y^2)$

34. $ab(2a^2 - 4ab - 6b^2)$

Objective B

Simplify.

35. $(x^2 + 3x + 2)(x + 1)$

36. $(x^2 - 2x + 7)(x - 2)$

37. $(a^2 - 3a + 4)(a - 3)$

38. $(x^2 - 3x + 5)(2x - 3)$

39. $(-2b^2 - 3b + 4)(b - 5)$

40. $(-a^2 + 3a - 2)(2a - 1)$

41. $(-2x^2 + 7x - 2)(3x - 5)$

42. $(-a^2 - 2a + 3)(2a - 1)$

43. $(x^2 + 5)(x - 3)$

44. $(y^2 - 2y)(2y + 5)$

45. $(x^3 - 3x + 2)(x - 4)$

46. $(y^3 + 4y^2 - 8)(2y - 1)$

47. $(5y^2 + 8y - 2)(3y - 8)$

48. $(3y^2 + 3y - 5)(4y - 3)$

49. $(5a^3 - 5a + 2)(a - 4)$

50. $(3b^3 - 5b^2 + 7)(6b - 1)$

51. $(y^3 + 2y^2 - 3y + 1)(y + 2)$

52. $(2a^3 - 3a^2 + 2a - 1)(2a - 3)$

Simplify.

53. $(x + 1)(x + 3)$

54. $(y + 2)(y + 5)$

55. $(a - 3)(a + 4)$

56. $(b - 6)(b + 3)$

57. $(y + 3)(y - 8)$

58. $(x + 10)(x - 5)$

59. $(y - 7)(y - 3)$

60. $(a - 8)(a - 9)$

61. $(2x + 1)(x + 7)$

62. $(y + 2)(5y + 1)$

63. $(3x - 1)(x + 4)$

64. $(7x - 2)(x + 4)$

65. $(4x - 3)(x - 7)$

66. $(2x - 3)(4x - 7)$

67. $(3y - 8)(y + 2)$

68. $(5y - 9)(y + 5)$

69. $(3x + 7)(3x + 11)$

70. $(5a + 6)(6a + 5)$

71. $(7a - 16)(3a - 5)$

72. $(5a - 12)(3a - 7)$

73. $(3a - 2b)(2a - 7b)$

74. $(5a - b)(7a - b)$

75. $(a - 9b)(2a + 7b)$

76. $(2a + 5b)(7a - 2b)$

77. $(10a - 3b)(10a - 7b)$

78. $(12a - 5b)(3a - 4b)$

79. $(5x + 12y)(3x + 4y)$

80. $(11x + 2y)(3x + 7y)$

81. $(2x + 15y)(7x - 4y)$

82. $(5x + 2y)(2x - 5y)$

83. $(8x - 3y)(7x - 5y)$

84. $(2x - 9y)(8x - 3y)$

Objective D

Simplify.

85. $(y - 5)(y + 5)$

86. $(y + 6)(y - 6)$

87. $(2x + 3)(2x - 3)$

88. $(4x - 7)(4x + 7)$

89. $(x + 1)^2$

90. $(y - 3)^2$

91. $(3a - 5)^2$

92. $(6x - 5)^2$

93. $(3x - 7)(3x + 7)$

94. $(9x - 2)(9x + 2)$

95. $(2a + b)^2$

96. $(x + 3y)^2$

97. $(x - 2y)^2$

98. $(2x - 3y)^2$

99. $(4 - 3y)(4 + 3y)$

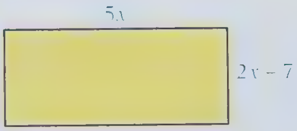
100. $(4x - 9y)(4x + 9y)$

101. $(5x + 2y)^2$

102. $(2a - 9b)^2$

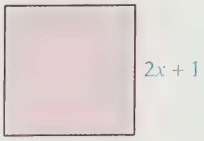
Objective E *Application Problems*

103. The length of a rectangle is $(5x)$ ft. The width is $(2x - 7)$ ft. Find the area of the rectangle in terms of the variable x.



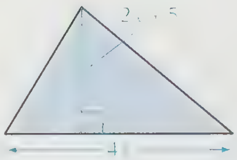
104. The width of a rectangle is $(3x + 1)$ in. The length of the rectangle is twice the width. Find the area of the rectangle in terms of the variable x.

105. The length of a side of a square is $(2x + 1)$ km. Find the area of the square in terms of the variable x.

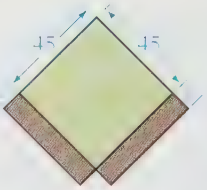


106. The radius of a circle is $(x + 4)$ cm. Find the area of the circle in terms of the variable x. Leave the answer in terms of π.

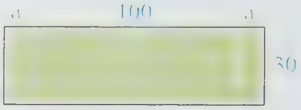
107. The base of a triangle is $(4x)$ m and the height is $(2x + 5)$ m. Find the area of the triangle in terms of the variable x.



108. A softball diamond has dimensions 45 ft by 45 ft. A base-path border x feet wide lies on both the first-base side and third-base side of the diamond. Express the total area of the softball diamond and the base paths in terms of the variable x.



109. An athletic field has dimensions 30 yd by 100 yd. An end zone that is w yards wide borders each end of the field. Express the total area of the field and the end zones in terms of the variable w.



APPLYING THE CONCEPTS

Simplify.

110. $(a + b)^2 - (a - b)^2$

111. $(x^2 + x - 3)^2$

112. $(a + 3)^3$

113. What polynomial has quotient $3x - 4$ when divided by $4x + 5$?

114. Add $x^2 + 2x - 3$ to the product of $2x - 5$ and $3x + 1$.

115. Subtract $4x^2 - x - 5$ from the product of $x^2 + x + 3$ and $x - 4$.

116. If a polynomial of degree 3 is multiplied by a polynomial of degree 2, what is the degree of the resulting polynomial?

117. Is it possible to multiply a polynomial of degree 2 by a polynomial of degree 2 and have the product be a polynomial of degree 3? If so, give an example. If not, explain why not.

4.4 Integer Exponents and Scientific Notation

To divide monomials

The quotient of two exponential expressions with the same base can be simplified by writing each expression in factored form, dividing by the common factors, and then writing the result with an exponent.

$$\frac{x^5}{x^2} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot x \cdot x \cdot x}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}}} = x^3$$

Note that subtracting the exponents gives the same result.

$$\frac{x^5}{x^2} = x^{5-2} = x^3$$

To divide two monomials with the same base, subtract the exponents of the like bases.

Simplify: $\frac{a^7}{a^3}$

$$\frac{a^7}{a^3} = a^{7-3}$$

- The bases are the same. Subtract the exponents.

$$= a^4$$

Simplify: $\frac{r^8t^6}{r^7t}$

$$\frac{r^8t^6}{r^7t} = r^{8-7}t^{6-1}$$

- Subtract the exponents of the like bases.

$$= rt^5$$

Simplify: $\frac{p^7}{z^4}$

Because the bases are not the same, $\frac{p^7}{z^4}$ is already in simplest form.

Consider the expression $\frac{x^4}{x^4}$, $x \neq 0$. This expression can be simplified, as shown below, by subtracting exponents or by dividing by common factors.

$$\frac{x^4}{x^4} = x^{4-4} = x^0 \qquad \frac{x^4}{x^4} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}}} = 1$$

The equations $\frac{x^4}{x^4} = x^0$ and $\frac{x^4}{x^4} = 1$ suggest the following definition of x^0.

Definition of Zero as an Exponent

If $x \neq 0$, then $x^0 = 1$. The expression 0^0 is not defined.

TAKE NOTE

In the example at the right, we indicated that $a \neq 0$. If we try to evaluate $(12a^3)^0$ when $a = 0$, we have

$$[12(0)^3]^0 = [12(0)]^0 = 0^0$$

However, 0^0 is not defined. Therefore, we must assume that $a \neq 0$. To avoid stating this for every example or exercise, we will assume that variables do not have values that result in the expression 0^0.

Simplify: $(12a^3)^0$, $a \neq 0$

$(12a^3)^0 = 1$

- Any nonzero expression to the zero power is 1.

Simplify: $-(4x^3y^7)^0$

$-(4x^3y^7)^0 = -(1) = -1$

Consider the expression $\frac{x^4}{x^6}$, $x \neq 0$. This expression can be simplified, as shown below, by subtracting exponents or by dividing by common factors.

$$\frac{x^4}{x^6} = x^{4-6} = x^{-2} \qquad \frac{x^4}{x^6} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x} = \frac{1}{x^2}$$

Point of Interest

In the 15th century, the expression $12^{2\overline{m}}$ was used to mean $12x^{-2}$. The use of $\overline{m}$ reflected an Italian influence. In Italy, *m* was used for minus and *p* was used for plus. It was understood that $2\overline{m}$ referred to an unnamed variable. Issac Newton, in the 17th century, advocated the negative exponent we currently use.

The equations $\frac{x^4}{x^6} = x^{-2}$ and $\frac{x^4}{x^6} = \frac{1}{x^2}$ suggest that $x^{-2} = \frac{1}{x^2}$.

Definition of a Negative Exponent

If $x \neq 0$ and n is a positive integer, then

$$x^{-n} = \frac{1}{x^n} \qquad \text{and} \qquad \frac{1}{x^{-n}} = x^n$$

TAKE NOTE

Note from the example at the right that 2^{-4} is a *positive* number. A negative exponent does not change the sign of a number.

An exponential expression is in simplest form when it is written with only positive exponents.

Evaluate 2^{-4}.

$2^{-4} = \frac{1}{2^4}$

- Use the Definition of a Negative Exponent.

$= \frac{1}{16}$

- Evaluate the expression.

TAKE NOTE

For the expression $3n^{-5}$ the exponent on n is -5 (*negative* 5). The n^{-5} is written in the denominator as n^5. The exponent on 3 is 1 (*positive* 1). The 3 remains in the numerator. Also, we indicated that $n \neq 0$. This is done because division by zero is not defined. In this textbook, we will assume that values of the variables are chosen so that division by zero does not occur.

Simplify: $3n^{-5}$, $n \neq 0$

$3n^{-5} = 3 \cdot \frac{1}{n^5} = \frac{3}{n^5}$

- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

Simplify: $\frac{2}{5a^{-4}}$

$\frac{2}{5a^{-4}} = \frac{2}{5} \cdot \frac{1}{a^{-4}} = \frac{2}{5} \cdot a^4 = \frac{2a^4}{5}$

- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

The expression $\left(\frac{x^4}{y^3}\right)^2$, $y \neq 0$, can be simplified by squaring $\frac{x^4}{y^3}$ or by multiplying each exponent in the quotient by the exponent outside the parentheses.

$$\left(\frac{x^4}{y^3}\right)^2 = \left(\frac{x^4}{y^3}\right)\left(\frac{x^4}{y^3}\right) = \frac{x^4 \cdot x^4}{y^3 \cdot y^3} = \frac{x^{4+4}}{y^{3+3}} = \frac{x^8}{y^6} \qquad \left(\frac{x^4}{y^3}\right)^2 = \frac{x^{4 \cdot 2}}{y^{3 \cdot 2}} = \frac{x^8}{y^6}$$

Rule for Simplifying Powers of Quotients

If m, n, and q are integers and $y \neq 0$, then $\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}$.

TAKE NOTE

As a reminder, although it is not stated, we are assuming that $a \neq 0$ and $b \neq 0$. This is done to ensure that we do not have division by zero.

Simplify: $\left(\frac{a^3}{b^2}\right)^{-2}$

$$\left(\frac{a^3}{b^2}\right)^{-2} = \frac{a^{3(-2)}}{b^{2(-2)}}$$

- Use the Rule for Simplifying Powers of Quotients.

$$= \frac{a^{-6}}{b^{-4}} = \frac{b^4}{a^6}$$

- Use the Definition of a Negative Exponent to write the expression with positive exponents.

The example above suggests the following rule.

Rule for Negative Exponents on Fractional Expressions

If $a \neq 0$, $b \neq 0$, and n is a positive integer, then

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

Now that zero as an exponent and negative exponents have been defined, a rule for dividing exponential expressions can be stated.

Rule for Dividing Exponential Expressions

If m and n are integers and $x \neq 0$, then $\frac{x^m}{x^n} = x^{m-n}$.

Evaluate $\frac{5^{-2}}{5}$.

$$\frac{5^{-2}}{5} = 5^{-2-1} = 5^{-3}$$

- Use the Rule for Dividing Exponential Expressions.

$$= \frac{1}{5^3} = \frac{1}{125}$$

- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent. Then evaluate.

Simplify: $\frac{x^4}{x^9}$

$$\frac{x^4}{x^9} = x^{4-9}$$

- Use the Rule for Dividing Exponential Expressions.

$$= x^{-5}$$

- Subtract the exponents.

$$= \frac{1}{x^5}$$

- Use the Definition of a Negative Exponent to rewrite the expression with a positive exponent.

The rules for simplifying exponential expressions and powers of exponential expressions are true for all integers. These rules are restated here, along with the rules for dividing exponential expressions.

Rules of Exponents

If m, n, and p are integers, then

$x^m \cdot x^n = x^{m+n}$ $\quad (x^m)^n = x^{mn}$ $\quad (x^m y^n)^p = x^{mp}y^{np}$

$\frac{x^m}{x^n} = x^{m-n}, x \neq 0$ $\quad \left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}, y \neq 0$ $\quad x^{-n} = \frac{1}{x^n}, x \neq 0$

$x^0 = 1, x \neq 0$

Simplify: $(3ab^{-4})(-2a^{-3}b^7)$

$$\begin{aligned}(3ab^{-4})(-2a^{-3}b^7) &= [3 \cdot (-2)](a^{1+(-3)}b^{-4+7}) \\ &= -6a^{-2}b^3 \\ &= -\frac{6b^3}{a^2}\end{aligned}$$

- When multiplying expressions, add the exponents on like bases

Simplify: $\left[\frac{6m^2n^3}{8m^7n^2}\right]^{-3}$

$$\left[\frac{6m^2n^3}{8m^7n^2}\right]^{-3} = \left[\frac{3m^{2-7}n^{3-2}}{4}\right]^{-3}$$

- Simplify inside the brackets.

$$= \left[\frac{3m^{-5}n}{4}\right]^{-3}$$

- Subtract the exponents.

$$= \frac{3^{-3}m^{15}n^{-3}}{4^{-3}}$$

- Use the Rule for Simplifying Powers of Quotients.

$$= \frac{4^3m^{15}}{3^3n^3} = \frac{64m^{15}}{27n^3}$$

- Use the Definition of a Negative Exponent to rewrite the expression with positive exponents. Then simplify.

Simplify: $\dfrac{4a^{-2}b^5}{6a^5b^2}$

$$\frac{4a^{-2}b^5}{6a^5b^2} = \frac{2a^{-2}b^5}{3a^5b^2}$$

- Divide the coefficients by their common factor.

$$= \frac{2a^{-2-5}b^{5-2}}{3}$$

- Use the Rule for Dividing Exponential Expressions.

$$= \frac{2a^{-7}b^3}{3} = \frac{2b^3}{3a^7}$$

- Use the Definition of a Negative Exponent to rewrite the expression with positive exponents.

Example 1 Simplify: $(-2x)(3x^{-2})^{-3}$

Solution

$$(-2x)(3x^{-2})^{-3} = (-2x)(3^{-3}x^6)$$

$$= \frac{-2x^{1+6}}{3^3}$$

$$= -\frac{2x^7}{27}$$

You Try It 1 Simplify: $(-2x^2)(x^{-3}y^{-4})^{-2}$

Your solution

Example 2

Simplify: $\dfrac{(2r^2t^{-1})^{-3}}{(r^{-3}t^4)^2}$

Solution

$$\frac{(2r^2t^{-1})^{-3}}{(r^{-3}t^4)^2} = \frac{2^{-3}r^{-6}t^3}{r^{-6}t^8}$$

$$= 2^{-3}r^{-6-(-6)}t^{3-8}$$

$$= 2^{-3}r^0t^{-5}$$

$$= \frac{1}{2^3t^5}$$

$$= \frac{1}{8t^5}$$

You Try It 2

Simplify: $\dfrac{(6a^{-2}b^3)^{-1}}{(4a^3b^{-2})^{-2}}$

Your solution

Example 3

Simplify: $\left[\dfrac{4a^{-2}b^3}{6a^4b^{-2}}\right]^{-3}$

Solution

$$\left[\frac{4a^{-2}b^3}{6a^4b^{-2}}\right]^{-3} = \left[\frac{2a^{-6}b^5}{3}\right]^{-3}$$

$$= \frac{2^{-3}a^{18}b^{-15}}{3^{-3}}$$

$$= \frac{27a^{18}}{8b^{15}}$$

You Try It 3

Simplify: $\left[\dfrac{6r^3s^{-3}}{9r^3s^{-1}}\right]^{-2}$

Your solution

Solutions on p. S9

To write a number in scientific notation

Very large and very small numbers abound in the natural sciences. For example, the mass of an electron is 0.000000000000000000000000000000911 kg. Numbers such as this are difficult to read, so a more convenient system called **scientific notation** is used. In scientific notation, a number is expressed as the product of two factors, one a number between 1 and 10, and the other a power of 10.

Point of Interest

An electron microscope uses wavelengths that are approximately 4×10^{-12} meter to make images of viruses.

The human eye can detect wavelengths between 4.3×10^{-7} meter and 6.9×10^{-7} meter. Although these are very short, they are approximately 10^5 times longer than the waves used in an electron microscope.

To express a number in scientific notation, write it in the form $a \times 10^n$, where a is a number between 1 and 10, and n is an integer.

For numbers greater than or equal to 10, move the decimal point to the right of the first digit. The exponent n is positive and equal to the number of places the decimal point has been moved.

$$240{,}000 = 2.4 \times 10^5$$

$$93{,}000{,}000 = 9.3 \times 10^7$$

For numbers less than 1, move the decimal point to the right of the first nonzero digit. The exponent n is negative. The absolute value of the exponent is equal to the number of places the decimal point has been moved.

$$0.0003 = 3 \times 10^{-4}$$

$$0.0000832 = 8.32 \times 10^{-5}$$

Changing a number written in scientific notation to decimal notation also requires moving the decimal point.

When the exponent is positive, move the decimal point to the right the same number of places as the exponent.

$$3.45 \times 10^6 = 3{,}450{,}000$$

$$2.3 \times 10^8 = 230{,}000{,}000$$

When the exponent is negative, move the decimal point to the left the same number of places as the absolute value of the exponent.

$$8.1 \times 10^{-3} = 0.0081$$

$$6.34 \times 10^{-7} = 0.000000634$$

Example 4 Write the number 824,300,000 in scientific notation.

Solution $824{,}300{,}000 = 8.243 \times 10^8$

You Try It 4 Write the number 0.000000961 in scientific notation.

Your solution

Example 5 Write the number 6.8×10^{-10} in decimal notation.

Solution $6.8 \times 10^{-10} = 0.00000000068$

You Try It 5 Write the number 7.329×10^6 in decimal notation.

Your solution

Solutions on p. S9

4.4 Exercises

Objective A

Replace the question marks to make a true statement.

1. $\frac{x^7}{x^5} = x^{?-?} = x^?$

2. $\frac{a^9}{a^{12}} = a^{?\ ?} = \frac{1}{a^?}$

Evaluate.

3. 5^{-2}

4. 3^{-3}

5. $\frac{1}{8^{\ 2}}$

6. $\frac{1}{12^{-1}}$

7. $\frac{3^{-2}}{3}$

8. $\frac{5^{-3}}{5}$

9. $\frac{2^{\ 2}}{2^{\ 3}}$

10. $\frac{3^2}{3^2}$

Simplify.

11. x^{-2}

12. y^{-10}

13. $\frac{1}{a^{-6}}$

14. $\frac{1}{b^{\ 4}}$

15. $4x^{-7}$

16. $-6y^{-1}$

17. $\frac{2}{3}z^{-2}$

18. $\frac{4}{5}a^{-4}$

19. $\frac{5}{b^{-8}}$

20. $\frac{-3}{v^{-3}}$

21. $\frac{1}{3x^{-2}}$

22. $\frac{2}{5c^{\ 6}}$

23. $(ab^5)^0$

24. $(32x^3y^4)^0$

25. $-(3p^2q^5)^0$

26. $-\left(\frac{2}{3}xy\right)^0$

27. $\frac{y^7}{y^3}$

28. $\frac{z^9}{z^2}$

29. $\frac{a^8}{a^5}$

30. $\frac{c^{12}}{c^5}$

31. $\frac{p^5}{p}$

32. $\frac{w^9}{w}$

33. $\frac{4x^8}{2x^5}$

34. $\frac{12z^7}{4z^3}$

35. $\frac{22k^5}{11k^4}$

36. $\frac{14m^{11}}{7m^{10}}$

37. $\frac{m^9n^7}{m^4n^5}$

38. $\frac{y^5z^6}{yz^3}$

39. $\frac{6r^4}{4r^2}$

40. $\frac{8x^9}{12x^6}$

41. $\frac{-16a^7}{24a^6}$

42. $\frac{-18b^5}{27b^4}$

43. $\frac{y^3}{y^8}$

44. $\frac{z^4}{z^6}$

45. $\frac{a^5}{a^{11}}$

46. $\frac{m}{m^7}$

47. $\frac{4x^2}{12x^5}$

48. $\frac{6y^8}{8y^9}$

49. $\frac{-12x}{-18x^6}$

50. $\frac{-24c^2}{-36c^{11}}$

51. $\frac{x^6y^5}{x^8y}$

52. $\frac{a^3b^2}{a^2b^3}$

53. $\frac{2m^6n^2}{5m^9n^{10}}$

54. $\frac{5r^3t^7}{6r^5t^7}$

55. $\frac{pq^3}{p^4q^4}$

56. $\frac{a^4b^5}{a^5b^6}$

57. $\frac{3x^4y^5}{6x^4y^8}$

58. $\frac{14a^3b^6}{21a^5b^6}$

59. $\frac{14x^4y^6z^2}{16x^3y^9z}$

60. $\frac{24a^2b^7c^9}{36a^7b^5c}$

61. $\frac{15mn^9p^3}{30m^4n^9p}$

62. $\frac{25x^4y^7z^2}{20x^5y^9z^{11}}$

63. $(-2xy^{-2})^3$

64. $(-3x^{-1}y^2)^2$

65. $(3x^{-1}y^{-2})^2$

66. $(5xy^{-3})^{-2}$

67. $(2x^{-1})(x^{-3})$

68. $(-2x^{-5})x^7$

69. $(-5a^2)(a^{-5})^2$

70. $(2a^{-3})(a^7b^{-1})^3$

71. $(-2ab^{-2})(4a^{-2}b)^{-2}$

72. $(3ab^{-2})(2a^{-1}b)^{-3}$

73. $(-5x^{-2}y)(-2x^{-2}y^2)$

74. $\frac{a^{-3}b^{-4}}{a^2b^2}$

75. $\frac{3x^{-2}y^2}{6xy^2}$

76. $\frac{2x^{-2}y}{8xy}$

77. $\frac{3x^{-2}y}{xy}$

78. $\frac{2x^{-1}y^4}{x^2y^3}$

79. $\frac{2x^{-1}y^{-4}}{4xy^2}$

80. $\frac{(x^{-1}y)^2}{xy^2}$

81. $\frac{(x^{-2}y)^2}{x^2y^3}$

82. $\frac{(x^{-3}y^{-2})^2}{x^6y^8}$

83. $\dfrac{(a^{-2}y^{3})^{-3}}{a^{2}y}$

84. $\dfrac{12a^{2}b^{3}}{-27a^{2}b^{2}}$

85. $\dfrac{-16xy^{4}}{96x^{4}y^{4}}$

86. $\dfrac{-8x^{2}y^{4}}{44y^{2}z^{5}}$

87. $\dfrac{22a^{2}b^{4}}{-132b^{3}c^{2}}$

88. $\dfrac{-(8a^{2}b^{4})^{3}}{64a^{3}b^{8}}$

89. $\dfrac{-(14ab^{4})^{2}}{28a^{4}b^{2}}$

90. $\dfrac{(2a^{-2}b^{3})^{-2}}{(4a^{2}b^{-4})^{-1}}$

91. $\dfrac{(3^{-1}r^{4}s^{-3})^{-2}}{(6r^{2}s^{-1}t^{-2})^{2}}$

92. $\left(\dfrac{6x^{-4}yz^{-1}}{14xy^{-4}z^{2}}\right)^{3}$

93. $\left(\dfrac{15m^{3}n^{-2}p^{-1}}{25m^{-2}n^{-4}}\right)^{3}$

94. $\left(\dfrac{18a^{4}b^{-2}c^{4}}{12ab^{-3}d^{2}}\right)^{2}$

[illegible]

95. Why might a number be written in scientific notation instead of decimal notation?

96. **a.** Explain how to write 0.00000076 in scientific notation.
b. Explain how to write 4.3×10^{8} in decimal notation.

Write in scientific notation.

97. 0.00000000324

98. 0.00000012

99. 0.000000000000000003

100. 1,800,000,000

101. 32,000,000,000,000,000

102. 76,700,000,000,000

103. 0.000000000000000000122

104. 0.00137

105. 547,000,000

Write in decimal notation.

106. 2.3×10^{-12}

107. 1.67×10^{-4}

108. 2×10^{15}

109. 6.8×10^{7}

110. 9×10^{-21}

111. 3.05×10^{-5}

112. 9.05×10^{11}

113. 1.02×10^{-9}

114. 7.2×10^{-3}

115. Avogadro's number is used in chemistry, and its value is approximately 602,300,000,000,000,000,000,000. Express this number in scientific notation.

116. 5,980,000,000,000,000,000,000,000 kg is the approximate mass of the planet Earth. Write the mass of Earth in scientific notation.

117. The length of an infrared light wave is approximately 0.0000037 m. Write this number in scientific notation.

118. Light travels approximately 16,000,000,000 mi in one day. Write this number in scientific notation.

119. One unit used to measure the speed of a computer is the picosecond. One picosecond is 0.000000001 s. Write this number in scientific notation.

120. One light-year is the distance traveled by light in 1 year. One light-year is 5,880,000,000,000 mi. Write this number in scientific notation.

121. The electric charge on an electron is 0.00000000000000000016 coulomb. Write this number in scientific notation.

122. Approximately 35 teragrams (3.5×10^{13} g) of sulfur in the atmosphere is converted to sulfate each year. Write this number in decimal notation.

APPLYING THE CONCEPTS

123. Evaluate 2^x when $x = -2, -1, 0, 1,$ and 2.

124. Evaluate 3^x when $x = -2, -1, 0, 1,$ and 2.

125. Evaluate 2^{-x} when $x = -2, -1, 0, 1,$ and 2.

126. Evaluate 3^{-x} when $x = -2, -1, 0, 1,$ and 2.

Determine whether each equation for Exercises 127 to 129 is true or false. If the equation is false, change the right-hand side of the equation to make a true equation.

127. $(2a)^{-3} = \dfrac{2}{a^3}$

128. $\dfrac{x^{-3}}{y^{-3}} = \left(\dfrac{x}{y}\right)^{-3}$

129. $(2 + 3)^{-1} = 2^{-1} + 3^{-1}$

130. Simplify: $\left(\dfrac{6x^4yz^3}{2x^2y^3}\right)\left(\dfrac{2x^2z^3}{4y^2z}\right) \div \left(\dfrac{6x^2y^3}{x^4y^2z}\right)$

131. If x is a nonzero real number, is x^{-2} always positive, always negative, or positive or negative depending on whether x is positive or negative? Explain your answer.

4.5 Division of Polynomials

Objective A

To divide a polynomial by a monomial

To divide a polynomial by a monomial, divide each term in the numerator by the denominator and write the sum of the quotients.

Simplify: $\dfrac{6x^3 - 3x^2 + 9x}{3x}$

$$\frac{6x^3 - 3x^2 + 9x}{3x} = \frac{6x^3}{3x} - \frac{3x^2}{3x} + \frac{9x}{3x}$$

- Divide each term of the polynomial by the monomial.

$$= 2x^2 - x + 3$$

- Simplify each expression.

Example 1

Simplify: $\dfrac{12x^2y - 6xy + 4x^2}{2xy}$

Solution

$$\frac{12x^2y - 6xy + 4x^2}{2xy} = \frac{12x^2y}{2xy} - \frac{6xy}{2xy} + \frac{4x^2}{2xy} = 6x - 3 + \frac{2x}{y}$$

You Try It 1

Simplify: $\dfrac{24x^2y^2 - 18xy + 6y}{6xy}$

Your solution

Solution on p. S9

Objective B

To divide polynomials

The procedure for dividing two polynomials is similar to the one for dividing whole numbers. The same equation used to check division of whole numbers is used to check polynomial division.

(Quotient × divisor) + remainder = dividend

Simplify: $(x^2 - 5x + 8) \div (x - 3)$

Step 1

$$\begin{array}{r} x\phantom{{}-5x+8} \\ x-3\overline{)x^2-5x+8} \\ \underline{x^2-3x}\phantom{{}+8} \\ -2x+8 \end{array}$$

- Think: $x\overline{)x^2} = \dfrac{x^2}{x} = x$
- Multiply: $x(x - 3) = x^2 - 3x$
- Subtract: $(x^2 - 5x) - (x^2 - 3x) = -2x$
Bring down the 8.

Step 2

$$\begin{array}{r} x-2\phantom{{}+8} \\ x-3\overline{)x^2-5x+8} \\ \underline{x^2-3x}\phantom{{}+8} \\ -2x+8 \\ \underline{-2x+6} \\ 2 \end{array}$$

- Think: $x\overline{)-2x} = \dfrac{-2x}{x} = -2$
- Multiply: $-2(x - 3) = -2x + 6$
- Subtract: $(-2x + 8) - (-2x + 6) = 2$
- The remainder is 2.

Check: $(x - 2)(x - 3) + 2 = x^2 - 5x + 6 + 2 = x^2 - 5x + 8$

$$(x^2 - 5x + 8) \div (x - 3) = x - 2 + \frac{2}{x - 3}$$

TAKE NOTE
Recall that a fraction bar means "divided by." Therefore, $6 \div 2$ can be written $\frac{6}{2}$, and $a \div b$ can be written $\frac{a}{b}$.

If a term is missing from the dividend, a zero can be inserted for that term. This helps keep like terms in the same column.

Simplify: $\dfrac{6x + 26 + 2x^3}{2 + x}$

$$\frac{2x^3 + 6x + 26}{x + 2}$$

- Arrange the terms of each polynomial in descending order.

$$
\begin{array}{r}
2x^2 - 4x + 14 \\
x + 2\overline{)2x^3 + 0 \quad + 6x + 26} \\
\underline{2x^3 + 4x^2} \qquad\qquad \\
-4x^2 + 6x \qquad \\
\underline{-4x^2 - 8x} \qquad \\
14x + 26 \\
\underline{14x + 28} \\
-2
\end{array}
$$

- There is no x^2 term in $2x^3 + 6x + 26$. Insert a zero for the missing term.

Check:

$(2x^2 - 4x + 14)(x + 2) + (-2) = (2x^3 + 6x + 28) + (-2) = 2x^3 + 6x + 26$

$(2x^3 + 6x + 26) \div (x + 2) = 2x^2 - 4x + 14 - \dfrac{2}{x + 2}$

Example 2

Simplify: $(8x^2 + 4x^3 + x - 4) \div (2x + 3)$

Solution

$$
\begin{array}{r}
2x^2 + x - 1 \\
2x + 3\overline{)4x^3 + 8x^2 + x - 4} \\
\underline{4x^3 + 6x^2} \qquad\quad \\
2x^2 + x \qquad \\
\underline{2x^2 + 3x} \qquad \\
-2x - 4 \\
\underline{-2x - 3} \\
-1
\end{array}
$$

- Write the dividend in descending powers of x.

$(4x^3 + 8x^2 + x - 4) \div (2x + 3)$

$= 2x^2 + x - 1 - \dfrac{1}{2x + 3}$

You Try It 2

Simplify: $(2x^3 + x^2 - 8x - 3) \div (2x - 3)$

Your solution

Example 3

Simplify: $\dfrac{x^2 - 1}{x + 1}$

Solution

$$
\begin{array}{r}
x - 1 \\
x + 1\overline{)x^2 + 0 - 1} \\
\underline{x^2 + x} \qquad \\
-x - 1 \\
\underline{-x - 1} \\
0
\end{array}
$$

- Insert a zero for the missing term.

$(x^2 - 1) \div (x + 1) = x - 1$

You Try It 3

Simplify: $\dfrac{x^3 - 2x + 1}{x - 1}$

Your solution

Solutions on p. S10

4.5 Exercises

[illegible]

Simplify.

1. $\dfrac{10a-25}{5}$

2. $\dfrac{16b-40}{8}$

3. $\dfrac{6y^2+4y}{y}$

4. $\dfrac{4b^3-3b}{b}$

5. $\dfrac{3x^2-6x}{3x}$

6. $\dfrac{10y^2-6y}{2y}$

7. $\dfrac{5x^2-10x}{-5x}$

8. $\dfrac{3y^2-27y}{-3y}$

9. $\dfrac{x^3+3x^2-5x}{x}$

10. $\dfrac{a^3-5a^2+7a}{a}$

11. $\dfrac{x^6-3x^4-x^2}{x^2}$

12. $\dfrac{a^8-5a^5-3a^3}{a^2}$

13. $\dfrac{5x^2y^2+10xy}{5xy}$

14. $\dfrac{8x^2y^2-24xy}{8xy}$

15. $\dfrac{9y^6-15y^3}{-3y^3}$

16. $\dfrac{4x^4-6x^2}{-2x^2}$

17. $\dfrac{3x^2-2x+1}{x}$

18. $\dfrac{8y^2+2y-3}{y}$

19. $\dfrac{-3x^2+7x-6}{x}$

20. $\dfrac{2y^2-6y+9}{y}$

21. $\dfrac{16a^2b-20ab+24ab^2}{4ab}$

22. $\dfrac{22a^2b-11ab-33ab^2}{11ab}$

23. $\dfrac{9x^2y+6xy-3xy^2}{xy}$

24. $\dfrac{5a^2b-15ab+30ab^2}{5ab}$

[illegible]

Replace the question marks to make a true statement.

25. If $\dfrac{x^2-x-6}{x-3}=x+2$, then
$x^2-x-6=(?)(?)$.

26. If $\dfrac{x^2+2x-3}{x-2}=x+4+\dfrac{5}{x-2}$, then
$x^2+2x-3=(?)(?)+?$.

Simplify.

27. $(b^2-14b+49)\div(b-7)$

28. $(x^2-x-6)\div(x-3)$

29. $(y^2+2y-35)\div(y+7)$

30. $(2x^2+5x+2)\div(x+2)$

31. $(2y^2-13y+21)\div(y-3)$

32. $(4x^2-16)\div(2x+4)$

33. $\dfrac{2y^2 + 7}{y - 3}$

34. $\dfrac{x^2 + 1}{x - 1}$

35. $\dfrac{x^2 + 4}{x + 2}$

36. $\dfrac{6x^2 - 7x}{3x - 2}$

37. $\dfrac{6y^2 + 2y}{2y + 4}$

38. $\dfrac{5x^2 + 7x}{x - 1}$

39. $(6x^2 - 5) \div (x + 2)$

40. $(a^2 + 5a + 10) \div (a + 2)$

41. $(b^2 - 8b - 9) \div (b - 3)$

42. $\dfrac{2y^2 - 9y + 8}{2y + 3}$

43. $\dfrac{3x^2 + 5x - 4}{x - 4}$

44. $(8x + 3 + 4x^2) \div (2x - 1)$

45. $(10 + 21y + 10y^2) \div (2y + 3)$

46. $\dfrac{15a^2 - 8a - 8}{3a + 2}$

47. $\dfrac{12a^2 - 25a - 7}{3a - 7}$

48. $(5 - 23x + 12x^2) \div (4x - 1)$

49. $(24 + 6a^2 + 25a) \div (3a - 1)$

50. $\dfrac{5x + 3x^2 + x^3 + 3}{x + 1}$

51. $\dfrac{7x + x^3 - 6x^2 - 2}{x - 1}$

52. $(x^4 - x^2 - 6) \div (x^2 + 2)$

53. $(x^4 + 3x^2 - 10) \div (x^2 - 2)$

APPLYING THE CONCEPTS

54. In your own words, explain how to divide exponential expressions.

55. The product of a monomial and $4b$ is $12ab^2$. Find the monomial.

Focus on Problem Solving

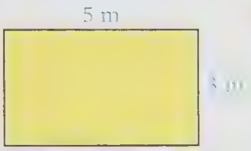
Dimensional Analysis

In solving application problems, it may be useful to include the units in order to organize the problem so that the answer is in the proper units. Using units to organize and check the correctness of an application is called **dimensional analysis.** We use the operations of multiplying units and dividing units in applying dimensional analysis to application problems.

The Rule for Multiplying Exponential Expressions states that we multiply two expressions with the same base by adding the exponents.

$$x^4 \cdot x^6 = x^{4+6} = x^{10}$$

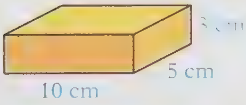
In calculations that involve quantities, the units are operated on algebraically.



A rectangle measures 3 m by 5 m. Find the area of the rectangle.

$A = LW = (3\text{ m})(5\text{ m}) = (3 \cdot 5)(\text{m} \cdot \text{m}) = 15\text{ m}^2$

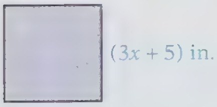
The area of the rectangle is 15 m^2 (square meters).



A box measures 10 cm by 5 cm by 3 cm. Find the volume of the box.

$V = LWH = (10\text{ cm})(5\text{ cm})(3\text{ cm}) = (10 \cdot 5 \cdot 3)(\text{cm} \cdot \text{cm} \cdot \text{cm}) = 150\text{ cm}^3$

The volume of the box is 150 cm^3 (cubic centimeters).



Find the area of a square whose side measures $(3x + 5)$ in.

$A = s^2 = [(3x + 5)\text{ in.}]^2 = (3x + 5)^2\text{ in}^2 = (9x^2 + 30x + 25)\text{ in}^2$

The area of the square is $(9x^2 + 30x + 25)$ in^2 (square inches).

Dimensional analysis is used in the conversion of units.

The following example converts the unit miles to feet. The equivalent measures 1 mi = 5280 ft are used to form the following rates, which are called conversion factors: $\frac{1\text{ mi}}{5280\text{ ft}}$ and $\frac{5280\text{ ft}}{1\text{ mi}}$. Because 1 mi = 5280 ft, both of the conversion factors $\frac{1\text{ mi}}{5280\text{ ft}}$ and $\frac{5280\text{ ft}}{1\text{ mi}}$ are equal to 1.

To convert 3 mi to feet, multiply 3 mi by the conversion factor $\frac{5280\text{ ft}}{1\text{ mi}}$.

$$3\text{ mi} = 3\text{ mi} \cdot \boxed{1} = \frac{3\text{ mi}}{1} \cdot \boxed{\frac{5280\text{ ft}}{1\text{ mi}}} = \frac{3\,\cancel{\text{mi}} \cdot 5280\text{ ft}}{1\,\cancel{\text{mi}}} = 3 \cdot 5280\text{ ft} = 15{,}840\text{ ft}$$

There are two important points in the above illustration. First, you can think of dividing the numerator and denominator by the common unit "mile" just as you would divide the numerator and denominator of a fraction by a common factor. Second, the conversion factor $\frac{5280\text{ ft}}{1\text{ mi}}$ is equal to 1, and multiplying an expression by 1 does not change the value of the expression.

In the application problem that follows, the units are kept in the problem while the problem is worked.

In 2000, a horse named Fusaichi Pegasus ran a 1.25-mile race in 2.02 min. Find Fusaichi Pegasus's average speed for that race in miles per hour. Round to the nearest tenth.

Strategy To find the average speed, use the formula $r = \frac{d}{t}$, where r is the speed, d is the distance, and t is the time. Use the conversion factor $\frac{60 \text{ min}}{1 \text{ h}}$.

Solution

$$r = \frac{d}{t} = \frac{1.25 \text{ mi}}{2.02 \text{ min}} = \frac{1.25 \text{ mi}}{2.02 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ h}}$$

$$= \frac{75 \text{ mi}}{2.02 \text{ h}} \approx 37.1 \text{ mph}$$

Fusaichi Pegasus's average speed was 37.1 mph.

Try each of the following problems. Round to the nearest tenth.

1. Convert 88 ft/s to miles per hour.
2. Convert 8 m/s to kilometers per hour (1 km = 1000 m).
3. A carpet is to be placed in a meeting hall that is 36 ft wide and 80 ft long. At $21.50 per square yard, how much will it cost to carpet the meeting hall?
4. A carpet is to be placed in a room that is 20 ft wide and 30 ft long. At $22.25 per square yard, how much will it cost to carpet the area?
5. Find the number of gallons of water in a fish tank that is 36 in. long and 24 in. wide and is filled to a depth of 16 in. (1 gal = 231 in^3).
6. Find the number of gallons of water in a fish tank that is 24 in. long and 18 in. wide and is filled to a depth of 12 in. (1 gal = 231 in^3).
7. A $\frac{1}{4}$-acre commercial lot is on sale for $2.15 per square foot. Find the sale price of the commercial lot (1 acre = 43,560 ft^2).
8. A 0.75-acre industrial parcel was sold for $98,010. Find the parcel's price per square foot (1 acre = 43,560 ft^2).
9. A new driveway will require 800 ft^3 of concrete. Concrete is ordered by the cubic yard. How much concrete should be ordered?
10. A piston-engined dragster traveled 440 yd in 4.936 s at Ennis, Texas, on October 9, 1988. Find the average speed of the dragster in miles per hour.
11. The Marianas Trench in the Pacific Ocean is the deepest part of the ocean. Its depth is 6.85 mi. The speed of sound under water is 4700 ft/s. Find the time it takes sound to travel from the surface to the bottom of the Marianas Trench and back.

Projects and Group Activities

Pascal's Triangle

Simplifying the power of a binomial is called *expanding the binomial*. The expansions of the first three powers of a binomial are shown below.

$$(a+b)^1 = a + b$$

$$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$$

$$(a+b)^3 = (a+b)^2(a+b) = (a^2 + 2ab + b^2)(a+b) = a^3 + 3a^2b + 3ab^2 + b^3$$

Find $(a+b)^4$. [*Hint*: $(a+b)^4 = (a+b)^3(a+b)$]

Find $(a+b)^5$. [*Hint*: $(a+b)^5 = (a+b)^4(a+b)$]

If we continue in this way, the results for $(a+b)^6$ are

$$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

Now expand $(a+b)^8$. Before you begin, see whether you can find a pattern that will help you write the expansion of $(a+b)^8$ without having to multiply it out. Here are some hints.

1. Write out the variable terms of each binomial expansion from $(a+b)^1$ through $(a+b)^6$. Observe how the exponents on the variables change.
2. Write out the coefficients of all the terms without the variable parts. It will be helpful to make a triangular arrangement as shown at the left. Note that each row begins and ends with a 1. Also note (in the two shaded regions, for example) that any number in a row is the sum of the two closest numbers above it. For instance, $1 + 5 = 6$ and $6 + 4 = 10$.

1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

The triangle of numbers shown at the left is called Pascal's Triangle. To find the expansion of $(a+b)^8$, you need to find the eighth row of Pascal's Triangle. First find row seven. Then find row eight and use the patterns you have observed to write the expansion $(a+b)^8$.

Pascal's Triangle has been the subject of extensive analysis, and many patterns have been found. See whether you can find some of them.

Point of Interest

Pascal did not invent the triangle of numbers known as Pascal's Triangle. It was known to mathematicians in China probably as early as 1050 A.D. But Pascal's *Traite du triangle arithmetique* (*Treatise Concerning the Arithmetical Triangle*) brought together all the different aspects of the numbers for the first time.

Chapter Summary

Key Words

A *monomial* is a number, a variable, or a product of a number and variables. [p. 165]

A *polynomial* is a variable expression in which the terms are monomials. A polynomial of one term is a *monomial*. A polynomial of two terms is a *binomial*. A polynomial of three terms is a *trinomial*. [p. 165]

The *degree of a polynomial* in one variable is the greatest of the degrees of any of its terms. [p. 165]

The *opposite of a polynomial* is the polynomial with the sign of every term changed. [p. 166]

A number written in *scientific notation* is a number written in the form $a \times 10^n$, where $1 \le a < 10$, and n is an integer. [p. 186]

Essential Rules

Addition of Polynomials [p.165]	To add polynomials, add the coefficients of the like terms.
Subtraction of Polynomials [p.166]	To subtract two polynomials, add the opposite of the second polynomial to the first.
Rule for Multiplying Exponential Expressions [p.169]	$x^m \cdot x^n = x^{m+n}$
Rule for Simplifying the Power of an Exponential Expression [p.170]	$(x^m)^n = x^{mn}$
The FOIL Method [p.174]	To multiply two binomials, add the products of the **F**irst terms, the **O**uter terms, the **I**nner terms, and the **L**ast terms.
The Sum and Difference of Two Terms [p.175]	$(a+b)(a-b) = a^2 - b^2$
The Square of a Binomial [p.175]	$(a+b)^2 = a^2 + 2ab + b^2$ $(a-b)^2 = a^2 - 2ab + b^2$
Rule for Simplifying Powers of Products [p.170]	$(x^m y^n)^p = x^{mp} y^{np}$
Definition of Zero as an Exponent [p.181]	For $x \neq 0$, $x^0 = 1$.
Definition of a Negative Exponent [p.182]	For $x \neq 0$, $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$.
Rule for Simplifying Powers of Quotients [p.183]	For $y \neq 0$, $\left(\frac{x^m}{y^n}\right)^p = \frac{x^{mp}}{y^{np}}$.
Rule for Negative Exponents on Fractional Expressions [p.183]	For $a \neq 0$, $b \neq 0$, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.
Rule for Dividing Exponential Expressions [p.183]	For $x \neq 0$, $\frac{x^m}{x^n} = x^{m-n}$.

Scientific Notation [p.186]

To express a number in scientific notation, write it in the form $a \times 10^n$, where a is a number between 1 and 10, and n is an integer. If the number is greater than 10, the exponent on 10 will be positive. If the number is less than 1, the exponent on 10 will be negative.	$367{,}000{,}000 = 3.67 \times 10^8$ $0.0000059 = 5.9 \times 10^{-6}$
To change a number written in scientific notation to decimal notation, move the decimal point to the right if the exponent on 10 is positive and to the left if the exponent on 10 is negative. Move the decimal point the same number of places as the absolute value of the exponent on 10.	$2.418 \times 10^7 = 24{,}180{,}000$ $9.06 \times 10^{-5} = 0.0000906$

Chapter Review

1. Simplify: $(2b - 3)(4b + 5)$

2. Simplify: $(12y^2 + 17y - 4) + (9y^2 - 13y + 3)$

3. Simplify: $(b^3 - 2b^2 - 33b - 7) \div (b - 7)$

4. Simplify: $(xy^5z^3)(x^3y^3z)$

5. Simplify: $-2x(4x^2 + 7x - 9)$

6. Simplify: $\dfrac{(4a^{-2}b^{-3})^2}{(2a^{-1}b^{-2})^4}$

7. Simplify: $(2^3)^2$

8. Simplify: $(5x^2 - 2x - 1) - (3x^2 - 5x + 7)$

9. Simplify: $\dfrac{8x^{12}}{12x^9}$

10. Simplify: $(5y - 7)^2$

11. Simplify: $(5a^7b^6)^2(4ab)$

12. Simplify: $\dfrac{12b^7 + 36b^5 - 3b^3}{3b^3}$

13. Simplify: $(13y^3 - 7y - 2) - (12y^2 - 2y - 1)$

14. Simplify: $(5a^2 + 6a - 11) + (5a^2 + 6a - 11)$

15. Simplify: $(3y^2 + 4y - 7)(2y + 3)$

16. Simplify: $\dfrac{-18a^6b}{27a^3b^4}$

17. Simplify: $2ab^3(4a^2 - 2ab + 3b^2)$

18. Simplify: $(6y^2 + 2y + 7) - (8y^2 + y + 12)$

19. Simplify: $(-3x^2y^3)^2$

20. Simplify: $(6b^3 - 2b^2 - 5)(2b^2 - 1)$

21. Simplify: $(2x^3 + 7x^2 + x) + (2x^2 - 4x - 12)$

22. Simplify: $\dfrac{16y^2 - 32y}{-4y}$

23. Simplify: $(a + 7)(a - 7)$

24. Simplify: $(2a^{12}b^3)(-9b^2c^6)(3ac)$

25. Simplify: $(6y^2 - 35y + 36) \div (3y - 4)$

26. Simplify: $(-3x^{-2}y^{-3})^{-2}$

27. Simplify: $(5xy^2)(-4x^2y^3)$

28. Simplify: $(5a - 7)(2a + 9)$

29. Write 0.000000127 in scientific notation.

30. Write 3.2×10^{-12} in decimal notation.

31. The length of a table-tennis table is 1 ft less than twice the width of the table. Let w represent the width of the table-tennis table. Express the area of the table in terms of the variable w.

32. The side of a checkerboard is $(3x - 2)$ in. Express the area of the checkerboard in terms of the variable x.

Chapter Test

1. Simplify: $2x(2x^2 - 3x)$

2. Simplify: $\dfrac{12x^3 - 3x^2 + 9}{3x^2}$

3. Simplify: $\dfrac{12x^2}{-3x^8}$

4. Simplify: $(-2xy^2)(3x^2y^4)$

5. Simplify: $(x^2 + 1) \div (x + 1)$

6. Simplify: $(x - 3)(x^2 - 4x + 5)$

7. Simplify: $(-2a^2b)^3$

8. Simplify: $\dfrac{(3x^{-2}y^3)^3}{3x^4y^{-1}}$

9. Simplify: $(a - 2b)(a + 5b)$

10. Simplify: $\dfrac{16x^5 - 8x^3 + 20x}{4x}$

11. Simplify: $(x^2 + 6x - 7) \div (x - 1)$

12. Simplify: $-3y^2(-2y^2 + 3y - 6)$

13. Simplify: $(-2x^3 + x^2 - 7)(2x - 3)$

14. Simplify: $(4y - 3)(4y + 3)$

15. Simplify: $(ab^2)(a^3b^5)$

16. Simplify: $\dfrac{2a^{-1}b}{2^{-2}a^{-2}b^{-3}}$

17. Simplify: $\dfrac{20a - 35}{5}$

18. Simplify: $(3a^2 - 2a - 7) - (5a^3 + 2a - 10)$

19. Simplify: $(2x - 5)^2$

20. Simplify: $(4x^2 - 7) \div (2x - 3)$

21. Simplify: $\dfrac{-(2x^2y)^3}{4x^3y^3}$

22. Simplify: $(2x - 7y)(5x - 4y)$

23. Simplify: $(3x^3 - 2x^2 - 4) + (8x^2 - 8x + 7)$

24. Write 0.00000000302 in scientific notation.

25. The radius of a circle is $(x - 5)$ m. Use the equation $A = \pi r^2$, where r is the radius, to find the area of the circle in terms of the variable x. Leave the answer in terms of π.

Cumulative Review

1. Simplify: $\frac{3}{16} - \left(-\frac{5}{8}\right) - \frac{7}{9}$

2. Evaluate $-3^2 \cdot \left(\frac{2}{3}\right)^3 \cdot \left(-\frac{5}{8}\right)$.

3. Simplify: $\left(-\frac{1}{2}\right)^3 \div \left(\frac{3}{8} - \frac{5}{6}\right) + 2$

4. Evaluate $\frac{b - (a - b)^2}{b^2}$ when $a = -2$ and $b = 3$.

5. Simplify: $-2x - (-xy) + 7x - 4xy$

6. Simplify: $(12x)\left(-\frac{3}{4}\right)$

7. Simplify: $-2[3x - 2(4 - 3x) + 2]$

8. Solve: $12 = -\frac{3}{4}x$

9. Solve: $2x - 9 = 3x + 7$

10. Solve: $2 - 3(4 - x) = 2x + 5$

11. 35.2 is what percent of 160?

12. Simplify: $(4b^3 - 7b^2 - 7) + (3b^2 - 8b + 3)$

13. Simplify: $(3y^3 - 5y + 8) - (-2y^2 + 5y + 8)$

14. Simplify: $(a^3b^5)^3$

15. Simplify: $(4xy^3)(-2x^2y^3)$

16. Simplify: $-2y^2(-3y^2 - 4y + 8)$

17. Simplify: $(2a - 7)(5a^2 - 2a + 3)$

18. Simplify: $(3b - 2)(5b - 7)$

19. Simplify: $\dfrac{(-2a^2b^3)^2}{8a^4b^8}$

20. Simplify: $(a^2 - 4a - 21) \div (a + 3)$

21. Write 6.09×10^{-5} in decimal notation.

22. Translate "the difference between eight times a number and twice the number is eighteen" into an equation and solve.

23. Fifty ounces of orange juice are added to 200 oz of a fruit punch that is 10% orange juice. What is the percent concentration of orange juice in the resulting mixture?

24. A car traveling at 50 mph overtakes a cyclist who, riding at 10 mph, has had a 2-hour head start. How far from the starting point does the car overtake the cyclist?

25. The width of a rectangle is 40% of the length. The perimeter of the rectangle is 42 m. Find the length and width of the rectangle.

Chapter 5

Factoring

Objectives

Section 5.1

- **A** To factor a monomial from a polynomial
- **B** To factor by grouping

Section 5.2

- **A** To factor a trinomial of the form $x^2 + bx + c$
- **B** To factor completely

Section 5.3

- **A** To factor a trinomial of the form $ax^2 + bx + c$ by using trial factors
- **B** To factor a trinomial of the form $ax^2 + bx + c$ by grouping

Section 5.4

- **A** To factor the difference of two squares and perfect-square trinomials
- **B** To factor completely

Section 5.5

- **A** To solve equations by factoring
- **B** To solve application problems

Coordinating the position of each skydiver in a jump involves a lot of careful planning. Position within the "pattern" is determined by when and where each shute is deployed, which is further dependent upon each diver's velocity during free fall. Factors such as the initial altitude of the jump, the size of the parachute, and the weight and body position of the diver affect velocity. The velocity of a falling object can be modeled by a quadratic equation, as shown in **Exercises 73 and 74 on page 243**.

Need help? For online student resources, such as section quizzes, visit this textbook's website at **math.college.hmco.com/students.**

Prep Test

1. Write 30 as a product of prime numbers.

2. Simplify: $-3(4y - 5)$

3. Simplify: $-(a - b)$

4. Simplify: $2(a - b) - 5(a - b)$

5. Solve: $4x = 0$

6. Solve: $2x + 1 = 0$

7. Multiply: $(x + 4)(x - 6)$

8. Multiply: $(2x - 5)(3x + 2)$

9. Simplify: $\frac{x^5}{x^2}$

10. $\frac{6x^4y^3}{2xy^2}$

Go Figure

Without using calculator (it probably won't work anyway), how many digits are in the product $4^{54} \cdot 5^{100}$?

5.1 Common Factors

Objective A

To factor a monomial from a polynomial

The **greatest common factor (GCF)** of two or more monomials is the product of the GCF of the coefficients and the common variable factors.

$6x^3y = 2 \cdot 3 \cdot x \cdot x \cdot x \cdot y$
$8x^2y^2 = 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y \cdot y$
$\text{GCF} = 2 \cdot x \cdot x \cdot y = 2x^2y$

Note that the exponent of each variable in the GCF is the same as the *smallest* exponent of that variable in either of the monomials.

The GCF of $6x^3y$ and $8x^2y^2$ is $2x^2y$.

⇒ Find the GCF of $12a^4b$ and $18a^2b^2c$.

The common variable factors are a^2 and b; c is not a common variable factor.

$12a^4b = 2 \cdot 2 \cdot 3 \cdot a^4 \cdot b$
$18a^2b^2c = 2 \cdot 3 \cdot 3 \cdot a^2 \cdot b^2 \cdot c$
$\text{GCF} = 2 \cdot 3 \cdot a^2 \cdot b = 6a^2b$

To **factor** a polynomial means to write the polynomial as a product of other polynomials.

$$\begin{tikzcd} \begin{matrix}\textbf{Polynomial}\\ 2x^2 + 10x\end{matrix} \arrow[rr, bend right, "\text{Factor}"'] & = & \begin{matrix}\textbf{Factors}\\ 2x(x+5)\end{matrix} \arrow[ll, bend right, "\text{Multiply}"'] \end{tikzcd}$$

In the example above, $2x$ is the GCF of the terms $2x^2$ and $10x$. It is a **common factor** of the terms.

⇒ Factor: $5x^3 - 35x^2 + 10x$

Find the GCF of the terms of the polynomial.

$5x^3 = 5 \cdot x^3$

$35x^2 = 5 \cdot 7 \cdot x^2$

$10x = 2 \cdot 5 \cdot x$

The GCF is $5x$.

Rewrite the polynomial, expressing each term as a product with the GCF as one of the factors.

$$\begin{aligned} 5x^3 - 35x^2 + 10x &= 5x(x^2) + 5x(-7x) + 5x(2) \\ &= 5x(x^2 - 7x + 2) \end{aligned}$$

- **Use the Distributive Property to write the polynomial as a product of factors.**

TAKE NOTE

At the right, the factors in parentheses are determined by dividing each term of the trinomial by the GCF, $5x$.

$\frac{5x^3}{5x} = x^2$,

$\frac{-35x^2}{5x} = -7x$, and

$\frac{10x}{5x} = 2$

Factor: $21x^2y^3 - 6xy^5 + 15x^4y^2$

Find the GCF of the terms of the polynomial.

$$21x^2y^3 = 3 \cdot 7 \cdot x^2 \cdot y^3$$
$$6xy^5 = 2 \cdot 3 \cdot x \cdot y^5$$
$$15x^4y^2 = 3 \cdot 5 \cdot x^4 \cdot y^2$$

The GCF is $3xy^2$.

Rewrite the polynomial, expressing each term as a product with the GCF as one of the factors.

$$\begin{aligned} 21x^2y^3 - 6xy^5 + 15x^4y^2 &= 3xy^2(7xy) + 3xy^2(-2y^3) + 3xy^2(5x^3) \\ &= 3xy^2(7xy - 2y^3 + 5x^3) \end{aligned}$$

- Use the Distributive Property to write the polynomial as a product of factors.

Example 1
Factor: $8x^2 + 2xy$

Solution
The GCF is $2x$.

$$\begin{aligned} 8x^2 + 2xy &= 2x(4x) + 2x(y) \\ &= 2x(4x + y) \end{aligned}$$

You Try It 1
Factor: $14a^2 - 21a^4b$

Your solution

Example 2
Factor: $n^3 - 5n^2 + 2n$

Solution
The GCF is n.

$$\begin{aligned} n^3 - 5n^2 + 2n &= n(n^2) + n(-5n) + n(2) \\ &= n(n^2 - 5n + 2) \end{aligned}$$

You Try It 2
Factor: $27b^2 + 18b + 9$

Your solution

Example 3
Factor: $16x^2y + 8x^4y^2 - 12x^4y^5$

Solution
The GCF is $4x^2y$.

$$\begin{aligned} &16x^2y + 8x^4y^2 - 12x^4y^5 \\ &\quad = 4x^2y(4) + 4x^2y(2x^2y) + 4x^2y(-3x^2y^4) \\ &\quad = 4x^2y(4 + 2x^2y - 3x^2y^4) \end{aligned}$$

You Try It 3
Factor: $6x^4y^2 - 9x^3y^2 + 12x^2y^4$

Your solution

Solutions on p. S10

To factor by grouping

In the examples at the right, the binomials in parentheses are called **binomial factors.**

$2a(a+b)^2$
$3xy(x-y)$

The Distributive Property is used to factor a common binomial factor from an expression.

The common binomial factor of the expression $6x(x-3) + y^2(x-3)$ is $(x-3)$. To factor that expression, use the Distributive Property to write the expression as a product of factors.

$$6x\underbrace{(x-3)} + y^2\underbrace{(x-3)} = \underbrace{(x-3)}(6x + y^2)$$

Consider the following simplification of $-(a-b)$.

$$-(a-b) = -1(a-b) = -a + b = b - a$$

Thus $\qquad b - a = -(a-b)$

This equation is sometimes used to factor a common binomial from an expression.

Factor: $2x(x-y) + 5(y-x)$

$$\begin{aligned} 2x(x-y) + 5(y-x) &= 2x(x-y) - 5(x-y) \\ &= (x-y)(2x-5) \end{aligned}$$

- **$5(y-x) = 5[(-1)(x-y)] = -5(x-y)$**

Some polynomials can be factored by grouping terms in such a way that a common binomial factor is found.

Factor: $ax + bx - ay - by$

$$ax + bx - ay - by = (ax + bx) - (ay + by)$$

- **Group the first two terms and the last two terms. Note that $-ay - by = -(ay + by)$.**

$$\begin{aligned} &= x(a+b) - y(a+b) \\ &= (a+b)(x-y) \end{aligned}$$

- **Factor the GCF, $(a+b)$, from each group.**

Factor: $6x^2 - 9x - 4xy + 6y$

$$6x^2 - 9x - 4xy + 6y = (6x^2 - 9x) - (4xy - 6y)$$

- **Group the first two terms and the last two terms. Note that $-4xy + 6y = -(4xy - 6y)$.**

$$\begin{aligned} &= 3x(2x-3) - 2y(2x-3) \\ &= (2x-3)(3x-2y) \end{aligned}$$

- **Factor the GCF, $(2x-3)$, from each group.**

Example 4

Factor: $4x(3x - 2) - 7(3x - 2)$

Solution

$4x(3x - 2) - 7(3x - 2)$

$= (3x - 2)(4x - 7)$

You Try It 4

Factor: $2y(5x - 2) - 3(2 - 5x)$

Your solution

Example 5

Factor: $9x^2 - 15x - 6xy + 10y$

Solution

$9x^2 - 15x - 6xy + 10y$

$= (9x^2 - 15x) - (6xy - 10y)$

$= 3x(3x - 5) - 2y(3x - 5)$

$= (3x - 5)(3x - 2y)$

You Try It 5

Factor: $a^2 - 3a + 2ab - 6b$

Your solution

Example 6

Factor: $3x^2y - 4x - 15xy + 20$

Solution

$3x^2y - 4x - 15xy + 20$

$= (3x^2y - 4x) - (15xy - 20)$

$= x(3xy - 4) - 5(3xy - 4)$

$= (3xy - 4)(x - 5)$

You Try It 6

Factor: $2mn^2 - n + 8mn - 4$

Your solution

Example 7

Factor: $4ab - 6 + 3b - 2ab^2$

Solution

$4ab - 6 + 3b - 2ab^2$

$= (4ab - 6) + (3b - 2ab^2)$

$= 2(2ab - 3) + b(3 - 2ab)$

$= 2(2ab - 3) - b(2ab - 3)$

$= (2ab - 3)(2 - b)$

You Try It 7

Factor: $3xy - 9y - 12 + 4x$

Your solution

Solutions on p. S10

5.1 Exercises

Objective A

1. Explain the meaning of "a common monomial factor of a polynomial."

2. Explain the meaning of "a factor" and the meaning of "to factor."

Factor.

3. $5a + 5$

4. $7b - 7$

5. $16 - 8a^2$

6. $12 + 12y^2$

7. $8x + 12$

8. $16a - 24$

9. $30a - 6$

10. $20b + 5$

11. $7x^2 - 3x$

12. $12y^2 - 5y$

13. $3a^2 + 5a^5$

14. $9x - 5x^2$

15. $14y^2 + 11y$

16. $6b^3 - 5b^2$

17. $2x^4 - 4x$

18. $3y^4 - 9y$

19. $10x^4 - 12x^2$

20. $12a^5 - 32a^2$

21. $8a^8 - 4a^5$

22. $16y^4 - 8y^7$

23. $x^2y^2 - xy$

24. $a^2b^2 + ab$

25. $3x^2y^4 - 6xy$

26. $12a^2b^5 - 9ab$

27. $x^2y - xy^3$

28. $3x^3 + 6x^2 + 9x$

29. $5y^3 - 20y^2 + 5y$

30. $2x^4 - 4x^3 + 6x^2$

31. $3y^4 - 9y^3 - 6y^2$

32. $2x^3 + 6x^2 - 14x$

33. $3y^3 - 9y^2 + 24y$

34. $2y^5 - 3y^4 + 7y^3$

35. $6a^5 - 3a^3 - 2a^2$

36. $x^3y - 3x^2y^2 + 7xy^3$

37. $2a^2b - 5a^2b^2 + 7ab^2$

38. $5y^3 + 10y^2 - 25y$

39. $4b^5 + 6b^3 - 12b$

40. $3a^2b^2 - 9ab^2 + 15b^2$

41. $8x^2y^2 - 4x^2y + x^2$

Objective B

Factor.

42. $x(b + 4) + 3(b + 4)$

43. $y(a + z) + 7(a + z)$

44. $a(y - x) - b(y - x)$

45. $3r(a - b) + s(a - b)$

46. $x(x - 2) + y(2 - x)$

47. $t(m - 7) + 7(7 - m)$

48. $2x(7 + b) - y(b + 7)$

49. $2y(4a - b) - (b - 4a)$

50. $8c(2m - 3n) + (3n - 2m)$

51. $x^2 + 2x + 2xy + 4y$

52. $x^2 - 3x + 4ax - 12a$

53. $p^2 - 2p - 3rp + 6r$

54. $t^2 + 4t - st - 4s$

55. $ab + 6b - 4a - 24$

56. $xy - 5y - 2x + 10$

57. $2z^2 - z + 2yz - y$

58. $2y^2 - 10y + 7xy - 35x$

59. $8v^2 - 12vy + 14v - 21y$

60. $21x^2 + 6xy - 49x - 14y$

61. $2x^2 - 5x - 6xy + 15y$

62. $4a^2 + 5ab - 10b - 8a$

63. $3y^2 - 6y - ay + 2a$

64. $2ra + a^2 - 2r - a$

65. $3xy - y^2 - y + 3x$

66. $2ab - 3b^2 - 3b + 2a$

67. $3st + t^2 - 2t - 6s$

68. $4x^2 + 3xy - 12y - 16x$

APPLYING THE CONCEPTS

A natural number is a perfect number if it is the sum of all its factors less than itself. For example, 6 is a perfect number because all the factors of 6 that are less than 6 are 1, 2, and 3, and $1 + 2 + 3 = 6$.

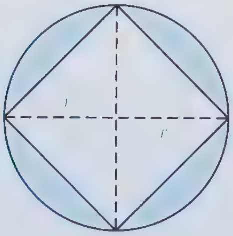
69. Find the one perfect number between 20 and 30.

70. Find the one perfect number between 490 and 500.

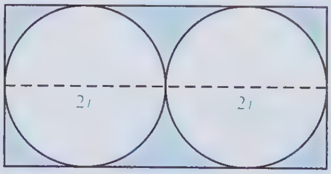
71. In the equation $P = 2L + 2W$, what is the effect on P when the quantity $L + W$ doubles?

72. Write an expression in factored form for each of the shaded portions in the following diagrams. Use the equation for the area of a rectangle ($A = LW$) and the equation for the area of a circle ($A = \pi r^2$).

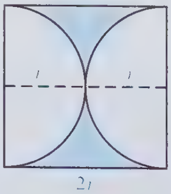
a.



b.



c.



5.2 Factoring Polynomials of the Form $x^2 + bx + c$

Objective A

To factor a trinomial of the form $x^2 + bx + c$

5

Trinomials of the form $x^2 + bx + c$, where b and c are integers, are shown at the right.

$$x^2 + 8x + 12;\ b = 8,\ c = 12$$
$$x^2 - 7x + 12;\ b = -7,\ c = 12$$
$$x^2 - 2x - 15;\ b = -2,\ c = -15$$

To factor a trinomial of this form means to express the trinomial as the product of two binomials.

Trinomials expressed as the product of binomials are shown at the right.

$$x^2 + 8x + 12 = (x + 6)(x + 2)$$
$$x^2 - 7x + 12 = (x - 3)(x - 4)$$
$$x^2 - 2x - 15 = (x + 3)(x - 5)$$

The method by which factors of a trinomial are found is based on FOIL. Consider the following binomial products, noting the relationship between the constant terms of the binomials and the terms of the trinomials.

The signs in the binomials are the same.

$$(x + 6)(x + 2) = x^2 + 2x + 6x + (6)(2) = x^2 + 8x + 12$$

Sum of 6 and 2
Product of 6 and 2

$$(x - 3)(x - 4) = x^2 - 4x - 3x + (-3)(-4) = x^2 - 7x + 12$$

Sum of -3 and -4
Product of -3 and -4

The signs in the binomials are opposites.

$$(x + 3)(x - 5) = x^2 - 5x + 3x + (3)(-5) = x^2 - 2x - 15$$

Sum of 3 and -5
Product of 3 and -5

$$(x - 4)(x + 6) = x^2 + 6x - 4x + (-4)(6) = x^2 + 2x - 24$$

Sum of -4 and 6
Product of -4 and 6

IMPORTANT RELATIONSHIPS

1. When the constant term of the trinomial is positive, the constant terms of the binomials have the same sign. They are both positive when the coefficient of the x term in the trinomial is positive. They are both negative when the coefficient of the x term in the trinomial is negative.
2. When the constant term of the trinomial is negative, the constant terms of the binomials have opposite signs.
3. In the trinomial, the coefficient of x is the sum of the constant terms of the binomials.
4. In the trinomial, the constant term is the product of the constant terms of the binomials.

Factor: $x^2 - 7x + 10$

Because the constant term is positive and the coefficient of x is negative, the binomial constants will be negative. Find two negative factors of 10 whose sum is -7. The results can be recorded in a table.

Negative Factors of 10	*Sum*
$-1, -10$	-11
$-2, -5$	-7

- These are the correct factors.

$x^2 - 7x + 10 = (x - 2)(x - 5)$

- Write the trinomial as a product of its factors.

You can check the proposed factorization by multiplying the two binomials.

TAKE NOTE
Always check your proposed factorization to ensure accuracy.

Check: $(x - 2)(x - 5) = x^2 - 5x - 2x + 10$
$= x^2 - 7x + 10$

Factor: $x^2 - 9x - 36$

The constant term is negative. The binomial constants will have opposite signs. Find two factors of -36 whose sum is -9.

Factors of −36	*Sum*
$+1, -36$	-35
$-1, +36$	35
$+2, -18$	-16
$-2, +18$	16
$+3, -12$	-9

- Once the correct factors are found, it is not necessary to try the remaining factors.

$x^2 - 9x - 36 = (x + 3)(x - 12)$

- Write the trinomial as a product of its factors.

Factor: $x^2 + 7x + 8$

Because the constant term is positive and the coefficient of x is positive, the binomial constants will be positive. Find two positive factors of 8 whose sum is 7.

Positive Factors of 8	*Sum*
1, 8	9
2, 4	6

There are no positive integer factors of 8 whose sum is 7. The trinomial $x^2 + 7x + 8$ is said to be **nonfactorable over the integers.**

Just as 17 is a prime number, $x^2 + 7x + 8$ is a **prime polynomial.** Binomials of the form $x - a$ and $x + a$ are also prime polynomials.

Example 1
Factor: $x^2 - 8x + 15$

Solution
Find two negative factors of 15 whose sum is -8.

Factors	*Sum*
$-1, -15$	-16
$-3, -5$	-8

$x^2 - 8x + 15 = (x - 3)(x - 5)$

You Try It 1
Factor: $x^2 + 9x + 20$

Your solution

Solution on p. S10

Example 2

Factor: $x^2 + 6x - 27$

Solution

Find two factors of -27 whose sum is 6.

Factors	*Sum*
$+1, -27$	-26
$-1, +27$	26
$+3, -9$	-6
$-3, +9$	6

$x^2 + 6x - 27 = (x - 3)(x + 9)$

You Try It 2

Factor: $x^2 + 7x - 18$

Your solution

Solution on p. S10

To factor completely 5

A polynomial is factored completely when it is written as a product of factors that are nonfactorable over the integers.

TAKE NOTE

The first step in *any* factoring problem is to determine whether the terms of the polynomial have a **common factor.** If they do, factor it out first.

Factor: $4y^3 - 4y^2 - 24y$

$$4y^3 - 4y^2 - 24y = 4y(y^2) - 4y(y) - 4y(6)$$

- The GCF is $4y$.

$$= 4y(y^2 - y - 6)$$

- Use the Distributive Property to factor out the GCF.

$$= 4y(y + 2)(y - 3)$$

- Factor $y^2 - y - 6$. The two factors of -6 whose sum is -1 are 2 and -3.

It is always possible to check the proposed factorization by multiplying the polynomials. Here is the check for the last example.

Check: $4y(y + 2)(y - 3) = 4y(y^2 - 3y + 2y - 6)$
$= 4y(y^2 - y - 6)$
$= 4y^3 - 4y^2 - 24y$

- This is the original polynomial.

Factor: $5x^2 + 60xy + 100y^2$

$$5x^2 + 60xy + 100y^2 = 5(x^2) + 5(12xy) + 5(20y^2)$$

- The GCF is 5.

$$= 5(x^2 + 12xy + 20y^2)$$

- Use the Distributive Property to factor out the GCF.

$$= 5(x + 2y)(x + 10y)$$

- Factor $x^2 + 12xy + 20y^2$. The two factors of 20 whose sum is 12 are 2 and 10.

TAKE NOTE

$2y$ and $10y$ are placed in the binomials. This is necessary so that the middle term contains xy and the last term contains y^2.

Note that $2y$ and $10y$ were placed in the binomials. The following check shows that this was necessary.

Check: $5(x + 2y)(x + 10y) = 5(x^2 + 10xy + 2xy + 20y^2)$
$= 5(x^2 + 12xy + 20y^2)$
$= 5x^2 + 60xy + 100y^2$

- This is the original polynomial.

Factor: $15 - 2x - x^2$

Because the coefficient of x^2 is -1, factor -1 from the trinomial and then write the resulting trinomial in descending order.

$$15 - 2x - x^2 = -(x^2 + 2x - 15)$$

- $15 - 2x - x^2 = -1(-15 + 2x + x^2)$ $= -(x^2 + 2x - 15)$

$$= -(x + 5)(x - 3)$$

- Factor $x^2 + 2x - 15$. The two factors of -15 whose sum is 2 are 5 and -3.

Check: $-(x + 5)(x - 3) = -(x^2 + 2x - 15)$
$= -x^2 - 2x + 15$
$= 15 - 2x - x^2$

- This is the original polynomial.

TAKE NOTE

When the coefficient of the highest power in a polynomial is negative, consider factoring out a negative GCF. Example 3 is another example of this technique.

Example 3

Factor: $-3x^3 + 9x^2 + 12x$

Solution

The GCF is $-3x$.

$-3x^3 + 9x^2 + 12x = -3x(x^2 - 3x - 4)$

Factor the trinomial $x^2 - 3x - 4$. Find two factors of -4 whose sum is -3.

Factors	*Sum*
+1, −4	−3

$-3x^3 + 9x^2 + 12x = -3x(x + 1)(x - 4)$

You Try It 3

Factor: $-2x^3 + 14x^2 - 12x$

Your solution

Example 4

Factor: $4x^2 - 40xy + 84y^2$

Solution

The GCF is 4.

$4x^2 - 40xy + 84y^2 = 4(x^2 - 10xy + 21y^2)$

Factor the trinomial $x^2 - 10xy + 21y^2$. Find two negative factors of 21 whose sum is -10.

Factors	*Sum*
−1, −21	−22
−3, −7	−10

$4x^2 - 40xy + 84y^2 = 4(x - 3y)(x - 7y)$

You Try It 4

Factor: $3x^2 - 9xy - 12y^2$

Your solution

Solutions on p. S11

5.2 Exercises

Objective A

1. Fill in the blank. In factoring a trinomial, if the constant term is positive, then the signs in both binomial factors will be ______.

2. Fill in the blanks. To factor $x^2 + 8x - 48$, we must find two numbers whose product is ______ and whose sum is ______.

Factor.

3. $x^2 + 3x + 2$

4. $x^2 + 5x + 6$

5. $x^2 - x \quad 2$

6. $x^2 + x - 6$

7. $a^2 + a - 12$

8. $a^2 - 2a - 35$

9. $a^2 - 3a + 2$

10. $a^2 - 5a + 4$

11. $a^2 + a - 2$

12. $a^2 - 2a - 3$

13. $b^2 - 6b + 9$

14. $b^2 + 8b + 16$

15. $b^2 + 7b - 8$

16. $y^2 - y - 6$

17. $y^2 + 6y - 55$

18. $z^2 - 4z - 45$

19. $y^2 - 5y + 6$

20. $y^2 - 8y + 15$

21. $z^2 - 14z + 45$

22. $z^2 - 14z + 49$

23. $z^2 - 12z - 160$

24. $p^2 + 2p - 35$

25. $p^2 + 12p + 27$

26. $p^2 - 6p + 8$

27. $x^2 + 20x + 100$

28. $x^2 + 18x + 81$

29. $b^2 + 9b + 20$

30. $b^2 + 13b + 40$

31. $x^2 - 11x - 42$

32. $x^2 + 9x - 70$

33. $b^2 - b - 20$

34. $b^2 + 3b - 40$

35. $y^2 - 14y - 51$

36. $y^2 - y - 72$

37. $p^2 - 4p - 21$

38. $p^2 + 16p + 39$

39. $y^2 - 8y + 32$

40. $y^2 - 9y + 81$

41. $x^2 - 20x + 75$

42. $x^2 - 12x + 11$

43. $p^2 + 24p + 63$

44. $x^2 - 15x + 56$

45. $x^2 + 21x + 38$

46. $x^2 + x - 56$

47. $x^2 + 5x - 36$

48. $a^2 - 21a - 72$

49. $a^2 - 7a - 44$

50. $a^2 - 15a + 36$

51. $a^2 - 21a + 54$

52. $z^2 - 9z - 136$

53. $z^2 + 14z - 147$

54. $c^2 - c - 90$

55. $c^2 - 3c - 180$

56. $z^2 + 15z + 44$

57. $p^2 + 24p + 135$

58. $c^2 + 19c + 34$

59. $c^2 + 11c + 18$

60. $x^2 - 4x - 96$

61. $x^2 + 10x - 75$

62. $x^2 - 22x + 112$

63. $x^2 + 21x - 100$

64. $b^2 + 8b - 105$

65. $b^2 - 22b + 72$

66. $a^2 - 9a - 36$

67. $a^2 + 42a - 135$

68. $b^2 - 23b + 102$

69. $b^2 - 25b + 126$

70. $a^2 + 27a + 72$

71. $z^2 + 24z + 144$

72. $x^2 + 25x + 156$

73. $x^2 - 29x + 100$

74. $x^2 - 10x - 96$

75. $x^2 + 9x - 112$

Factor.

76. $2x^2 + 6x + 4$

77. $3x^2 + 15x + 18$

78. $18 + 7x - x^2$

79. $12 - 4x - x^2$

80. $ab^2 + 2ab - 15a$

81. $ab^2 + 7ab - 8a$

82. $xy^2 - 5xy + 6x$

83. $xy^2 + 8xy + 15x$

84. $z^3 - 7z^2 + 12z$

85. $-2a^3 - 6a^2 - 4a$

86. $-3y^3 + 15y^2 - 18y$

87. $4y^3 + 12y^2 - 72y$

88. $3x^2 + 3x - 36$

89. $2x^3 - 2x^2 + 4x$

90. $5z^2 - 15z - 140$

91. $6z^2 + 12z - 90$

92. $2a^3 + 8a^2 - 64a$

93. $3a^3 - 9a^2 - 54a$

94. $x^2 - 5xy + 6y^2$

95. $x^2 + 4xy - 21y^2$

96. $a^2 - 9ab + 20b^2$

97. $a^2 - 15ab + 50b^2$

98. $x^2 - 3xy - 28y^2$

99. $s^2 + 2st - 48t^2$

100. $y^2 - 15yz - 41z^2$

101. $y^2 + 85xy + 36z^2$

102. $z^4 - 12z^3 + 35z^2$

103. $z^4 + 2z^3 - 80z^2$

104. $b^4 - 22b^3 + 120b^2$

105. $b^4 - 3b^3 - 10b^2$

106. $2y^4 - 26y^3 - 96y^2$

107. $3y^4 + 54y^3 + 135y^2$

108. $-x^4 - 7x^3 + 8x^2$

109. $-x^4 + 11x^3 + 12x^2$

110. $4x^2y + 20xy - 56y$

111. $3x^2y - 6xy - 45y$

112. $c^3 + 18c^2 - 40c$

113. $-3x^3 + 36x^2 - 81x$

114. $-4x^3 - 4x^2 + 24x$

115. $x^2 - 8xy + 15y^2$

116. $y^2 - 7xy - 8x^2$

117. $a^2 - 13ab + 42b^2$

118. $y^2 + 4yz - 21z^2$

119. $y^2 + 8yz + 7z^2$

120. $y^2 - 16yz + 15z^2$

121. $3x^2y + 60xy - 63y$

122. $4x^2y - 68xy - 72y$

123. $3x^3 + 3x^2 - 36x$

124. $4x^3 + 12x^2 - 160x$

125. $4z^3 + 32z^2 - 132z$

126. $5z^3 - 50z^2 - 120z$

127. $4x^3 + 8x^2 - 12x$

128. $5x^3 + 30x^2 + 40x$

129. $5p^2 + 25p - 420$

130. $4p^2 - 28p - 480$

131. $p^4 + 9p^3 - 36p^2$

132. $p^4 + p^3 - 56p^2$

133. $t^2 - 12ts + 35s^2$

134. $a^2 - 10ab + 25b^2$

135. $a^2 - 8ab - 33b^2$

APPLYING THE CONCEPTS

Factor.

136. $2 + c^2 + 9c$

137. $x^2y - 54y - 3xy$

138. $45a^2 + a^2b^2 - 14a^2b$

Find all integers k such that the trinomial can be factored over the integers.

139. $x^2 + kx + 35$

140. $x^2 + kx + 18$

141. $x^2 + kx + 21$

Determine the positive integer values of k for which the following polynomials are factorable over the integers.

142. $y^2 + 4y + k$

143. $z^2 + 7z + k$

144. $a^2 - 6a + k$

145. $c^2 - 7c + k$

146. $x^2 - 3x + k$

147. $y^2 + 5y + k$

148. In Exercises 142 to 147, there was the stated requirement that $k > 0$. If k is allowed to be any integer, how many different values of k are possible for each polynomial?

5.3 Factoring Polynomials of the Form $ax^2 + bx + c$

Objective A

To factor a trinomial of the form $ax^2 + bx + c$ by using trial factors

Trinomials of the form $ax^2 + bx + c$, where a, b, and c are integers, are shown at the right.

$3x^2 - x + 4$; $a = 3$, $b = -1$, $c = 4$
$6x^2 + 2x - 3$; $a = 6$, $b = 2$, $c = -3$

These trinomials differ from those in the preceding section in that the coefficient of x^2 is not 1. There are various methods of factoring these trinomials. The method described in this objective is factoring polynomials using trial factors.

To reduce the number of trial factors that must be considered, remember the following:

1. Use the signs of the constant term and the coefficient of x in the trinomial to determine the signs of the binomial factors. If the constant term is positive, the signs of the binomial factors will be the same as the sign of the coefficient of x in the trinomial. If the sign of the constant term is negative, the constant terms in the binomials have opposite signs.
2. If the terms of the trinomial do not have a common factor, then the terms of neither of the binomial factors will have a common factor.

Factor: $2x^2 - 7x + 3$

The terms have no common factor. The constant term is positive. The coefficient of x is negative. The binomial constants will be negative.

Positive Factors of 2 (coefficient of x^2)	*Negative Factors of 3* (constant term)
1, 2	−1, −3

Write trial factors. Use the **O**uter and **I**nner products of FOIL to determine the middle term, $-7x$, of the trinomial.

Trial Factors	*Middle Term*
$(x - 1)(2x - 3)$	$-3x - 2x = -5x$
$(x - 3)(2x - 1)$	$-x - 6x = -7x$

Write the factors of the trinomial.

$2x^2 - 7x + 3 = (x - 3)(2x - 1)$

Factor: $3x^2 + 14x + 15$

The terms have no common factor. The constant term is positive. The coefficient of x is positive. The binomial constants will be positive.

Positive Factors of 3 (coefficient of x^2)	*Positive Factors of 15* (constant term)
1, 3	1, 15
	3, 5

Write trial factors. Use the **O**uter and **I**nner products of FOIL to determine the middle term, $14x$, of the trinomial.

Trial Factors	*Middle Term*
$(x + 1)(3x + 15)$	Common factor
$(x + 15)(3x + 1)$	$x + 45x = 46x$
$(x + 3)(3x + 5)$	$5x + 9x = 14x$
$(x + 5)(3x + 3)$	Common factor

Write the factors of the trinomial.

$3x^2 + 14x + 15 = (x + 3)(3x + 5)$

Factor: $6x^3 + 14x^2 - 12x$.

Factor the GCF, $2x$, from the terms.

$$6x^3 + 14x^2 - 12x = 2x(3x^2 + 7x - 6)$$

Factor the trinomial. The constant term is negative. The binomial constants will have opposite signs.

Positive Factors of 3	*Factors of −6*
1, 3	1, −6 −1, 6 2, −3 −2, 3

Write trial factors. Use the **O**uter and **I**nner products of FOIL to determine the middle term, $7x$, of the trinomial.

It is not necessary to test trial factors that have a common factor.

Trial Factors	*Middle Term*
$(x + 1)(3x - 6)$	Common factor
$(x - 6)(3x + 1)$	$x - 18x = -17x$
$(x - 1)(3x + 6)$	Common factor
$(x + 6)(3x - 1)$	$-x + 18x = 17x$
$(x + 2)(3x - 3)$	Common factor
$(x - 3)(3x + 2)$	$2x - 9x = -7x$
$(x - 2)(3x + 3)$	Common factor
$(x + 3)(3x - 2)$	$-2x + 9x = 7x$

Write the factors of the trinomial.

$$6x^3 + 14x^2 - 12x = 2x(x + 3)(3x - 2)$$

For this example, all the trial factors were listed. Once the correct factors have been found, however, the remaining trial factors can be omitted. For the examples and solutions in this text, all trial factors except those that have a common factor will be listed.

Example 1

Factor: $3x^2 + x - 2$

Solution

Positive factors of 3: 1, 3

Factors of −2: −1, −2
−1, −2

Trial Factors	*Middle Term*
$(x + 1)(3x - 2)$	$-2x + 3x = x$
$(x - 2)(3x + 1)$	$x - 6x = -5x$
$(x - 1)(3x + 2)$	$2x - 3x = -x$
$(x + 2)(3x - 1)$	$-x + 6x = 5x$

$3x^2 + x - 2 = (x + 1)(3x - 2)$

You Try It 1

Factor: $2x^2 - x - 3$

Your solution

Example 2

Factor: $-12x^3 - 32x^2 + 12x$

Solution

The GCF is $-4x$.

$-12x^3 - 32x^2 + 12x = -4x(3x^2 + 8x - 3)$

Factor the trinomial.

Positive factors of 3: 1, 3

Factors of −3: −1, −3
−1, −3

Trial Factors	*Middle Term*
$(x - 3)(3x + 1)$	$x - 9x = -8x$
$(x + 3)(3x - 1)$	$-x + 9x = 8x$

$-12x^3 - 32x^2 + 12x = -4x(x + 3)(3x - 1)$

You Try It 2

Factor: $-45y^3 + 12y^2 + 12y$

Your solution

Solutions on p.S11

[illegible]

To factor a trinomial of the form $ax^2 + bx + c$ by grouping

In the preceding objective, trinomials of the form $ax^2 + bx + c$ were factored by using trial factors. In this objective, these trinomials will be factored by grouping.

To factor $ax^2 + bx + c$, first find two factors of $a \cdot c$ whose sum is b. Then use factoring by grouping to write the factorization of the trinomial.

Factor: $2x^2 + 13x + 15$

Find two positive factors of 30 $(2 \cdot 15)$ whose sum is 13.

Positive Factors of 30	*Sum*
1, 30	31
2, 15	17
3, 10	13

- Once the required sum has been found, the remaining factors need not be checked.

$$\begin{aligned} 2x^2 + 13x + 15 &= 2x^2 + 3x + 10x + 15 \\ &= (2x^2 + 3x) + (10x + 15) \\ &= x(2x + 3) + 5(2x + 3) \\ &= (2x + 3)(x + 5) \end{aligned}$$

- Use the factors of 30 whose sum is 13 to write $13x$ as $3x + 10x$.
- Factor by grouping.

Check your answer. $(2x + 3)(x + 5) = 2x^2 + 10x + 3x + 15$
$= 2x^2 + 13x + 15$

Factor: $6x^2 - 11x - 10$

Find two factors of -60 $[6(-10)]$ whose sum is -11.

Factors of −60	*Sum*
1, −60	−59
−1, 60	59
2, −30	−28
−2, 30	28
3, −20	−17
−3, 20	17
4, −15	−11

$$\begin{aligned} 6x^2 - 11x - 10 &= 6x^2 + 4x - 15x - 10 \\ &= (6x^2 + 4x) - (15x + 10) \\ &= 2x(3x + 2) - 5(3x + 2) \\ &= (3x + 2)(2x - 5) \end{aligned}$$

- Use the factors of -60 whose sum is -11 to write $-11x$ as $4x - 15x$.
- Factor by grouping. Recall that $-15x - 10 = -(15x + 10)$.

Check your answer. $(3x + 2)(2x - 5) = 6x^2 - 15x + 4x - 10$
$= 6x^2 - 11x - 10$

Factor: $3x^2 - 2x - 4$

Find two factors of -12 $[3(-4)]$ whose sum is -2.

Factors of −12	*Sum*
1, −12	−11
−1, 12	11
2, −6	−4
−2, 6	4
3, −4	−1
−3, 4	1

TAKE NOTE

$3x^2 - 2x - 4$ is a prime polynomial.

Because no integer factors of -12 have a sum of -2, $3x^2 - 2x - 4$ is nonfactorable over the integers.

Example 3

Factor: $2x^2 + 19x - 10$

Solution

Factors of −20 [2(−10)]	*Sum*
−1, 20	19

$$\begin{aligned}2x^2 + 19x - 10 &= 2x^2 - x + 20x - 10\\ &= (2x^2 - x) + (20x - 10)\\ &= x(2x - 1) + 10(2x - 1)\\ &= (2x - 1)(x + 10)\end{aligned}$$

You Try It 3

Factor: $2a^2 + 13a - 7$

Your solution

Example 4

Factor: $24x^2y - 76xy + 40y$

Solution

The GCF is $4y$.

$24x^2y - 76xy + 40y = 4y(6x^2 - 19x + 10)$

Negative Factors of 60 [6(10)]	*Sum*
−1, −60	−61
−2, −30	−32
−3, −20	−23
−4, −15	−19

$$\begin{aligned}6x^2 - 19x + 10 &= 6x^2 - 4x - 15x + 10\\ &= (6x^2 - 4x) - (15x - 10)\\ &= 2x(3x - 2) - 5(3x - 2)\\ &= (3x - 2)(2x - 5)\end{aligned}$$

$$\begin{aligned}24x^2y - 76xy + 40y &= 4y(6x^2 - 19x + 10)\\ &= 4y(3x - 2)(2x - 5)\end{aligned}$$

You Try It 4

Factor: $15x^3 + 40x^2 - 80x$

Your solution

Solutions on p. S11

5.3 Exercises

[illegible]

Factor by using trial factors.

1. $2x^2 + 3x + 1$

2. $5x^2 + 6x + 1$

3. $2y^2 + 7y + 3$

4. $3y^2 + 7y + 2$

5. $2a^2 - 3a + 1$

6. $3a^2 - 4a + 1$

7. $2b^2 - 11b + 5$

8. $3b^2 - 13b + 4$

9. $2x^2 + x - 1$

10. $4x^2 - 3x - 1$

11. $2x^2 - 5x - 3$

12. $3x^2 + 5x - 2$

13. $2t^2 - t - 10$

14. $2t^2 + 5t - 12$

15. $3p^2 - 16p + 5$

16. $6p^2 + 5p + 1$

17. $12y^2 - 7y + 1$

18. $6y^2 - 5y + 1$

19. $6z^2 - 7z + 3$

20. $9z^2 + 3z + 2$

21. $6t^2 - 11t + 4$

22. $10t^2 + 11t + 3$

23. $8x^2 + 33x + 4$

24. $7x^2 + 50x + 7$

25. $5x^2 - 62x - 7$

26. $9x^2 - 13x - 4$

27. $12y^2 + 19y + 5$

28. $5y^2 - 22y + 8$

29. $7a^2 + 47a - 14$

30. $11a^2 - 54a - 5$

31. $3b^2 - 16b + 16$

32. $6b^2 - 19b + 15$

33. $2z^2 - 27z - 14$

34. $4z^2 + 5z - 6$

35. $3p^2 + 22p - 16$

36. $7p^2 + 19p + 10$

37. $4x^2 + 6x + 2$

38. $12x^2 + 33x - 9$

39. $15y^2 - 50y + 35$

40. $30y^2 + 10y - 20$

41. $2x^3 - 11x^2 + 5x$

42. $2x^3 - 3x^2 - 5x$

43. $3a^2b - 16ab + 16b$

44. $2a^2b - ab - 21b$

45. $3z^2 + 95z + 10$

46. $8z^2 - 36z + 1$

47. $36x - 3x^2 - 3x^3$

48. $-2x^3 + 2x^2 + 4x$

49. $80y^2 - 36y + 4$

50. $24y^2 - 24y - 18$

51. $8z^3 + 14z^2 + 3z$

52. $6z^3 - 23z^2 + 20z$

53. $6x^2y - 11xy - 10y$

54. $8x^2y - 27xy + 9y$

55. $10t^2 - 5t - 50$

56. $16t^2 + 40t - 96$

57. $3p^3 - 16p^2 + 5p$

58. $6p^3 + 5p^2 + p$

59. $26z^2 + 98z - 24$

60. $30z^2 - 87z + 30$

61. $10y^3 - 44y^2 + 16y$

62. $14y^3 + 94y^2 - 28y$

63. $4yz^3 + 5yz^2 - 6yz$

64. $12a^3 + 14a^2 - 48a$

65. $42a^3 + 45a^2 - 27a$

66. $36p^2 - 9p^3 - p^4$

67. $9x^2y - 30xy^2 + 25y^3$

68. $8x^2y - 38xy^2 + 35y^3$

69. $9x^3y - 24x^2y^2 + 16xy^3$

70. $9x^3y + 12x^2y + 4xy$

[illegible]

Factor by grouping.

71. $6x^2 - 17x + 12$

72. $15x^2 - 19x + 6$

73. $5b^2 + 33b - 14$

74. $8x^2 - 30x + 25$

75. $6a^2 + 7a - 24$

76. $14a^2 + 15a - 9$

77. $4z^2 + 11z + 6$

78. $6z^2 - 25z + 14$

79. $22p^2 + 51p - 10$

80. $14p^2 - 41p + 15$

81. $8y^2 + 17y + 9$

82. $12y^2 - 145y + 12$

83. $18t^2 - 9t - 5$

84. $12t^2 + 28t - 5$

85. $6b^2 + 71b - 12$

86. $8b^2 + 65b + 8$

87. $9x^2 + 12x + 4$

88. $25x^2 - 30x + 9$

89. $6b^2 - 13b + 6$

90. $20b^2 + 37b + 15$

91. $33b^2 + 34b - 35$

92. $15b^2 - 43b + 22$

93. $18y^2 - 39y + 20$

94. $24y^2 + 41y + 12$

95. $15a^2 + 26a - 21$

96. $6a^2 + 23a + 21$

97. $8y^2 - 26y + 15$

98. $18y^2 - 27y + 4$

99. $8z^2 + 2z - 15$

100. $10z^2 + 3z - 4$

101. $15x^2 - 82x + 24$

102. $13z^2 + 49z - 8$

103. $10z^2 - 29z + 10$

104. $15z^2 - 44z + 32$

105. $36z^2 + 72z + 35$

106. $16z^2 + 8z - 35$

107. $3x^2 + xy - 2y^2$

108. $6x^2 + 10xy + 4y^2$

109. $3a^2 + 5ab - 2b^2$

110. $2a^2 - 9ab + 9b^2$

111. $4y^2 - 11yz + 6z^2$

112. $2y^2 + 7yz + 5z^2$

113. $28 + 3z - z^2$

114. $15 - 2z - z^2$

115. $8 - 7x - x^2$

116. $12 + 11x - x^2$

117. $9x^2 + 33x - 60$

118. $16x^2 - 16x - 12$

119. $24x^2 - 52x + 24$

120. $60x^2 + 95x + 20$

121. $35a^4 + 9a^3 - 2a^2$

122. $15a^4 + 26a^3 + 7a^2$

123. $15b^2 - 115b + 70$

124. $25b^2 + 35b - 30$

125. $3x^2 - 26xy + 35y^2$

126. $4x^2 + 16xy + 15y^2$

127. $216y^2 - 3y - 3$

128. $360y^2 + 4y - 4$

129. $21 - 20x - x^2$

130. $18 + 17x - x^2$

APPLYING THE CONCEPTS

131. In your own words, explain how the signs of the last terms of the two binomial factors of a trinomial are determined.

Factor.

132. $(x + 1)^2 - (x + 1) - 6$

133. $(x - 2)^2 + 3(x - 2) + 2$

134. $(y + 3)^2 - 5(y + 3) + 6$

135. $2(y + 2)^2 - (y + 2) - 3$

136. $3(a + 2)^2 - (a + 2) - 4$

137. $4(y - 1)^2 - 7(y - 1) - 2$

Find all integers k such that the trinomial can be factored over the integers.

138. $2x^2 + kx + 3$

139. $2x^2 + kx - 3$

140. $3x^2 + kx + 2$

141. $3x^2 + kx - 2$

142. $2x^2 + kx + 5$

143. $2x^2 + kx - 5$

5.4 Special Factoring

Objective A

To factor the difference of two squares and perfect-square trinomials

Recall that the product of the sum and difference of the same terms equals the square of the first term minus the square of the second term.

Sum and difference of two terms		**Difference of two squares**
$(a+b)(a-b)$	$=$	a^2-b^2

This suggests that the difference of two squares can be factored as

$$a^2-b^2=(a+b)(a-b)$$

Note that the polynomial x^2+y^2 is the *sum* of two squares. The sum of two squares is nonfactorable over the integers.

Factor: x^2-16

$$\begin{aligned} x^2-16 &= (x)^2-(4)^2 \\ &= (x+4)(x-4) \end{aligned}$$

- Write x^2-16 as the difference of two squares. Then factor.

Check: $(x+4)(x-4)=x^2-4x+4x-16$

$$=x^2-16$$

Factor: $8x^3-18x$

$$\begin{aligned} 8x^3-18x &= 2x(4x^2-9) \\ &= 2x[(2x)^2-3^2] \\ &= 2x(2x+3)(2x-3) \end{aligned}$$

- The GCF is $2x$.
- Factor the difference of two squares.

You should check the factorization.

Factor: x^2-10

Because 10 cannot be written as the square of an integer, x^2-10 is nonfactorable over the integers.

Recall from an earlier discussion the pattern for finding the square of a binomial. The result is a **perfect-square trinomial.**

The Square of a Binomial

$$\begin{aligned} (a+b)^2 = (a+b)(a+b) &= a^2+ab+ab+b^2 \\ &= \mathbf{a^2+2ab+b^2} \end{aligned}$$

Square of the first term → a^2

Twice the product of the two terms → $2ab$

Square of the last term → b^2

This pattern is used to factor a perfect-square trinomial.

Factor: $4x^2 - 20x + 25$

Because the first and last terms are squares $[(2x)^2 = 4x^2; 5^2 = 25]$, try to factor this as the square of a binomial. Check the factorization.

$$4x^2 - 20x + 25 = (2x - 5)^2$$

Check: $(2x - 5)^2 = (2x)^2 + 2(2x)(-5) + 5^2$
$= 4x^2 - 20x + 25$

- **The factorization is correct.**

$$4x^2 - 20x + 25 = (2x - 5)^2$$

Factor: $4x^2 + 37x + 9$

Because the first and last terms are squares $[(2x)^2 = 4x^2; 3^2 = 9]$, try to factor this as the square of a binomial. Check the proposed factorization.

$$4x^2 + 37x + 9 = (2x + 3)^2$$

Check: $(2x + 3)^2 = (2x)^2 + 2(2x)(3) + 3^2$
$= 4x^2 + 12x + 9$

Because $4x^2 + 12x + 9 \neq 4x^2 + 37x + 9$, the proposed factorization is not correct. In this case, the polynomial is not a perfect-square trinomial. It may, however, still factor. In fact, $4x^2 + 37x + 9 = (4x + 1)(x + 9)$.

Example 1
Factor: $16x^2 - y^2$

Solution
$16x^2 - y^2 = (4x)^2 - y^2 = (4x + y)(4x - y)$

You Try It 1
Factor: $25a^2 - b^2$

Your solution

Example 2
Factor: $z^4 - 16$

Solution
$$\begin{aligned} z^4 - 16 &= (z^2)^2 - 4^2 = (z^2 + 4)(z^2 - 4) \\ &= (z^2 + 4)(z^2 - 2^2) \\ &= (z^2 + 4)(z + 2)(z - 2) \end{aligned}$$

You Try It 2
Factor: $n^4 - 81$

Your solution

Example 3
Factor: $9x^2 - 30x + 25$

Solution
$9x^2 = (3x)^2$, $25 = (5)^2$
$9x^2 - 30x + 25 = (3x - 5)^2$

Check: $(3x - 5)^2 = (3x)^2 + 2(3x)(-5) + 5^2$
$= 9x^2 - 30x + 25$

You Try It 3
Factor: $16y^2 + 8y + 1$

Your solution

Solutions on p. S12

Example 4

Factor: $9x^2 + 40x + 16$

Solution

Because $9x^2 = (3x)^2$, $16 = 4^2$, and $40x \neq 2(3x)(4)$, the trinomial is not a perfect-square trinomial.

Try to factor by another method.

$9x^2 + 40x + 16 = (9x + 4)(x + 4)$

You Try It 4

Factor: $x^2 + 15x + 36$

Your solution

Example 5

Factor: $(r + 2)^2 - 4$

Solution

$$\begin{aligned}(r + 2)^2 - 4 &= (r + 2)^2 - 2^2\\ &= (r + 2 + 2)(r + 2 - 2)\\ &= r(r + 4)\end{aligned}$$

You Try It 5

Factor: $(x^2 - 6x + 9) - y^2$

Your solution

Solutions on p. S12

To factor completely

Questions to Ask when Factoring a Polynomial

1. Is there a common factor? If so, factor out the common factor.
2. Is the polynomial the difference of two perfect squares? If so, factor.
3. Is the polynomial a perfect-square trinomial? If so, factor.
4. Is the polynomial a trinomial that is the product of two binomials? If so, factor.
5. Does the polynomial contain four terms? If so, try factoring by grouping.
6. Is each binomial factor a prime polynomial over the integers? If not, factor.

Factor: $z^3 + 4z^2 - 9z - 36$

$$\begin{aligned}z^3 + 4z^2 - 9z - 36 &= (z^3 + 4z^2) - (9z + 36)\\ &= z^2(z + 4) - 9(z + 4)\\ &= (z + 4)(z^2 - 9)\\ &= (z + 4)(z + 3)(z - 3)\end{aligned}$$

- Factor by grouping. Recall that $-9z - 36 = -(9z + 36)$.
- $z^3 + 4z^2 = z^2(z + 4)$
  $9z + 36 = 9(z + 4)$
- Factor out the common binomial factor $(z + 4)$.
- Factor the difference of squares.

Example 6

Factor: $3x^2 - 48$

Solution

The GCF is 3.

$$\begin{aligned} 3x^2 - 48 &= 3(x^2 - 16) \\ &= 3(x + 4)(x - 4) \end{aligned}$$

You Try It 6

Factor: $12x^3 - 75x$

Your solution

Example 7

Factor: $x^3 - 3x^2 - 4x + 12$

Solution

Factor by grouping.

$$\begin{aligned} x^3 - 3x^2 - 4x + 12 &= (x^3 - 3x^2) - (4x - 12) \\ &= x^2(x - 3) - 4(x - 3) \\ &= (x - 3)(x^2 - 4) \\ &= (x - 3)(x + 2)(x - 2) \end{aligned}$$

You Try It 7

Factor: $a^2b - 7a^2 - b + 7$

Your solution

Example 8

Factor: $4x^2y^2 + 12xy^2 + 9y^2$

Solution

The GCF is y^2.

$$\begin{aligned} 4x^2y^2 + 12xy^2 + 9y^2 &= y^2(4x^2 + 12x + 9) \\ &= y^2(2x + 3)^2 \end{aligned}$$

You Try It 8

Factor: $4x^3 + 28x^2 - 120x$

Your solution

Solutions on p. S12

5.4 Exercises

Objective A

1. Provide an example of **(a)** a binomial that is the difference of two squares and **(b)** a perfect-square trinomial.

2. Explain why a binomial that is the sum of two squares is nonfactorable over the integers.

Factor.

3. $x^2 - 4$

4. $x^2 - 9$

5. $a^2 - 81$

6. $a^2 - 49$

7. $y^2 + 2y + 1$

8. $y^2 + 14y + 49$

9. $a^2 - 2a + 1$

10. $x^2 - 12x + 36$

11. $4x^2 - 1$

12. $9x^2 - 16$

13. $x^6 - 9$

14. $y^{12} - 4$

15. $x^2 + 8x - 16$

16. $z^2 - 18z - 81$

17. $x^2 + 2xy + y^2$

18. $x^2 + 6xy + 9y^2$

19. $4a^2 + 4a + 1$

20. $25x^2 + 10x + 1$

21. $9x^2 - 1$

22. $1 - 49x^2$

23. $1 - 64x^2$

24. $t^2 + 36$

25. $x^2 + 64$

26. $64a^2 - 16a + 1$

27. $9a^2 + 6a + 1$

28. $x^4 - y^2$

29. $b^4 - 16a^2$

30. $16b^2 + 8b + 1$

31. $4a^2 - 20a + 25$

32. $4b^2 + 28b + 49$

33. $9a^2 - 42a + 49$

34. $9x^2 - 16y^2$

35. $25z^2 - y^2$

36. $x^2y^2 - 4$

37. $a^2b^2 - 25$

38. $16 - x^2y^2$

39. $25x^2 - 1$

40. $25a^2 + 30ab + 9b^2$

41. $4a^2 - 12ab + 9b^2$

42. $49x^2 + 28xy + 4y^2$

43. $4y^2 - 36yz + 81z^2$

44. $64y^2 - 48yz + 9z^2$

45. $\frac{1}{x^2} - 4$

46. $\frac{9}{a^2} - 16$

47. $9a^2b^2 - 6ab + 1$

48. $16x^2y^2 - 24xy + 9$

[illegible]

Factor.

49. $8y^2 - 2$

50. $12n^2 - 48$

51. $3a^3 + 6a^2 + 3a$

52. $4rs^2 - 4rs + r$

53. $m^4 - 256$

54. $81 - t^4$

55. $9x^2 + 13x + 4$

56. $x^2 + 10x + 16$

57. $16y^4 + 48y^3 + 36y^2$

58. $36c^4 - 48c^3 + 16c^2$

59. $y^8 - 81$

60. $32s^4 - 2$

61. $25 - 20p + 4p^2$

62. $9 + 24a + 16a^2$

63. $(4x - 3)^2 - y^2$

64. $(2x + 5)^2 - 25$

65. $(x^2 - 4x + 4) - y^2$

66. $(4x^2 + 12x + 9) - 4y^2$

67. $5x^2 - 5$

68. $2x^2 - 18$

69. $x^3 + 4x^2 + 4x$

70. $y^3 - 10y^2 + 25y$

71. $x^4 + 2x^3 - 35x^2$

72. $a^4 - 11a^3 + 24a^2$

73. $5b^2 + 75b + 180$

74. $6y^2 - 48y + 72$

75. $3a^2 + 36a + 10$

76. $5a^2 - 30a + 4$

77. $2x^2y + 16xy - 66y$

78. $3a^2b + 21ab - 54b$

79. $x^3 - 6x^2 - 5x$

80. $b^3 - 8b^2 - 7b$

81. $3y^2 - 36$

82. $3y^2 - 147$

83. $20a^2 + 12a + 1$

84. $12a^2 - 36a + 27$

85. $x^2y^2 - 7xy^2 - 8y^2$

86. $a^2b^2 + 3a^2b - 88a^2$

87. $10a^2 - 5ab - 15b^2$

88. $16x^2 - 32xy + 12y^2$

89. $50 - 2x^2$

90. $72 - 2x^2$

91. $a^2b^2 - 10ab^2 + 25b^2$

92. $a^2b^2 + 6ab^2 + 9b^2$

93. $12a^3b - a^2b^2 - ab^3$

94. $2x^3y - 7x^2y^2 + 6xy^3$

95. $12a^3 - 12a^2 + 3a$

96. $18a^3 + 24a^2 + 8a$

97. $243 + 3a^2$

98. $75 + 27y^2$

99. $12a^3 - 46a^2 + 40a$

100. $24x^3 - 66x^2 + 15x$

101. $4a^3 + 20a^2 + 25a$

102. $2a^3 - 8a^2b + 8ab^2$

103. $27a^2b - 18ab + 3b$

104. $a^2b^2 - 6ab^2 + 9b^2$

105. $48 - 12x - 6x^2$

106. $21x^2 - 11x^3 - 2x^4$

107. $x^4 - x^2y^2$

108. $b^4 - a^2b^2$

109. $18a^3 + 24a^2 + 8a$

110. $32xy^2 - 48xy + 18x$

111. $2b + ab - 6a^2b$

112. $15y^2 - 2xy^2 - x^2y^2$

113. $4x^4 - 38x^3 + 48x^2$

114. $3x^2 - 27y^2$

115. $x^4 - 25x^2$

116. $y^3 - 9y$

117. $a^4 - 16$

118. $15x^4y^2 - 13x^3y^3 - 20x^2y^4$

119. $45y^2 - 42y^3 - 24y^4$

120. $a(2x - 2) + b(2x - 2)$

121. $4a(x - 3) - 2b(x - 3)$

122. $x^2(x - 2) - (x - 2)$

123. $y^2(a - b) - (a - b)$

124. $a(x^2 - 4) + b(x^2 - 4)$

125. $x(a^2 - b^2) - y(a^2 - b^2)$

126. $4(x - 5) - x^2(x - 5)$

APPLYING THE CONCEPTS

Find all integers k such that the trinomial is a perfect-square trinomial.

127. $4x^2 - kx + 9$

128. $x^2 + 6x + k$

129. $64x^2 + kxy + y^2$

130. $x^2 - 2x + k$

131. $25x^2 - kx + 1$

132. $x^2 + 10x + k$

133. Select any odd integer greater than 1, square it, and then subtract 1. Is the result evenly divisible by 8? Prove that this procedure always produces a number divisible by 8. (*Suggestion:* Any odd integer greater than 1 can be expressed as $2n + 1$, where n is a natural number.)

5.5 Solving Equations

[illegible]

To solve equations by factoring

The Multiplication Property of Zero states that the product of a number and zero is zero. This property is stated below.

If a is a real number, then $a \cdot 0 = 0 \cdot a = 0$.

Now consider $x \cdot y = 0$. For this to be a true equation, then either $x = 0$ or $y = 0$.

Principle of Zero Products

If the product of two factors is zero, then at least one of the factors must be zero.

If $a \cdot b = 0$, then $a = 0$ or $b = 0$.

The Principle of Zero Products is used to solve some equations.

Solve: $(x - 2)(x - 3) = 0$

$$(x - 2)(x - 3) = 0$$
$$x - 2 = 0 \qquad x - 3 = 0$$

- Let each factor equal zero (the Principle of Zero Products).

$$x = 2 \qquad x = 3$$

- Rewrite each equation in the form *variable = constant*.

Check:

$$\begin{array}{r|l} \multicolumn{2}{c}{(x - 2)(x - 3) = 0} \\ \hline (2 - 2)(2 - 3) & 0 \\ 0(-1) & 0 \\ \multicolumn{2}{r}{0 = 0} \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{(x - 2)(x - 3) = 0} \\ \hline (3 - 2)(3 - 3) & 0 \\ (1)(0) & 0 \\ \multicolumn{2}{r}{0 = 0} \end{array}$$

A true equation A true equation

The solutions are 2 and 3.

An equation of the form $ax^2 + bx + c = 0, a \neq 0$, is a **quadratic equation.** A quadratic equation is in **standard form** when the polynomial is in descending order and equal to zero. The quadratic equations at the right are in standard form.

$$3x^2 + 2x + 1 = 0$$
$$4x^2 - 3x + 2 = 0$$

Solve: $2x^2 + x = 6$

$$2x^2 + x = 6$$

$$2x^2 + x - 6 = 0$$

- Write the equation in standard form.

$$(2x - 3)(x + 2) = 0$$

- Factor.

$$2x - 3 = 0 \qquad x + 2 = 0$$

- Use the Principle of Zero Products.

$$2x = 3 \qquad x = -2$$

$$x = \frac{3}{2}$$

- Rewrite each equation in the form *variable = constant.*

Check: $\frac{3}{2}$ and -2 check as solutions.

The solutions are $\frac{3}{2}$ and -2.

Example 1

Solve: $x(x - 3) = 0$

Solution

$$x(x - 3) = 0$$

$$x = 0 \qquad x - 3 = 0$$

$$x = 3$$

The solutions are 0 and 3.

You Try It 1

Solve: $2x(x + 7) = 0$

Your solution

Example 2

Solve: $2x^2 - 50 = 0$

Solution

$$2x^2 - 50 = 0$$

$$2(x^2 - 25) = 0$$

$$2(x + 5)(x - 5) = 0$$

$$x + 5 = 0 \qquad x - 5 = 0$$

$$x = -5 \qquad x = 5$$

The solutions are -5 and 5.

You Try It 2

Solve: $4x^2 - 9 = 0$

Your solution

Example 3

Solve: $(x - 3)(x - 10) = -10$

Solution

$$(x - 3)(x - 10) = -10$$

$$x^2 - 13x + 30 = -10$$

- Multiply $(x - 3)(x - 10)$.

$$x^2 - 13x + 40 = 0$$

- Add 10 to each side of the equation. The equation is now in standard form.

$$(x - 8)(x - 5) = 0$$

$$x - 8 = 0 \qquad x - 5 = 0$$

$$x = 8 \qquad x = 5$$

The solutions are 8 and 5.

You Try It 3

Solve: $(x + 2)(x - 7) = 52$

Your solution

Solutions on p. S12

Objective B To solve application problems

Example 4

The sum of the squares of two consecutive positive even integers is equal to 100. Find the two integers.

Strategy

First positive even integer: n
Second positive even integer: $n + 2$

The sum of the square of the first positive even integer and the square of the second positive even integer is 100.

Solution

$$\begin{aligned} n^2 + (n + 2)^2 &= 100 \\ n^2 + n^2 + 4n + 4 &= 100 \\ 2n^2 + 4n + 4 &= 100 \\ 2n^2 + 4n - 96 &= 0 \\ 2(n^2 + 2n - 48) &= 0 \\ 2(n - 6)(n + 8) &= 0 \end{aligned}$$

$$n - 6 = 0 \qquad n + 8 = 0$$
$$n = 6 \qquad n = -8$$

Because -8 is not a positive even integer, it is not a solution.

$n = 6$
$n + 2 = 6 + 2 = 8$

The two integers are 6 and 8.

You Try It 4

The sum of the squares of two consecutive positive integers is 61. Find the two integers.

Your strategy

Your solution

Solution on p. S12

Example 5

A stone is thrown into a well with an initial speed of 4 ft/s. The well is 420 ft deep. How many seconds later will the stone hit the bottom of the well? Use the equation $d = vt + 16t^2$, where d is the distance in feet that the stone travels in t seconds when its initial speed is v feet per second.

Strategy

To find the time for the stone to drop to the bottom of the well, replace the variables d and v by their given values and solve for t.

Solution

$$\begin{aligned} d &= vt + 16t^2 \\ 420 &= 4t + 16t^2 \\ 0 &= -420 + 4t + 16t^2 \\ 0 &= 16t^2 + 4t - 420 \\ 0 &= 4(4t^2 + t - 105) \\ 0 &= 4(4t + 21)(t - 5) \end{aligned}$$

$$\begin{aligned} 4t + 21 &= 0 \\ 4t &= -21 \\ t &= -\frac{21}{4} \end{aligned} \qquad \begin{aligned} t - 5 &= 0 \\ t &= 5 \end{aligned}$$

Because the time cannot be a negative number, $-\frac{21}{4}$ is not a solution.

The time is 5 s.

You Try It 5

The length of a rectangle is 4 in. longer than twice the width. The area of the rectangle is 96 in^2. Find the length and width of the rectangle.

Your strategy

Your solution

Solution on p. S12

5.5 Exercises

[illegible]

1. In your own words, explain the Principle of Zero Products.

2. Fill in the blanks. If $(x + 5)(2x - 7) = 0$, then ________ $= 0$ or ________ $= 0$.

Solve.

3. $(y + 3)(y + 2) = 0$

4. $(y - 3)(y - 5) = 0$

5. $(z - 7)(z - 3) = 0$

6. $(z + 8)(z - 9) = 0$

7. $x(x - 5) = 0$

8. $x(x + 2) = 0$

9. $a(a - 9) = 0$

10. $a(a + 12) = 0$

11. $y(2y + 3) = 0$

12. $t(4t - 7) = 0$

13. $2a(3a - 2) = 0$

14. $4b(2b + 5) = 0$

15. $(b + 2)(b - 5) = 0$

16. $(b - 8)(b + 3) = 0$

17. $x^2 - 81 = 0$

18. $x^2 - 121 = 0$

19. $4x^2 - 49 = 0$

20. $16x^2 - 1 = 0$

21. $9x^2 - 1 = 0$

22. $16x^2 - 49 = 0$

23. $x^2 + 6x + 8 = 0$

24. $x^2 - 8x + 15 = 0$

25. $z^2 - 5z - 14 = 0$

26. $z^2 - z - 72 = 0$

27. $2a^2 - 9a - 5 = 0$

28. $3a^2 + 14a + 8 = 0$

29. $6z^2 + 5z + 1 = 0$

30. $6y^2 - 19y + 15 = 0$

31. $x^2 - 3x = 0$

32. $a^2 - 5a = 0$

33. $x^2 - 7x = 0$

34. $2a^2 - 8a = 0$

35. $a^2 + 5a = -4$

36. $a^2 - 5a = 24$

37. $y^2 - 5y = -6$

38. $y^2 - 7y = 8$

39. $2t^2 + 7t = 4$

40. $3t^2 + t = 10$

41. $3t^2 - 13t = -4$

42. $5t^2 - 16t = -12$

43. $x(x - 12) = -27$

44. $x(x - 11) = 12$

45. $y(y - 7) = 18$

46. $y(y + 8) = -15$

47. $p(p + 3) = -2$ **48.** $p(p - 1) = 20$ **49.** $y(y + 4) = 45$ **50.** $y(y - 8) = -15$

51. $x(x + 3) = 28$ **52.** $p(p - 14) = 15$ **53.** $(x + 8)(x - 3) = -30$ **54.** $(x + 4)(x - 1) = 14$

55. $(z - 5)(z + 4) = 52$ **56.** $(z - 8)(z + 4) = -35$ **57.** $(z - 6)(z + 1) = -10$

58. $(a + 3)(a + 4) = 72$ **59.** $(a - 4)(a + 7) = -18$ **60.** $(2x + 5)(x + 1) = -1$

[illegible] Application Problems

61. The square of a positive number is six more than five times the positive number. Find the number.

62. The square of a negative number is fifteen more than twice the negative number. Find the number.

63. The sum of two numbers is six. The sum of the squares of the two numbers is twenty. Find the two numbers.

64. The sum of two numbers is eight. The sum of the squares of the two numbers is thirty-four. Find the two numbers.

65. The sum of the squares of two consecutive positive integers is forty-one. Find the two integers.

66. The sum of the squares of two consecutive positive even integers is one hundred. Find the two integers.

67. The sum of two numbers is ten. The product of the two numbers is twenty-one. Find the two numbers.

68. The sum of two numbers is thirteen. The product of the two numbers is forty. Find the two numbers.

The formula $S = \frac{n^2 + n}{2}$ gives the sum, S, of the first n natural numbers. Use this formula for Exercises 69 and 70.

69. How many consecutive natural numbers beginning with 1 will give a sum of 78?

70. How many consecutive natural numbers beginning with 1 will give a sum of 171?

The formula $N = \frac{t^2 - t}{2}$ gives the number, N, of football games that must be scheduled in a league with t teams if each team is to play every other team once. Use this formula for Exercises 71 and 72.

71. How many teams are in a league that schedules 15 games in such a way that each team plays every other team once?

72. How many teams are in a league that schedules 45 games in such a way that each team plays every other team once?

The distance, s, in feet, that an object will fall (neglecting air resistance) in t seconds is given by $s = vt + 16t^2$, where v is the initial velocity of the object in feet per second. Use this formula for Exercises 73 and 74.

73. An object is released from the top of a building 192 ft high. The initial velocity is 16 ft/s, and air resistance is neglected. How many seconds later will the object hit the ground?

74. An object is released from the top of a building 320 ft high. The initial velocity is 16 ft/s, and air resistance is neglected. How many seconds later will the object hit the ground?

The height, h, in feet, an object will attain (neglecting air resistance) in t seconds is given by $h = vt - 16t^2$, where v is the initial velocity of the object in feet per second. Use this formula for Exercises 75 and 76.

75. A golf ball is thrown onto a cement surface and rebounds straight up. The initial velocity of the rebound is 60 ft/s. How many seconds later will the golf ball return to the ground?

76. A foul ball leaves a bat and travels straight up with an initial velocity of 64 ft/s. How many seconds later will the ball be 64 ft above the ground?

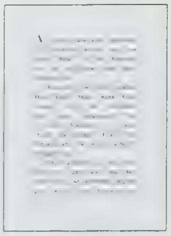
77. The length of a rectangle is 5 in. more than twice its width. Its area is 75 in^2. Find the length and width of the rectangle.

78. The width of a rectangle is 5 ft less than the length. The area of the rectangle is 176 ft^2. Find the length and width of the rectangle.

79. The height of a triangle is 4 m more than twice the length of the base. The area of the triangle is 35 m^2. Find the height of the triangle.

80. The length of each side of a square is extended 5 in. The area of the resulting square is 64 in^2. Find the length of a side of the original square.

81. The page of a book measures 6 in. by 9 in. A uniform border around the page leaves 28 in^2 for type. What are the dimensions of the type area?



82. A small garden measures 8 ft by 10 ft. A uniform border around the garden increases the total area to 143 ft^2. What is the width of the border?

83. The radius of a circle is increased by 3 in.; this increases the area by 100 in^2. Find the radius of the original circle. Round to the nearest hundredth.

84. A circle has a radius of 10 in. Find the increase in area that occurs when the radius is increased by 2 in. Round to the nearest hundredth.

APPLYING THE CONCEPTS

85. Explain the error made in solving the equation at the right. Solve the equation correctly.

$$\begin{array}{cc} \multicolumn{2}{c}{(x+2)(x-3)=6} \\ x+2=6 & x-3=6 \\ x=4 & x=9 \end{array}$$

86. Explain the error made in solving the equation at the right. Solve the equation correctly.

$$\begin{aligned} x^2 &= x \\ \frac{x^2}{x} &= \frac{x}{x} \\ x &= 1 \end{aligned}$$

87. Find $3n^2$ if $n(n+5)=-4$.

88. Find $2n^2$ if $n(n+3)=4$.

Solve.

89. $2y(y+4)=-5(y+3)$

90. $(b+5)^2=16$

91. $p^3=9p^2$

92. $(x+3)(2x-1)=(3-x)(5-3x)$

Focus on Problem Solving

Making a Table

Sometimes a table can be used in organizing information so that it is in a useful form. In the chapter Solving Equations we used tables in the applications, to organize the data. Tables are also useful in applications that require you to find all possible combinations for a given situation.

A basketball player scored 11 points in a game. The player can score 1 point for making a free throw, 2 points for making a field goal within the three-point line, and 3 points for making a field goal outside the three-point line. Find the possible combinations in which the player could have scored 11 points.

The following table lists the possible combinations for scoring 11 points.

Free throws	0	2	1	3	5	0	2	4	6	8	1	3	5	7	9	11
2-point field goal	1	0	2	1	0	4	3	2	1	0	5	4	3	2	1	0
3-point field goal	3	3	2	2	2	1	1	1	1	1	0	0	0	0	0	0
Total points	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11

There are 16 possible ways in which the basketball player could have scored 11 points.

1. A football team scores 17 points. A touchdown counts as 6 points, an extra point as 1 point, a field goal as 3 points, and a safety as 2 points. Find the possible combinations in which the team can score 17 points. Remember that the number of extra points cannot exceed the number of touchdowns scored.

2. Repeat Exercise 1. Assume that no safety was scored.

3. Repeat Exercise 1. Assume that no safety was scored and that the team scored two field goals.

4. Find the number of possible combinations of nickels, dimes, and quarters that one might get when receiving $.85 in change.

5. Repeat Exercise 4. Assume no combination contains coins that could be exchanged for a larger coin. That is, the combination of three quarters and two nickels would not be allowed, because the two nickels could be exchanged for a dime.

6. Find the number of possible combinations of $1, $5, $10, and $20 bills that one might get when receiving $33 in change.

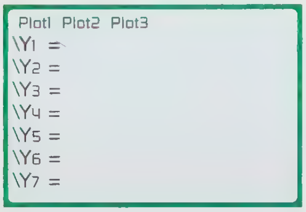
Projects and Group Activities

Evaluating Polynomials Using a Graphing Calculator

A graphing calculator can be used to evaluate a polynomial. To illustrate the method, consider the polynomial $2x^3 - 3x^2 + 4x - 7$. The keystrokes below are for a TI-83 Plus calculator, but the keystrokes for other calculators will closely follow these keystrokes.

TAKE NOTE
Once the polynomial has been entered in Y1, there are several methods to evaluate it. We will show just one option.

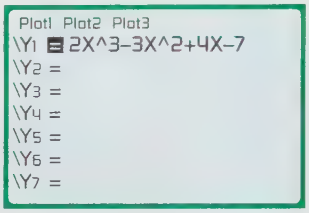
Press the Y= key. You will see a screen similar to the one below. Press CLEAR to erase any expression next to Y1.



```
Plot1 Plot2 Plot3
\Y1 =
\Y2 =
\Y3 =
\Y4 =
\Y5 =
\Y6 =
\Y7 =
```

Enter the polynomial as follows. The ^ key is used to enter an exponent.

2 X,T,ϑ,n ^ 3 − 3 X,T,ϑ,n ^ 2 + 4 X,T,ϑ,n − 7



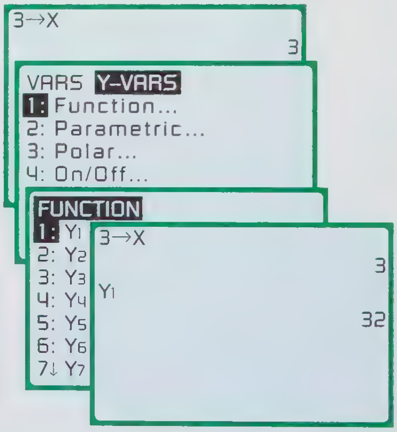
```
Plot1 Plot2 Plot3
\Y1 =2X^3-3X^2+4X-7
\Y2 =
\Y3 =
\Y4 =
\Y5 =
\Y6 =
\Y7 =
```

To evaluate the polynomial when $x = 3$, first return to what is called the HOME screen by pressing 2nd QUIT.

Enter the following keystrokes. Sample screens are shown at the right.

(1) 3 STO▶ X,T,ϑ,n ENTER
(2) VARS ▶ ENTER
(3) ENTER
(4) ENTER

The value of the polynomial when $x = 3$ is 32.



```
3→X
                3
```

```
VARS Y-VARS
1: Function...
2: Parametric...
3: Polar...
4: On/Off...
```

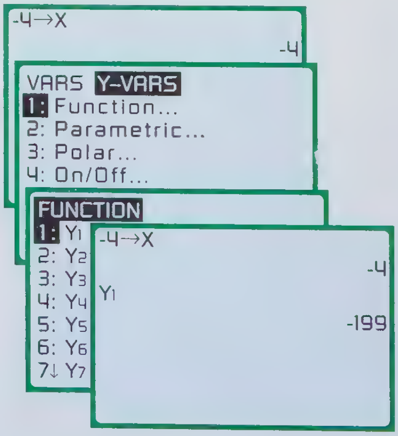
```
FUNCTION
1: Y1
2: Y2
3: Y3
4: Y4
5: Y5
6: Y6
7↓ Y7
```

```
3→X
                3
Y1
               32
```

To evaluate the polynomial at a different value of x, repeat Steps 1 through 4. For instance, to evaluate the polynomial when $x = -4$, we would have

(1) (−) 4 STO▶ X,T,ϑ,n ENTER
(2) VARS ▶ ENTER
(3) ENTER
(4) ENTER

The value of the polynomial when $x = -4$ is -199.



```
-4→X
               -4
```

```
VARS Y-VARS
1: Function...
2: Parametric...
3: Polar...
4: On/Off...
```

```
FUNCTION
1: Y1
2: Y2
3: Y3
4: Y4
5: Y5
6: Y6
7↓ Y7
```

```
-4→X
               -4
Y1
             -199
```

Here are some practice exercises. Evaluate for the given value.

1. $2x^2 - 3x + 7; x = 4$

2. $3x^2 + 7x - 12; x = -3$

3. $3x^3 - 2x^2 + 6x - 8; x = 3$

4. $2x^3 + 4x^2 - x - 2; x = 2$

5. $x^4 - 3x^3 + 6x^2 + 5x - 1; x = 2$

6. $x^5 - x^3 + 2x - 7; x = -4$

Search the World Wide Web

At the address **http://www.utm.edu/research/primes/mersenne.shtml**, you can find the history, theorems, and lists of Mersenne Primes. When $2^n - 1$ is prime, it is said to be a Mersenne Prime. An interesting note is that the 25th and 26th Mersenne Primes were found by high school students Laura Nickel and Curt Noll. In 2000, the largest known Mersenne Prime had more than 2 million digits. Nayan Hajratwala, the person who found this prime, received $50,000 for his efforts.

Would you like to find the answer to questions such as these?

1. What are perfect numbers and how are they used?
2. Is zero a prime number, a composite number, or neither?
3. What is the difference between zero and nothing?
4. When something is divided by zero, why is the answer undefined?
5. What is infinity plus one?
6. Is infinity positive or negative?

These and many other questions are answered at the address

http://mathforum.org/dr.math/

The answers to these questions are presented in a clever and interesting fashion. The answer to the question "What is the purpose of the number zero?" is given below.

> The invention of zero was one of the most important breakthroughs in the history of civilization. More important, in my opinion, than the invention of the wheel. I think that it's a fairly deep concept.
>
> One crucial purpose that zero holds is as a placeholder in our system of notation. When we write the number 408, we're really using a shorthand notation. What we really mean by 408 is "4 times 100, plus 0 times 10, plus 8 times 1." Without the number zero, we wouldn't be able to tell the numbers 408, 48, 480, 408,000, and 4800 apart. So yes, zero is important.
>
> Another crucial role that zero plays in mathematics is that of an "additive identity element." What this means is that when you add zero to any number, you get the number that you started with. For instance, $5 + 0 = 5$. That may seem obvious and trivial, but it's actually quite important to have such a number. For instance, when you're manipulating some numerical quantity and you want to change its form but not its value, you might add some fancy version of zero to it, like this:
>
> $$\begin{aligned} x^2 + y^2 &= x^2 + y^2 + 2xy - 2xy \\ &= x^2 + 2xy + y^2 - 2xy \\ &= (x + y)^2 - 2xy \end{aligned}$$
>
> Now if we wanted to, we could use this as a proof that $(x + y)^2$ is always greater than $2xy$; the expression we started with was positive, so the one we ended up with must be positive, too. Therefore, subtracting $2xy$ from $(x + y)^2$ must leave us with a positive number. Neat stuff.*

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The Internet is a good source for the history of and interesting facts about mathematical concepts. You can use the address

http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Prime_numbers.html

to find the history of prime numbers. This web address also contains a list of unsolved problems on prime numbers.

There is an online mathematics dictionary at the address

http://www.math.com/school/glossary/glossindex.html

This site contains several hundred mathematical definitions as well as an online glossary of technical notation.

Chapter Summary

Key Words

The *greatest common factor* (GCF) of two or more monomials is the product of the GCF of the coefficients and the common variable factors. [p. 207]

To *factor a polynomial* means to write the polynomial as a product of other polynomials. [p. 207]

To *factor a trinomial* of the form $ax^2 + bx + c$ means to express the trinomial as the product of two binomials. [p. 213]

A polynomial that does not factor using only integers is *nonfactorable over the integers.* [p. 214]

An equation of the form $ax^2 + bx + c = 0$ is a *quadratic equation*. A quadratic equation is in *standard form* when the polynomial is in descending order and equal to zero. The quadratic equation $ax^2 + bx + c = 0$ is in standard form. [p. 237]

Essential Rules

Sum and Difference of Two Terms = Difference of Two Squares [p. 229]

$$(a + b)(a - b) = a^2 - b^2$$

Square of a Binomial = Perfect-Square Trinomial [p. 229]

$$(a + b)^2 = a^2 + 2ab + b^2$$

Principle of Zero Products [p. 237]

If the product of two factors is zero, then at least one of the factors must be zero.

If $a \cdot b = 0$, then $a = 0$ or $b = 0$.

General Factoring Strategy [p. 231]

1. Is there a common factor? If so, factor out the common factor.
2. Is the polynomial the difference of two perfect squares? If so, factor.
3. Is the polynomial a perfect-square trinomial? If so, factor.
4. Is the polynomial a trinomial that is the product of two binomials? If so, factor.
5. Does the polynomial contain four terms? If so, try factoring by grouping.
6. Is each binomial factor a prime polynomial over the integers? If not, factor.

Chapter Review

1. Factor: $b^2 - 13b + 30$

2. Factor: $4x(x - 3) - 5(3 - x)$

3. Factor $2x^2 - 5x + 6$ by using trial factors.

4. Factor: $5x^3 + 10x^2 + 35x$

5. Factor: $14y^9 - 49y^6 + 7y^3$

6. Factor: $y^2 + 5y - 36$

7. Factor $6x^2 - 29x + 28$ by using trial factors.

8. Factor: $12a^2b + 3ab^2$

9. Factor: $a^6 - 100$

10. Factor: $n^4 - 2n^3 - 3n^2$

11. Factor $12y^2 + 16y - 3$ by using trial factors.

12. Factor: $12b^3 - 58b^2 + 56b$

13. Factor: $9y^4 - 25z^2$

14. Factor: $c^2 + 8c + 12$

15. Factor $18a^2 - 3a - 10$ by grouping.

16. Solve: $4x^2 + 27x = 7$

17. Factor: $4x^3 - 20x^2 - 24x$

18. Factor: $3a^2 - 15a - 42$

19. Factor $2a^2 - 19a - 60$ by grouping.

20. Solve: $(x + 1)(x - 5) = 16$

21. Factor: $21ax - 35bx - 10by + 6ay$

22. Factor: $a^2b^2 - 1$

23. Factor: $10x^2 + 25x + 4xy + 10y$

24. Factor: $5x^2 - 5x - 30$

25. Factor: $3x^2 + 36x + 108$

26. Factor $3x^2 - 17x + 10$ by grouping.

27. The length of a hockey field is 20 yd less than twice the width of the hockey field. The area of the hockey field is 6000 yd². Find the length and width of the hockey field.

28. The size, S, of an image from a slide projector depends on the distance, d, of the screen from the projector and is given by $S = d^2$. Find the distance between the projector and the screen when the size of the picture is 400 ft².

29. A rectangular photograph has dimensions 15 in. by 12 in. A picture frame around the photograph increases the total area to 270 in². What is the width of the frame?

30. The length of each side of a square garden plot is extended 4 ft. The area of the resulting square is 576 ft². Find the length of a side of the original garden plot.

Chapter Test

1. Factor: $ab + 6a - 3b - 18$

2. Factor: $2y^4 - 14y^3 - 16y^2$

3. Factor $8x^2 + 20x - 48$ by grouping.

4. Factor $6x^2 + 19x + 8$ by using trial factors.

5. Factor: $a^2 - 19a + 48$

6. Factor: $6x^3 - 8x^2 + 10x$

7. Factor: $x^2 + 2x - 15$

8. Solve: $4x^2 - 1 = 0$

9. Factor: $5x^2 - 45x - 15$

10. Factor: $p^2 + 12p + 36$

11. Solve: $x(x - 8) = -15$

12. Factor: $3x^2 + 12xy + 12y^2$

13. Factor: $b^2 - 16$

14. Factor $6x^2y^2 + 9xy^2 + 3y^2$ by grouping.

15. Factor: $p^2 + 5p + 6$

16. Factor: $a(x - 2) + b(x - 2)$

17. Factor: $x(p + 1) - (p + 1)$

18. Factor: $3a^2 - 75$

19. Factor $2x^2 + 4x - 5$ by using trial factors.

20. Factor: $x^2 - 9x - 36$

21. Factor: $4a^2 - 12ab + 9b^2$

22. Factor: $4x^2 - 49y^2$

23. Solve: $(2a - 3)(a + 7) = 0$

24. The sum of two numbers is ten. The sum of the squares of the two numbers is fifty-eight. Find the two numbers.

25. The length of a rectangle is 3 cm longer than twice its width. The area of the rectangle is 90 cm². Find the length and width of the rectangle.

Cumulative Review

1. Subtract: $-2 - (-3) - 5 - (-11)$

2. Simplify: $(3 - 7)^2 \div (-2) - 3 \cdot (-4)$

3. Evaluate $-2a^2 \div (2b) - c$ when $a = -4$, $b = 2$, and $c = -1$.

4. Simplify: $-\frac{3}{4}(-20x^2)$

5. Simplify: $-2[4x - 2(3 - 2x) - 8x]$

6. Solve: $-\frac{5}{7}x = -\frac{10}{21}$

7. Solve: $3x - 2 = 12 - 5x$

8. Solve: $-2 + 4[3x - 2(4 - x) - 3] = 4x + 2$

9. 120% of what number is 54?

10. Simplify: $(-3a^3b^2)^2$

11. Simplify: $(x + 2)(x^2 - 5x + 4)$

12. Simplify: $(8x^2 + 4x - 3) \div (2x - 3)$

13. Simplify: $(x^{-4}y^3)^2$

14. Factor: $3a - 3b - ax + bx$

15. Factor: $15xy^2 - 20xy^4$

16. Factor: $x^2 - 5xy - 14y^2$

17. Factor: $p^2 - 9p - 10$

18. Factor: $18a^3 + 57a^2 + 30a$

19. Factor: $36a^2 - 49b^2$

20. Factor: $4x^2 + 28xy + 49y^2$

21. Factor: $9x^2 + 15x - 14$

22. Factor: $18x^2 - 48xy + 32y^2$

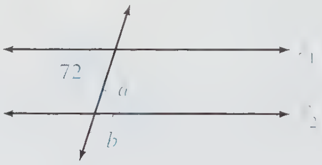
23. Factor: $3y(x - 3) - 2(x - 3)$

24. Solve: $3x^2 + 19x - 14 = 0$

25. A board 10 ft long is cut into two pieces. Four times the length of the shorter piece is 2 ft less than three times the length of the longer piece. Find the length of each piece.

26. A stereo that regularly sells for \$165 is on sale for \$99. Find the discount rate. Use the formula $S = R - rR$.

27. Given that lines ℓ_1 and ℓ_2 are parallel, find the measures of angles a and b.



28. A family drove to a resort at an average speed of 42 mph and later returned over the same road at an average speed of 56 mph. Find the distance to the resort if the total driving time was 7 h.

29. Find three consecutive even integers such that five times the middle integer is twelve more than twice the sum of the first and third integers.

30. The length of the base of a triangle is three times the height. The area of the triangle is 24 in². Find the length of the base of the triangle.

Chapter 6 Rational Expressions

In order to monitor species that are or are becoming endangered, scientists need to determine the present population of that species. Scientists catch and tag a certain number of the animals and then release them. Later, a group of the animals from that same habitat is caught and the number tagged is counted. A proportion is used to estimate the total population size in that region, as shown in **Exercises 26 and 27 on page 290.** Tracking the tagged animals also assists scientists in learning more about the habits of that species.

Objectives

Section 6.1

A To simplify a rational expression
B To multiply rational expressions
C To divide rational expressions

Section 6.2

A To find the least common multiple (LCM) of two or more polynomials
B To express two fractions in terms of the LCM of their denominators

Section 6.3

A To add or subtract rational expressions with the same denominators
B To add or subtract rational expressions with different denominators

Section 6.4

A To simplify a complex fraction

Section 6.5

A To solve an equation containing fractions

Section 6.6

A To solve a proportion
B To solve application problems
C To solve problems involving similar triangles

Section 6.7

A To solve a literal equation for one of the variables

Section 6.8

A To solve work problems
B To solve uniform motion problems

Need help? For online student resources, such as section quizzes, visit this textbook's website at **math.college.hmco.com/students.**

Prep Test

1. Find the least common multiple (LCM) of 12 and 18.

2. Simplify: $\dfrac{9x^3y^4}{3x^2y^7}$

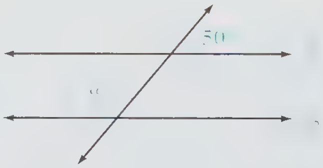
3. Subtract: $\dfrac{3}{4} - \dfrac{8}{9}$

4. Divide: $\left(-\dfrac{8}{11}\right) \div \dfrac{4}{5}$

5. If a is a nonzero number, are the following two quantities equal: $\dfrac{0}{a}$ and $\dfrac{a}{0}$?

6. Solve: $\dfrac{2}{3}x - \dfrac{3}{4} = \dfrac{5}{6}$

7. Line l_1 is parallel to line l_2. Find the measure of angle a.



8. Factor: $x^2 - 4x - 12$

9. Factor: $2x^2 - x - 3$

10. At 9:00 AM, Anthony begins jogging on a park trail at a rate of 9 ft/min. Ten minutes later his sister Jean begins the same trail jogging in pursuit of her brother at a rate of 12 ft/min. At what time will Jean catch up to Anthony?

Go Figure

A mouse begins at corner A of a 12 ft by 12 ft square maze traveling clockwise at a constant speed of 2 feet per second. Six seconds later a second mouse starts from the same corner A and travels clockwise at a constant speed of 3 feet per second. How far apart are the two mice 18 seconds after the second mouse begins?

6.1 Multiplication and Division of Rational Expressions

To simplify a rational expression

A fraction in which the numerator or denominator is a polynomial is called a **rational expression.** Examples of rational expressions are shown at the right.

$$\frac{5}{z}, \qquad \frac{x^2+1}{2x-1}, \qquad \frac{y^2+y-1}{4y^2+1}$$

Care must be exercised with a rational expression to ensure that when the variables are replaced with numbers, the resulting denominator is not zero.

Consider the rational expression at the right. The value of x cannot be 3 because the denominator would then be zero.

$$\frac{4x^2-9}{2x-6}$$

$$\frac{4(3)^2-9}{2(3)-6} = \frac{27}{0} \quad \text{Not a real number}$$

A rational expression is in **simplest form** when the numerator and denominator have no common factors. The Multiplication Property of One is used to write a rational expression in simplest form.

Simplify: $\dfrac{x^2-4}{x^2-2x-8}$

$$\begin{aligned}\frac{x^2-4}{x^2-2x-8} &= \frac{(x-2)(x+2)}{(x-4)(x+2)}\\ &= \frac{x-2}{x-4}\cdot\boxed{\frac{x+2}{x+2}} = \frac{x-2}{x-4}\cdot 1\\ &= \frac{x-2}{x-4},\ x \neq -2, 4\end{aligned}$$

- Factor the numerator and denominator.
- The restrictions, $x \neq -2$ or 4, are necessary to prevent division by zero.

This simplification is usually shown with slashes through the common factors. The last simplification would be shown as

$$\begin{aligned}\frac{x^2-4}{x^2-2x-8} &= \frac{(x-2)\overset{1}{\cancel{(x+2)}}}{(x-4)\underset{1}{\cancel{(x+2)}}}\\ &= \frac{x-2}{x-4},\ x \neq -2, 4\end{aligned}$$

- Factor the numerator and denominator.
- Divide by the common factors. The restrictions, $x \neq -2$ or 4, are necessary to prevent division by zero.

Simplify: $\dfrac{10+3x-x^2}{x^2-4x-5}$

$$\begin{aligned}\frac{10+3x-x^2}{x^2-4x-5} &= \frac{-(x^2-3x-10)}{x^2-4x-5}\\ &= \frac{-\overset{1}{\cancel{(x-5)}}(x+2)}{\underset{1}{\cancel{(x-5)}}(x+1)}\\ &= -\frac{x+2}{x+1},\ x \neq -1, 5\end{aligned}$$

- Because the coefficient of x^2 in the numerator is −1, factor −1 from the numerator.
- Factor the numerator and denominator.

For the remaining examples, we will omit the restrictions on the variables that prevent division by zero and assume the values of the variables are such that division by zero is not possible.

Example 1

Simplify: $\frac{4x^3y^4}{6x^4y}$

Solution

$\frac{4x^3y^4}{6x^4y} = \frac{2y^3}{3x}$

- Use rules of exponents.

You Try It 1

Simplify: $\frac{6x^5y}{12x^2y^3}$

Your solution

Example 2

Simplify: $\frac{9 - x^2}{x^2 + x - 12}$

Solution

$$\frac{9 - x^2}{x^2 + x - 12} = \frac{\overset{-1}{\cancel{(3 - x)}}(3 + x)}{\underset{1}{\cancel{(x - 3)}}(x + 4)} = -\frac{x + 3}{x + 4}$$

You Try It 2

Simplify: $\frac{x^2 + 2x - 24}{16 - x^2}$

Your solution

Example 3

Simplify: $\frac{x^2 + 2x - 15}{x^2 - 7x + 12}$

Solution

$$\frac{x^2 + 2x - 15}{x^2 - 7x + 12} = \frac{(x + 5)\overset{1}{\cancel{(x - 3)}}}{\underset{1}{\cancel{(x - 3)}}(x - 4)} = \frac{x + 5}{x - 4}$$

You Try It 3

Simplify: $\frac{x^2 + 4x - 12}{x^2 - 3x + 2}$

Your solution

Solutions on p. S13

Objective B

To multiply rational expressions

The product of two fractions is a fraction whose numerator is the product of the numerators of the two fractions and whose denominator is the product of the denominators of the two fractions.

Rule for Multiplying Fractions

If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions and $b \neq 0$, $d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

$$\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15} \qquad \frac{3x}{y} \cdot \frac{2}{z} = \frac{6x}{yz} \qquad \frac{x + 2}{x} \cdot \frac{3}{x - 2} = \frac{3x + 6}{x^2 - 2x}$$

➡ Simplify: $\dfrac{x^2 + 3x}{x^2 - 3x - 4} \cdot \dfrac{x^2 - 5x + 4}{x^2 + 2x - 3}$

$$\frac{x^2 + 3x}{x^2 - 3x - 4} \cdot \frac{x^2 - 5x + 4}{x^2 + 2x - 3}$$

$$= \frac{x(x + 3)}{(x - 4)(x + 1)} \cdot \frac{(x - 4)(x - 1)}{(x + 3)(x - 1)}$$

- Factor the numerator and denominator of each fraction.

$$= \frac{x\overset{1}{\cancel{(x + 3)}}\overset{1}{\cancel{(x - 4)}}\overset{1}{\cancel{(x - 1)}}}{\underset{1}{\cancel{(x - 4)}}(x + 1)\underset{1}{\cancel{(x + 3)}}\underset{1}{\cancel{(x - 1)}}}$$

- Multiply. Then divide by the common factors.

= [illegible]

- Write the answer in simplest form.

Example 4

Simplify: $\dfrac{10x^2 - 15x}{12x - 8} \cdot \dfrac{3x - 2}{20x - 25}$

Solution

$$\frac{10x^2 - 15x}{12x - 8} \cdot \frac{3x - 2}{20x - 25}$$

$$= \frac{5x(2x - 3)}{4(3x - 2)} \cdot \frac{(3x - 2)}{5(4x - 5)}$$

$$= \frac{\overset{1}{\cancel{5}}x(2x - 3)\overset{1}{\cancel{(3x - 2)}}}{4\underset{1}{\cancel{(3x - 2)}}\underset{1}{\cancel{5}}(4x - 5)} = \frac{x(2x - 3)}{4(4x - 5)}$$

You Try It 4

Simplify: $\dfrac{12x^2 + 3x}{10x - 15} \cdot \dfrac{8x - 12}{9x + 18}$

Your solution

Example 5

Simplify: $\dfrac{x^2 + x - 6}{x^2 + 7x + 12} \cdot \dfrac{x^2 + 3x - 4}{4 - x^2}$

Solution

$$\frac{x^2 + x - 6}{x^2 + 7x + 12} \cdot \frac{x^2 + 3x - 4}{4 - x^2}$$

$$= \frac{(x + 3)(x - 2)}{(x + 3)(x + 4)} \cdot \frac{(x + 4)(x - 1)}{(2 - x)(2 + x)}$$

$$= \frac{\overset{1}{\cancel{(x + 3)}}\overset{-1}{\cancel{(x - 2)}}\overset{1}{\cancel{(x + 4)}}(x - 1)}{\underset{1}{\cancel{(x + 3)}}\underset{1}{\cancel{(x + 4)}}\underset{1}{\cancel{(2 - x)}}(2 + x)} = -\frac{x - 1}{x + 2}$$

You Try It 5

Simplify: $\dfrac{x^2 + 2x - 15}{9 - x^2} \cdot \dfrac{x^2 - 3x - 18}{x^2 - 7x + 6}$

Your solution

Solutions on p. S13

Objective C

To divide rational expressions

The reciprocal of a fraction is a fraction with the numerator and denominator interchanged.

$$\text{Fraction}\left\{\begin{array}{ll} \dfrac{a}{b} & \dfrac{b}{a} \\ x^2 = \dfrac{x^2}{1} & \dfrac{1}{x^2} \\ \dfrac{x+2}{x} & \dfrac{x}{x+2} \end{array}\right\}\text{Reciprocal}$$

Rule for Dividing Fractions

Divide fractions by multiplying the dividend by the reciprocal of the divisor.
$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

$$\frac{4}{x} \div \frac{y}{5} = \frac{4}{x} \cdot \frac{5}{y} = \frac{20}{xy} \qquad \frac{x+4}{x} \div \frac{x-2}{4} = \frac{x+4}{x} \cdot \frac{4}{x-2} = \frac{4(x+4)}{x(x-2)}$$

The basis for the division rule is shown at the right.

$$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b}}{\frac{c}{d}} \cdot \frac{\frac{d}{c}}{\frac{d}{c}} = \frac{\frac{a}{b} \cdot \frac{d}{c}}{1} = \frac{a}{b} \cdot \frac{d}{c}$$

Example 6

Simplify: $\dfrac{xy^2 - 3x^2y}{z^2} \div \dfrac{6x^2 - 2xy}{z^3}$

Solution

$$\frac{xy^2 - 3x^2y}{z^2} \div \frac{6x^2 - 2xy}{z^3}$$
$$= \frac{xy^2 - 3x^2y}{z^2} \cdot \frac{z^3}{6x^2 - 2xy}$$
$$= \frac{xy\overset{-1}{\cancel{(y - 3x)}} \cdot z^3}{z^2 \cdot 2x\underset{1}{\cancel{(3x - y)}}} = -\frac{yz}{2}$$

You Try It 6

Simplify: $\dfrac{a^2}{4bc^2 - 2b^2c} \div \dfrac{a}{6bc - 3b^2}$

Your solution

Example 7

Simplify: $\dfrac{2x^2 + 5x + 2}{2x^2 + 3x - 2} \div \dfrac{3x^2 + 13x + 4}{2x^2 + 7x - 4}$

Solution

$$\frac{2x^2 + 5x + 2}{2x^2 + 3x - 2} \div \frac{3x^2 + 13x + 4}{2x^2 + 7x - 4}$$
$$= \frac{2x^2 + 5x + 2}{2x^2 + 3x - 2} \cdot \frac{2x^2 + 7x - 4}{3x^2 + 13x + 4}$$
$$= \frac{(2x + 1)\overset{1}{\cancel{(x + 2)}} \cdot \overset{1}{\cancel{(2x - 1)}}\overset{1}{\cancel{(x + 4)}}}{\underset{1}{\cancel{(2x - 1)}}\underset{1}{\cancel{(x + 2)}} \cdot (3x + 1)\underset{1}{\cancel{(x + 4)}}} = \frac{2x + 1}{3x + 1}$$

You Try It 7

Simplify: $\dfrac{3x^2 + 26x + 16}{3x^2 - 7x - 6} \div \dfrac{2x^2 + 9x - 5}{x^2 + 2x - 15}$

Your solution

Solutions on p. S13

6.1 Exercises

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1. Explain the procedure for writing a rational expression in simplest form.

2. Explain why the following simplification is incorrect.

$$\frac{x+3}{x} = \frac{\overset{1}{\cancel{x}} + 3}{\underset{1}{\cancel{x}}} = 4$$

Simplify.

3. $\dfrac{9x^3}{12x^4}$ **4.** $\dfrac{16x^2y}{24xy^3}$ **5.** $\dfrac{(x+3)^2}{(x+3)^3}$ **6.** $\dfrac{(2x-1)^5}{(2x-1)^4}$

7. $\dfrac{3n-4}{4-3n}$ **8.** $\dfrac{5-2x}{2x-5}$ **9.** $\dfrac{6y(y+2)}{9y^2(y+2)}$ **10.** $\dfrac{12x^2(3-x)}{18x(3-x)}$

11. $\dfrac{6x(x-5)}{8x^2(5-x)}$ **12.** $\dfrac{14x^3(7-3x)}{21x(3x-7)}$ **13.** $\dfrac{a^2+4a}{ab+4b}$ **14.** $\dfrac{x^2-3x}{2x-6}$

15. $\dfrac{4-6x}{3x^2-2x}$ **16.** $\dfrac{5xy-3y}{9-15x}$ **17.** $\dfrac{y^2-3y+2}{y^2-4y+3}$ **18.** $\dfrac{x^2+5x+6}{x^2+8x+15}$

19. $\dfrac{x^2+3x-10}{x^2+2x-8}$ **20.** $\dfrac{a^2+7a-8}{a^2+6a-7}$ **21.** $\dfrac{x^2+x-12}{x^2-6x+9}$ **22.** $\dfrac{x^2+8x+16}{x^2-2x-24}$

23. $\dfrac{x^2-3x-10}{25-x^2}$ **24.** $\dfrac{4-y^2}{y^2-3y-10}$ **25.** $\dfrac{2x^3+2x^2-4x}{x^3+2x^2-3x}$ **26.** $\dfrac{3x^3-12x}{6x^3-24x^2+24x}$

27. $\dfrac{6x^2-7x+2}{6x^2+5x-6}$ **28.** $\dfrac{2n^2-9n+4}{2n^2-5n-12}$ **29.** $\dfrac{x^2+3x-28}{24-2x-x^2}$

[illegible]

Simplify.

30. $\dfrac{8x^2}{9y^3} \cdot \dfrac{3y^2}{4x^3}$

31. $\dfrac{14a^2b^3}{15x^5y^2} \cdot \dfrac{25x^3y}{16ab}$

32. $\dfrac{12x^3y^4}{7a^2b^3} \cdot \dfrac{14a^3b^4}{9x^2y^2}$

33. $\dfrac{18a^4b^2}{25x^2y^3} \cdot \dfrac{50x^5y^6}{27a^6b^2}$

34. $\dfrac{3x - 6}{5x - 20} \cdot \dfrac{10x - 40}{27x - 54}$

35. $\dfrac{8x - 12}{14x + 7} \cdot \dfrac{42x + 21}{32x - 48}$

36. $\dfrac{3x^2 + 2x}{2xy - 3y} \cdot \dfrac{2xy^3 - 3y^3}{3x^3 + 2x^2}$

37. $\dfrac{4a^2x - 3a^2}{2by + 5b} \cdot \dfrac{2b^3y + 5b^3}{4ax - 3a}$

38. $\dfrac{x^2 + 5x + 4}{x^3y^2} \cdot \dfrac{x^2y^3}{x^2 + 2x + 1}$

39. $\dfrac{x^2 + x - 2}{xy^2} \cdot \dfrac{x^3y}{x^2 + 5x + 6}$

40. $\dfrac{x^4y^2}{x^2 + 3x - 28} \cdot \dfrac{x^2 - 49}{xy^4}$

41. $\dfrac{x^5y^3}{x^2 + 13x + 30} \cdot \dfrac{x^2 + 2x - 3}{x^7y^2}$

42. $\dfrac{2x^2 - 5x}{2xy + y} \cdot \dfrac{2xy^2 + y^2}{5x^2 - 2x^3}$

43. $\dfrac{3a^3 + 4a^2}{5ab - 3b} \cdot \dfrac{3b^3 - 5ab^3}{3a^2 + 4a}$

44. $\dfrac{x^2 - 2x - 24}{x^2 - 5x - 6} \cdot \dfrac{x^2 + 5x + 6}{x^2 + 6x + 8}$

45. $\dfrac{x^2 - 8x + 7}{x^2 + 3x - 4} \cdot \dfrac{x^2 + 3x - 10}{x^2 - 9x + 14}$

46. $\dfrac{x^2 + 2x - 35}{x^2 + 4x - 21} \cdot \dfrac{x^2 + 3x - 18}{x^2 + 9x + 18}$

47. $\dfrac{y^2 + y - 20}{y^2 + 2y - 15} \cdot \dfrac{y^2 + 4y - 21}{y^2 + 3y - 28}$

48. $\dfrac{x^2-3x-4}{x^2+6x+5} \cdot \dfrac{x^2+5x+6}{8+2x-x^2}$

49. $\dfrac{25-n^2}{n^2-2n-35} \cdot \dfrac{n^2-8n-20}{n^2-3n-10}$

50. $\dfrac{12x^2-6x}{x^2+6x+5} \cdot \dfrac{2x^4+10x^3}{4x^2-1}$

51. $\dfrac{8x^3+4x^2}{x^2-3x+2} \cdot \dfrac{x^2-4}{16x^2+8x}$

52. $\dfrac{16+6x-x^2}{x^2-10x-24} \cdot \dfrac{x^2-6x-27}{x^2-17x+72}$

53. $\dfrac{x^2-11x+28}{x^2-13x+42} \cdot \dfrac{x^2+7x+10}{20-x-x^2}$

54. $\dfrac{2x^2+5x+2}{2x^2+7x+3} \cdot \dfrac{x^2-7x-30}{x^2-6x-40}$

55. $\dfrac{x^2-4x-32}{x^2-8x-48} \cdot \dfrac{3x^2+17x+10}{3x^2-22x-16}$

Objective C

56. What is the reciprocal of a rational expression?

57. Explain how to divide rational expressions.

Simplify.

58. $\dfrac{4x^2y^3}{15a^2b^3} \div \dfrac{6xy}{5a^3b^5}$

59. $\dfrac{9x^3y^4}{16a^4b^2} \div \dfrac{45x^4y^2}{14a^7b}$

60. $\dfrac{6x-12}{8x+32} \div \dfrac{18x-36}{10x+40}$

61. $\dfrac{28x+14}{45x-30} \div \dfrac{14x+7}{30x-20}$

62. $\dfrac{6x^3+7x^2}{12x-3} \div \dfrac{6x^2+7x}{36x-9}$

63. $\dfrac{5a^2y+3a^2}{2x^3+5x^2} \div \dfrac{10ay+6a}{6x^3+15x^2}$

64. $\dfrac{x^2+4x+3}{x^2y} \div \dfrac{x^2+2x+1}{xy^2}$

65. $\dfrac{x^3y^2}{x^2-3x-10} \div \dfrac{xy^4}{x^2-x-20}$

66. $\dfrac{x^2-49}{x^4y^3} \div \dfrac{x^2-14x+49}{x^4y^3}$

67. $\dfrac{x^2y^5}{x^2-11x+30} \div \dfrac{xy^6}{x^2-7x+10}$

68. $\dfrac{4ax - 8a}{c^2} \div \dfrac{2y - xy}{c^3}$

69. $\dfrac{3x^2y - 9xy}{a^2b} \div \dfrac{3x^2 - x^3}{ab^2}$

70. $\dfrac{x^2 - 5x + 6}{x^2 - 9x + 18} \div \dfrac{x^2 - 6x + 8}{x^2 - 9x + 20}$

71. $\dfrac{x^2 + 3x - 40}{x^2 + 2x - 35} \div \dfrac{x^2 + 2x - 48}{x^2 + 3x - 18}$

72. $\dfrac{x^2 + 2x - 15}{x^2 - 4x - 45} \div \dfrac{x^2 + x - 12}{x^2 - 5x - 36}$

73. $\dfrac{y^2 - y - 56}{y^2 + 8y + 7} \div \dfrac{y^2 - 13y + 40}{y^2 - 4y - 5}$

74. $\dfrac{8 + 2x - x^2}{x^2 + 7x + 10} \div \dfrac{x^2 - 11x + 28}{x^2 - x - 42}$

75. $\dfrac{x^2 - x - 2}{x^2 - 7x + 10} \div \dfrac{x^2 - 3x - 4}{40 - 3x - x^2}$

76. $\dfrac{2x^2 - 3x - 20}{2x^2 - 7x - 30} \div \dfrac{2x^2 - 5x - 12}{4x^2 + 12x + 9}$

77. $\dfrac{6n^2 + 13n + 6}{4n^2 - 9} \div \dfrac{6n^2 + n - 2}{4n^2 - 1}$

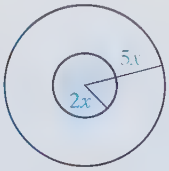
APPLYING THE CONCEPTS

78. Given the expression $\dfrac{9}{x^2 + 1}$, choose some values of x and evaluate the expression for those values. Is it possible to choose a value of x for which the value of the expression is greater than 10? If so, what is that value of x? If not, explain why it is not possible.

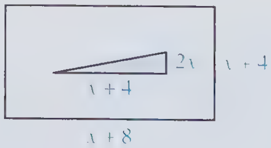
79. Given the expression $\dfrac{1}{y - 3}$, choose some values of y and evaluate the expression for those values. Is it possible to choose a value of y for which the value of the expression is greater than 10,000,000? If so, what is that value of y? If not, explain why it is not possible.

Write in simplest form the ratio of the shaded area of the figure to the total area of the figure.

80.



81.



Complete each simplification.

82. $\dfrac{8x}{9y} \div \underline{\ ?\ } = \dfrac{10y}{3}$

83. $\dfrac{n}{n + 3} \div \underline{\ ?\ } = \dfrac{n}{n - 2}$

84. $\underline{\ ?\ } \div \dfrac{n - 1}{4n^3} = 2n^2(n + 1)$

6.2 Expressing Fractions in Terms of the Least Common Multiple (LCM)

[illegible]

To find the least common multiple (LCM) of two or more polynomials

The **least common multiple (LCM)** of two or more numbers is the smallest number that contains the prime factorization of each number.

The LCM of 12 and 18 is 36 because 36 contains the prime factors of 12 and the prime factors of 18.

$$12 = 2 \cdot 2 \cdot 3$$
$$18 = 2 \cdot 3 \cdot 3$$
$$\text{LCM} = 36 = \rlap{$\overbrace{}^{\text{Factors of 12}}$}2 \cdot \underbrace{2 \cdot 3 \cdot 3}_{\text{Factors of 18}}$$

The least common multiple of two or more polynomials is the polynomial of least degree that contains the factors of each polynomial.

To find the LCM of two or more polynomials, first factor each polynomial completely. The LCM is the product of each factor the greatest number of times it occurs in any one factorization.

TAKE NOTE
The LCM must contain the factors of each polynomial. As shown with the braces at the right, the LCM contains the factors of $4x^2 + 4x$ and the factors of $x^2 + 2x + 1$.

⇒ Find the LCM of $4x^2 + 4x$ and $x^2 + 2x + 1$.

The LCM of the polynomials is the product of the LCM of the numerical coefficients and each variable factor the greatest number of times it occurs in any one factorization.

$$4x^2 + 4x = 4x(x + 1) = 2 \cdot 2 \cdot x(x + 1)$$
$$x^2 + 2x + 1 = (x + 1)(x + 1)$$
$$\text{LCM} = \overbrace{2 \cdot 2 \cdot x\rlap{$\underbrace{}_{\text{Factors of } x^2 + 2x + 1}$}(x + 1)}^{\text{Factors of } 4x^2 + 4x}(x + 1) = 4x(x + 1)(x + 1)$$

Example 1
Find the LCM of $4x^2y$ and $6xy^2$.

Solution
$4x^2y = 2 \cdot 2 \cdot x \cdot x \cdot y$
$6xy^2 = 2 \cdot 3 \cdot x \cdot y \cdot y$
$\text{LCM} = 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y = 12x^2y^2$

You Try It 1
Find the LCM of $8uv^2$ and $12uw$.

Your solution

Example 2
Find the LCM of $x^2 - x - 6$ and $9 - x^2$.

Solution
$x^2 - x - 6 = (x - 3)(x + 2)$
$9 - x^2 = -(x^2 - 9) = -(x + 3)(x - 3)$
$\text{LCM} = (x - 3)(x + 2)(x + 3)$

You Try It 2
Find the LCM of $m^2 - 6m + 9$ and $m^2 - 2m - 3$.

Your solution

Solutions on p. S13

[illegible]

To express two fractions in terms of the LCM of their denominators

When adding and subtracting fractions, it is frequently necessary to express two or more fractions in terms of a common denominator. This common denominator is the LCM of the denominators of the fractions.

➡ Write the fractions $\frac{x+1}{4x^2}$ and $\frac{x-3}{6x^2-12x}$ in terms of the LCM of the denominators.

Find the LCM of the denominators.

The LCM is $12x^2(x-2)$.

For each fraction, multiply the numerator and the denominator by the factors whose product with the denominator is the LCM.

$$\frac{x+1}{4x^2}=\frac{x+1}{4x^2}\cdot\frac{3(x-2)}{3(x-2)}=\frac{3x^2-3x-6}{12x^2(x-2)}$$

$$\frac{x-3}{6x^2-12x}=\frac{x-3}{6x(x-2)}\cdot\frac{2x}{2x}=\frac{2x^2-6x}{12x^2(x-2)}$$

LCM

Example 3

Write the fractions $\frac{x+2}{3x^2}$ and $\frac{x-1}{8xy}$ in terms of the LCM of the denominators.

Solution

The LCM is $24x^2y$.

$$\frac{x+2}{3x^2}=\frac{x+2}{3x^2}\cdot\frac{8y}{8y}=\frac{8xy+16y}{24x^2y}$$

$$\frac{x-1}{8xy}=\frac{x-1}{8xy}\cdot\frac{3x}{3x}=\frac{3x^2-3x}{24x^2y}$$

You Try It 3

Write the fractions $\frac{x-3}{4xy^2}$ and $\frac{2x+1}{9y^2z}$ in terms of the LCM of the denominators.

Your solution

Example 4

Write the fractions $\frac{2x-1}{2x-x^2}$ and $\frac{x}{x^2+x-6}$ in terms of the LCM of the denominators.

Solution

$$\frac{2x-1}{2x-x^2}=\frac{2x-1}{-(x^2-2x)}=-\frac{2x-1}{x^2-2x}$$

The LCM is $x(x-2)(x+3)$.

$$\frac{2x-1}{2x-x^2}=-\frac{2x-1}{x(x-2)}\cdot\frac{x+3}{x+3}=-\frac{2x^2+5x-3}{x(x-2)(x+3)}$$

$$\frac{x}{x^2+x-6}=\frac{x}{(x-2)(x+3)}\cdot\frac{x}{x}=\frac{x^2}{x(x-2)(x+3)}$$

You Try It 4

Write the fractions $\frac{x+4}{x^2-3x-10}$ and $\frac{2x}{25-x^2}$ in terms of the LCM of the denominators.

Your solution

Solutions on p. S13

6.2 Exercises

[illegible]

Find the LCM of the expressions.

1. $8x^3y$
$12xy^2$

2. $6ab^2$
$18ab^3$

3. $10x^4y^2$
$15x^3y$

4. $12a^2b$
$18ab^3$

5. $8x^2$
$4x^2 + 8x$

6. $6y^2$
$4y + 12$

7. $2x^2y$
$3x^2 + 12x$

8. $4xy^2$
$6xy^2 + 12y^2$

9. $9x(x + 2)$
$12(x + 2)^2$

10. $8x^2(x - 1)^2$
$10x^3(x - 1)$

11. $3x + 3$
$2x^2 + 4x + 2$

12. $4x - 12$
$2x^2 - 12x + 18$

13. $(x - 1)(x + 2)$
$(x - 1)(x + 3)$

14. $(2x - 1)(x + 4)$
$(2x + 1)(x + 4)$

15. $(2x + 3)^2$
$(2x + 3)(x - 5)$

16. $(x - 7)(x + 2)$
$(x - 7)^2$

17. $x - 1$
$x - 2$
$(x - 1)(x - 2)$

18. $(x + 4)(x - 3)$
$x + 4$
$x - 3$

19. $x^2 - x - 6$
$x^2 + x - 12$

20. $x^2 + 3x - 10$
$x^2 + 5x - 14$

21. $x^2 + 5x + 4$
$x^2 - 3x - 28$

22. $x^2 - 10x + 21$
$x^2 - 8x + 15$

23. $x^2 - 2x - 24$
$x^2 - 36$

24. $x^2 + 7x + 10$
$x^2 - 25$

25. $x^2 - 7x - 30$
$x^2 - 5x - 24$

26. $2x^2 - 7x + 3$
$2x^2 + x - 1$

27. $3x^2 - 11x + 6$
$3x^2 + 4x - 4$

28. $2x^2 - 9x + 10$
$2x^2 + x - 15$

29. $6 + x - x^2$
$x + 2$
$x - 3$

30. $15 + 2x - x^2$
$x - 5$
$x + 3$

31. $5 + 4x - x^2$
$x - 5$
$x + 1$

32. $x^2 + 3x - 18$
$3 - x$
$x + 6$

33. $x^2 - 5x + 6$
$1 - x$
$x - 6$

[illegible]

Write each fraction in terms of the LCM of the denominators.

34. $\frac{4}{x}, \frac{3}{x^2}$

35. $\frac{5}{ab^2}, \frac{6}{ab}$

36. $\frac{x}{3y^2}, \frac{z}{4y}$

37. $\frac{5y}{6x^2}, \frac{7}{9xy}$

38. $\frac{y}{x(x-3)}, \frac{6}{x^2}$

39. $\frac{a}{y^2}, \frac{6}{y(y+5)}$

40. $\frac{9}{(x-1)^2}, \frac{6}{x(x-1)}$

41. $\frac{a^2}{y(y+7)}, \frac{a}{(y+7)^2}$

42. $\frac{3}{x-3}, \frac{5}{x(3-x)}$

43. $\frac{b}{y(y-4)}, \frac{b^2}{4-y}$

44. $\frac{3}{(x-5)^2}, \frac{2}{5-x}$

45. $\frac{3}{7-y}, \frac{2}{(y-7)^2}$

46. $\frac{3}{x^2+2x}, \frac{4}{x^2}$

47. $\frac{2}{y-3}, \frac{3}{y^3-3y^2}$

48. $\frac{x-2}{x+3}, \frac{x}{x-4}$

49. $\frac{x^2}{2x-1}, \frac{x+1}{x+4}$

50. $\frac{3}{x^2+x-2}, \frac{x}{x+2}$

51. $\frac{3x}{x-5}, \frac{4}{x^2-25}$

52. $\frac{x}{x^2+x-6}, \frac{2x}{x^2-9}$

53. $\frac{x-1}{x^2+2x-15}, \frac{x}{x^2+6x+5}$

APPLYING THE CONCEPTS

54. When is the LCM of two expressions equal to their product?

Write each expression in terms of the LCM of the denominators.

55. $\frac{8}{10^3}, \frac{9}{10^5}$

56. $3, \frac{2}{n}$

57. $x, \frac{x}{x^2-1}$

58. $\frac{x^2+1}{(x-1)^3}, \frac{x+1}{(x-1)^2}, \frac{1}{x-1}$

59. $\frac{c}{6c^2+7cd+d^2}, \frac{d}{3c^2-3d^2}$

60. $\frac{1}{ab+3a-3b-b^2}, \frac{1}{ab+3a+3b+b^2}$

6.3 Addition and Subtraction of Rational Expressions

[illegible]

To add or subtract rational expressions with the same denominators

6

When adding rational expressions in which the denominators are the same, add the numerators. The denominator of the sum is the common denominator.

Rule for Adding Fractions

If $\frac{a}{b}$ and $\frac{c}{b}$ are fractions and $b \neq 0$, then $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$.

$$\frac{5x}{18} + \frac{7x}{18} = \frac{5x + 7x}{18} = \frac{12x}{18} = \frac{2x}{3}$$

$$\frac{x}{x^2 - 1} + \frac{1}{x^2 - 1} = \frac{x + 1}{x^2 - 1} = \frac{\overset{1}{\cancel{(x+1)}}}{(x-1)\underset{1}{\cancel{(x+1)}}} = \frac{1}{x - 1}$$

Note that the sum is written in simplest form.

When subtracting rational expressions with like denominators, subtract the numerators. The denominator of the difference is the common denominator. Write the answer in simplest form.

$$\frac{2x}{x - 2} - \frac{4}{x - 2} = \frac{2x - 4}{x - 2} = \frac{2\overset{1}{\cancel{(x-2)}}}{\underset{1}{\cancel{x-2}}} = 2$$

$$\frac{3x - 1}{x^2 - 5x + 4} - \frac{2x + 3}{x^2 - 5x + 4} = \frac{(3x - 1) - (2x + 3)}{x^2 - 5x + 4} = \frac{3x - 1 - 2x - 3}{x^2 - 5x + 4}$$

$$= \frac{x - 4}{x^2 - 5x + 4} = \frac{\overset{1}{\cancel{(x-4)}}}{\underset{1}{\cancel{(x-4)}}(x-1)} = \frac{1}{x - 1}$$

Example 1

Simplify: $\frac{3x^2}{x^2 - 1} - \frac{x + 4}{x^2 - 1}$

Solution

$$\frac{3x^2}{x^2 - 1} - \frac{x + 4}{x^2 - 1} = \frac{3x^2 - (x + 4)}{x^2 - 1}$$

$$= \frac{3x^2 - x - 4}{x^2 - 1}$$

$$= \frac{(3x - 4)\overset{1}{\cancel{(x+1)}}}{(x - 1)\underset{1}{\cancel{(x+1)}}} = \frac{3x - 4}{x - 1}$$

You Try It 1

Simplify: $\frac{2x^2}{x^2 - x - 12} - \frac{7x + 4}{x^2 - x - 12}$

Your solution

Solution on p. S14

Example 2

Simplify:

$$\frac{2x^2+5}{x^2+2x-3}-\frac{x^2-3x}{x^2+2x-3}+\frac{x-2}{x^2+2x-3}$$

Solution

$$\frac{2x^2+5}{x^2+2x-3}-\frac{x^2-3x}{x^2+2x-3}+\frac{x-2}{x^2+2x-3}$$

$$=\frac{(2x^2+5)-(x^2-3x)+(x-2)}{x^2+2x-3}$$

$$=\frac{2x^2+5-x^2+3x+x-2}{x^2+2x-3}$$

$$=\frac{x^2+4x+3}{x^2+2x-3}=\frac{\overset{1}{\cancel{(x+3)}}(x+1)}{\underset{1}{\cancel{(x+3)}}(x-1)}=\frac{x+1}{x-1}$$

You Try It 2

Simplify:

$$\frac{x^2-1}{x^2-8x+12}-\frac{2x+1}{x^2-8x+12}+\frac{x}{x^2-8x+12}$$

Your solution

Solution on p. S14

Objective B

To add or subtract rational expressions with different denominators

Before two fractions with unlike denominators can be added or subtracted, each fraction must be expressed in terms of a common denominator. This common denominator is the LCM of the denominators of the fractions.

Simplify: $\dfrac{x-3}{x^2-2x}+\dfrac{6}{x^2-4}$

The LCM is $x(x-2)(x+2)$.

- Find the LCM of the denominators.

$$\frac{x-3}{x^2-2x}+\frac{6}{x^2-4}=\frac{x-3}{x(x-2)}\cdot\frac{x+2}{x+2}+\frac{6}{(x-2)(x+2)}\cdot\frac{x}{x}$$

- Write each fraction in terms of the LCM.

$$=\frac{x^2-x-6}{x(x-2)(x+2)}+\frac{6x}{x(x-2)(x+2)}$$

- Multiply the factors in the numerator.

$$=\frac{(x^2-x-6)+6x}{x(x-2)(x+2)}$$

- Add the fractions.

$$=\frac{x^2+5x-6}{x(x-2)(x+2)}$$

$$=\frac{(x+6)(x-1)}{x(x-2)(x+2)}$$

The last step is to factor the numerator to determine whether there are common factors in the numerator and denominator. For this example, there are no common factors, so the answer is in simplest form.

Example 3

Simplify: $\dfrac{y}{x} - \dfrac{4y}{3x} + \dfrac{3y}{4x}$

Solution

The LCM of the denominators is $12x$.

$$\begin{aligned}\frac{y}{x} - \frac{4y}{3x} + \frac{3y}{4x} &= \frac{y}{x}\cdot\frac{12}{12} - \frac{4y}{3x}\cdot\frac{4}{4} + \frac{3y}{4x}\cdot\frac{3}{3}\\ &= \frac{12y}{12x} - \frac{16y}{12x} + \frac{9y}{12x}\\ &= \frac{12y - 16y + 9y}{12x} = \frac{5y}{12x}\end{aligned}$$

You Try It 3

Simplify: $\dfrac{z}{8y} - \dfrac{4z}{3y} + \dfrac{5z}{4y}$

Your solution

Example 4

Simplify: $\dfrac{2x}{x-3} - \dfrac{5}{3-x}$

Solution

Remember $3 - x = -(x - 3)$.

Therefore, $\dfrac{5}{3-x} = \dfrac{5}{-(x-3)} = \dfrac{-5}{x-3}$.

$$\begin{aligned}\frac{2x}{x-3} - \frac{5}{3-x} &= \frac{2x}{x-3} - \frac{-5}{x-3}\\ &= \frac{2x-(-5)}{x-3} = \frac{2x+5}{x-3}\end{aligned}$$

You Try It 4

Simplify: $\dfrac{5x}{x-2} - \dfrac{3}{2-x}$

Your solution

Example 5

Simplify: $\dfrac{2x}{2x-3} - \dfrac{1}{x+1}$

Solution

The LCM is $(2x - 3)(x + 1)$.

$$\begin{aligned}&\frac{2x}{2x-3} - \frac{1}{x+1}\\ &= \frac{2x}{2x-3}\cdot\frac{x+1}{x+1} - \frac{1}{x+1}\cdot\frac{2x-3}{2x-3}\\ &= \frac{2x^2+2x}{(2x-3)(x+1)} - \frac{2x-3}{(2x-3)(x+1)}\\ &= \frac{(2x^2+2x)-(2x-3)}{(2x-3)(x+1)} = \frac{2x^2+2x-2x+3}{(2x-3)(x+1)}\\ &= \frac{2x^2+3}{(2x-3)(x+1)}\end{aligned}$$

You Try It 5

Simplify: $\dfrac{4x}{3x-1} - \dfrac{9}{x+4}$

Your solution

Solutions on p.S14

Example 6

Simplify: $1 + \dfrac{3}{x^2}$

Solution

The LCM is x^2.

$$1 + \frac{3}{x^2} = 1 \cdot \frac{x^2}{x^2} + \frac{3}{x^2}$$
$$= \frac{x^2}{x^2} + \frac{3}{x^2}$$
$$= \frac{x^2 + 3}{x^2}$$

You Try It 6

Simplify: $2 - \dfrac{1}{x - 3}$

Your solution

Example 7

Simplify: $\dfrac{x + 3}{x^2 - 2x - 8} + \dfrac{3}{4 - x}$

Solution

The LCM is $(x - 4)(x + 2)$.

Recall: $\dfrac{3}{4 - x} = \dfrac{-3}{x - 4}$

$$\frac{x + 3}{x^2 - 2x - 8} + \frac{3}{4 - x}$$
$$= \frac{x + 3}{(x - 4)(x + 2)} + \frac{(-3)}{x - 4}$$
$$= \frac{x + 3}{(x - 4)(x + 2)} + \frac{(-3)}{x - 4} \cdot \frac{x + 2}{x + 2}$$
$$= \frac{x + 3}{(x - 4)(x + 2)} + \frac{(-3)(x + 2)}{(x - 4)(x + 2)}$$
$$= \frac{(x + 3) + (-3)(x + 2)}{(x - 4)(x + 2)}$$
$$= \frac{x + 3 - 3x - 6}{(x - 4)(x + 2)} = \frac{-2x - 3}{(x - 4)(x + 2)}$$

You Try It 7

Simplify: $\dfrac{2x - 1}{x^2 - 25} + \dfrac{2}{5 - x}$

Your solution

Example 8

Simplify: $\dfrac{3x + 2}{2x^2 - x - 1} - \dfrac{3}{2x + 1} + \dfrac{4}{x - 1}$

Solution

The LCM is $(2x + 1)(x - 1)$.

$$\frac{3x + 2}{2x^2 - x - 1} - \frac{3}{2x + 1} + \frac{4}{x - 1}$$
$$= \frac{3x + 2}{(2x + 1)(x - 1)} - \frac{3}{2x + 1} \cdot \frac{x - 1}{x - 1} + \frac{4}{x - 1} \cdot \frac{2x + 1}{2x + 1}$$
$$= \frac{3x + 2}{(2x + 1)(x - 1)} - \frac{3x - 3}{(2x + 1)(x - 1)} + \frac{8x + 4}{(2x + 1)(x - 1)}$$
$$= \frac{(3x + 2) - (3x - 3) + (8x + 4)}{(2x + 1)(x - 1)}$$
$$= \frac{3x + 2 - 3x + 3 + 8x + 4}{(2x + 1)(x - 1)} = \frac{8x + 9}{(2x + 1)(x - 1)}$$

You Try It 8

Simplify: $\dfrac{2x - 3}{3x^2 - x - 2} + \dfrac{5}{3x + 2} - \dfrac{1}{x - 1}$

Your solution

Solutions on p. S14

6.3 Exercises

[illegible]

Simplify.

1. $\dfrac{3}{y^2} + \dfrac{8}{y^2}$

2. $\dfrac{6}{ab} - \dfrac{2}{ab}$

3. $\dfrac{3}{x+4} - \dfrac{10}{x+4}$

4. $\dfrac{x}{x+6} - \dfrac{2}{x+6}$

5. $\dfrac{3x}{2x+3} + \dfrac{5x}{2x+3}$

6. $\dfrac{6y}{4y+1} - \dfrac{11y}{4y+1}$

7. $\dfrac{2x+1}{x-3} + \dfrac{3x+6}{x-3}$

8. $\dfrac{4x+3}{2x-7} + \dfrac{3x-8}{2x-7}$

9. $\dfrac{5x-1}{x+9} - \dfrac{3x+4}{x+9}$

10. $\dfrac{6x-5}{x-10} - \dfrac{3x-4}{x-10}$

11. $\dfrac{x-7}{2x+7} - \dfrac{4x-3}{2x+7}$

12. $\dfrac{2n}{3n+4} - \dfrac{5n-3}{3n+4}$

13. $\dfrac{x}{x^2+2x-15} - \dfrac{3}{x^2+2x-15}$

14. $\dfrac{3x}{x^2+3x-10} - \dfrac{6}{x^2+3x-10}$

15. $\dfrac{2x+3}{x^2-x-30} - \dfrac{x-2}{x^2-x-30}$

16. $\dfrac{3x-1}{x^2+5x-6} - \dfrac{2x-7}{x^2+5x-6}$

17. $\dfrac{4y+7}{2y^2+7y-4} - \dfrac{y-5}{2y^2+7y-4}$

18. $\dfrac{x+1}{2x^2-5x-12} + \dfrac{x+2}{2x^2-5x-12}$

19. $\dfrac{2x^2+3x}{x^2-9x+20} + \dfrac{2x^2-3}{x^2-9x+20} - \dfrac{4x^2+2x+1}{x^2-9x+20}$

20. $\dfrac{2x^2+3x}{x^2-2x-63} - \dfrac{x^2-3x+21}{x^2-2x-63} - \dfrac{x-7}{x^2-2x-63}$

21. Explain the process of writing equivalent rational expressions with the LCM of the denominators of the rational expressions as the new denominator.

22. Explain the process of adding rational expressions with different denominators.

Simplify.

23. $\frac{4}{x} + \frac{5}{y}$

24. $\frac{7}{a} + \frac{5}{b}$

25. $\frac{12}{x} - \frac{5}{2x}$

26. $\frac{5}{3a} - \frac{3}{4a}$

27. $\frac{1}{2x} - \frac{5}{4x} + \frac{7}{6x}$

28. $\frac{7}{4y} + \frac{11}{6y} - \frac{8}{3y}$

29. $\frac{5}{3x} - \frac{2}{x^2} + \frac{3}{2x}$

30. $\frac{6}{y^2} + \frac{3}{4y} - \frac{2}{5y}$

31. $\frac{2}{x} - \frac{3}{2y} + \frac{3}{5x} - \frac{1}{4y}$

32. $\frac{5}{2a} + \frac{7}{3b} - \frac{2}{b} - \frac{3}{4a}$

33. $\frac{2x + 1}{3x} + \frac{x - 1}{5x}$

34. $\frac{4x - 3}{6x} + \frac{2x + 3}{4x}$

35. $\frac{x - 3}{6x} + \frac{x + 4}{8x}$

36. $\frac{2x - 3}{2x} + \frac{x + 3}{3x}$

37. $\frac{2x + 9}{9x} - \frac{x - 5}{5x}$

38. $\frac{3y - 2}{12y} - \frac{y - 3}{18y}$

39. $\frac{x + 4}{2x} - \frac{x - 1}{x^2}$

40. $\frac{x - 2}{3x^2} - \frac{x + 4}{x}$

41. $\frac{x - 10}{4x^2} + \frac{x + 1}{2x}$

42. $\frac{x + 5}{3x^2} + \frac{2x + 1}{2x}$

43. $\frac{4}{x + 4} - x$

44. $2x + \frac{1}{x}$

45. $5 - \frac{x - 2}{x + 1}$

46. $3 + \frac{x - 1}{x + 1}$

47. $\frac{x+3}{6x} - \frac{x-3}{8x^2}$

48. $\frac{x+2}{xy} - \frac{3x-2}{x^2y}$

49. $\frac{3x-1}{xy^2} - \frac{2x+3}{xy}$

50. $\frac{4x-3}{3x^2y} + \frac{2x+1}{4xy^2}$

51. $\frac{5x+7}{6xy^2} - \frac{4x-3}{8x^2y}$

52. $\frac{x-2}{8x^2} - \frac{x+7}{12xy}$

53. $\frac{3x-1}{6y^2} - \frac{x+5}{9xy}$

54. $\frac{4}{x-2} + \frac{5}{x+3}$

55. $\frac{2}{x-3} + \frac{5}{x-4}$

56. $\frac{6}{x-7} - \frac{4}{x+3}$

57. $\frac{3}{y+6} - \frac{4}{y-3}$

58. $\frac{2x}{x+1} + \frac{1}{x-3}$

59. $\frac{3x}{x-4} + \frac{2}{x+6}$

60. $\frac{4x}{2x-1} - \frac{5}{x-6}$

61. $\frac{6x}{x+5} - \frac{3}{2x+3}$

62. $\frac{2a}{a-7} + \frac{5}{7-a}$

63. $\frac{4x}{6-x} + \frac{5}{x-6}$

64. $\frac{x}{x^2-9} + \frac{3}{x-3}$

65. $\frac{y}{y^2-16} + \frac{1}{y-4}$

66. $\frac{2x}{x^2-x-6} - \frac{3}{x+2}$

67. $\frac{(x-1)^2}{(x+1)^2} - 1$

68. $1 - \frac{(y-2)^2}{(y+2)^2}$

69. $\frac{x}{1-x^2} - 1 + \frac{x}{1+x}$

70. $\frac{y}{x-y} + 2 - \frac{x}{y-x}$

71. $\frac{3x - 1}{x^2 - 10x + 25} - \frac{3}{x - 5}$

72. $\frac{2a + 3}{a^2 - 7a + 12} - \frac{2}{a - 3}$

73. $\frac{x + 4}{x^2 - x - 42} + \frac{3}{7 - x}$

74. $\frac{x + 3}{x^2 - 3x - 10} + \frac{2}{5 - x}$

75. $\frac{1}{x + 1} + \frac{x}{x - 6} - \frac{5x - 2}{x^2 - 5x - 6}$

76. $\frac{x}{x - 4} + \frac{5}{x + 5} - \frac{11x - 8}{x^2 + x - 20}$

77. $\frac{3x + 1}{x - 1} - \frac{x - 1}{x - 3} + \frac{x + 1}{x^2 - 4x + 3}$

78. $\frac{4x + 1}{x - 8} - \frac{3x + 2}{x + 4} - \frac{49x + 4}{x^2 - 4x - 32}$

79. $\frac{2x + 9}{3 - x} + \frac{x + 5}{x + 7} - \frac{2x^2 + 3x - 3}{x^2 + 4x - 21}$

80. $\frac{3x + 5}{x + 5} - \frac{x + 1}{2 - x} - \frac{4x^2 - 3x - 1}{x^2 + 3x - 10}$

APPLYING THE CONCEPTS

81. Suppose that you drive about 12,000 mi per year and that the cost of gasoline averages \$1.60 per gallon.

a. Let x represent the number of miles per gallon your car gets. Write a variable expression for the amount you spend on gasoline in 1 year.
b. Write and simplify a variable expression for the amount of money you will save each year if you can increase your gas mileage by 5 miles per gallon.
c. If you currently get 25 mi per gallon and you increase your gas mileage by 5 miles per gallon, how much will you save in 1 year?

82. A recommended percent of oxygen (by volume) in the air that a deep-sea diver breathes is given by $\frac{660}{d + 33}$, where d is the depth, in feet, at which the diver is working.

a. What is the recommended percent (to the nearest percent) of oxygen for a diver working at a depth of 50 ft?
b. As the depth of the diver increases, does the recommended amount of oxygen increase or decrease?
c. At sea level, the oxygen content of air is approximately 21%. Is this less than or more than the recommended amount of oxygen for a diver working at the water's surface?

6.4 Complex Fractions

To simplify a complex fraction 6

A **complex fraction** is a fraction whose numerator or denominator contains one or more fractions. Examples of complex fractions are shown at the right.

$$\frac{3}{2-\frac{1}{2}}, \quad \frac{4+\frac{1}{x}}{3+\frac{2}{x}}, \quad \frac{\frac{1}{x-1}+x+3}{x-3+\frac{1}{x+4}}$$

Point of Interest

There are many instances of complex fractions in application problems. The fraction $\frac{1}{\frac{1}{r_1}+\frac{1}{r_2}}$ is used to determine the total resistance in certain electric circuits.

Simplify: $\dfrac{1-\frac{4}{x^2}}{1+\frac{2}{x}}$

The LCM of x and x^2 is x^2.

- **Find the LCM of the denominators of the fractions in the numerator and denominator.**

$$\frac{1-\frac{4}{x^2}}{1+\frac{2}{x}} = \frac{1-\frac{4}{x^2}}{1+\frac{2}{x}} \cdot \frac{x^2}{x^2}$$

- **Multiply the numerator and denominator by the LCM.**

$$= \frac{1 \cdot x^2 - \frac{4}{x^2} \cdot x^2}{1 \cdot x^2 + \frac{2}{x} \cdot x^2}$$

- **Simplify.**

$$= \frac{x^2-4}{x^2+2x} = \frac{(x-2)\overset{1}{\cancel{(x+2)}}}{x\underset{1}{\cancel{(x+2)}}}$$

$$= \frac{x-2}{x}$$

Example 1

Simplify: $\dfrac{\frac{1}{x}+\frac{1}{2}}{\frac{1}{x^2}-\frac{1}{4}}$

Solution

The LCM of x, 2, x^2, and 4 is $4x^2$.

$$\frac{\frac{1}{x}+\frac{1}{2}}{\frac{1}{x^2}-\frac{1}{4}} = \frac{\frac{1}{x}+\frac{1}{2}}{\frac{1}{x^2}-\frac{1}{4}} \cdot \frac{4x^2}{4x^2} = \frac{\frac{1}{x}\cdot 4x^2+\frac{1}{2}\cdot 4x^2}{\frac{1}{x^2}\cdot 4x^2-\frac{1}{4}\cdot 4x^2}$$

$$= \frac{4x+2x^2}{4-x^2} = \frac{2x\overset{1}{\cancel{(2+x)}}}{(2-x)\underset{1}{\cancel{(2+x)}}} = \frac{2x}{2-x}$$

You Try It 1

Simplify: $\dfrac{\frac{1}{3}-\frac{1}{x}}{\frac{1}{9}-\frac{1}{x^2}}$

Your solution

Solution on p. S15

Example 2

Simplify: $\dfrac{1 - \dfrac{2}{x} - \dfrac{15}{x^2}}{1 - \dfrac{11}{x} + \dfrac{30}{x^2}}$

Solution

The LCM of x and x^2 is x^2.

$$\frac{1 - \dfrac{2}{x} - \dfrac{15}{x^2}}{1 - \dfrac{11}{x} + \dfrac{30}{x^2}} = \frac{1 - \dfrac{2}{x} - \dfrac{15}{x^2}}{1 - \dfrac{11}{x} + \dfrac{30}{x^2}} \cdot \frac{x^2}{x^2}$$

$$= \frac{1 \cdot x^2 - \dfrac{2}{x} \cdot x^2 - \dfrac{15}{x^2} \cdot x^2}{1 \cdot x^2 - \dfrac{11}{x} \cdot x^2 + \dfrac{30}{x^2} \cdot x^2}$$

$$= \frac{x^2 - 2x - 15}{x^2 - 11x + 30}$$

$$= \frac{\overset{1}{\cancel{(x-5)}}(x+3)}{\underset{1}{\cancel{(x-5)}}(x-6)} = \frac{x+3}{x-6}$$

You Try It 2

Simplify: $\dfrac{1 + \dfrac{4}{x} + \dfrac{3}{x^2}}{1 + \dfrac{10}{x} + \dfrac{21}{x^2}}$

Your solution

Example 3

Simplify: $\dfrac{x - 8 + \dfrac{20}{x+4}}{x - 10 + \dfrac{24}{x+4}}$

Solution

The LCM is $x + 4$.

$$\frac{x - 8 + \dfrac{20}{x+4}}{x - 10 + \dfrac{24}{x+4}}$$

$$= \frac{x - 8 + \dfrac{20}{x+4}}{x - 10 + \dfrac{24}{x+4}} \cdot \frac{x+4}{x+4}$$

$$= \frac{(x-8)(x+4) + \dfrac{20}{x+4} \cdot (x+4)}{(x-10)(x+4) + \dfrac{24}{x+4} \cdot (x+4)}$$

$$= \frac{x^2 - 4x - 32 + 20}{x^2 - 6x - 40 + 24} = \frac{x^2 - 4x - 12}{x^2 - 6x - 16}$$

$$= \frac{(x-6)\overset{1}{\cancel{(x+2)}}}{(x-8)\underset{1}{\cancel{(x+2)}}} = \frac{x-6}{x-8}$$

You Try It 3

Simplify: $\dfrac{x + 3 - \dfrac{20}{x-5}}{x + 8 + \dfrac{30}{x-5}}$

Your solution

Solutions on p. S15

6.4 Exercises

Simplify.

1. $\dfrac{1+\dfrac{3}{x}}{1-\dfrac{9}{x^2}}$

2. $\dfrac{1+\dfrac{4}{x}}{1-\dfrac{16}{x^2}}$

3. $\dfrac{2-\dfrac{8}{x+4}}{3-\dfrac{12}{x+4}}$

4. $\dfrac{5-\dfrac{25}{x+5}}{1-\dfrac{3}{x+5}}$

5. $\dfrac{1+\dfrac{5}{y-2}}{1-\dfrac{2}{y-2}}$

6. $\dfrac{2-\dfrac{11}{2x-1}}{3-\dfrac{17}{2x-1}}$

7. $\dfrac{4-\dfrac{2}{x+7}}{5+\dfrac{1}{x+7}}$

8. $\dfrac{5+\dfrac{3}{x-8}}{2-\dfrac{1}{x-8}}$

9. $\dfrac{1-\dfrac{1}{x}-\dfrac{6}{x^2}}{1-\dfrac{9}{x^2}}$

10. $\dfrac{1+\dfrac{4}{x}+\dfrac{4}{x^2}}{1-\dfrac{2}{x}-\dfrac{8}{x^2}}$

11. $\dfrac{1-\dfrac{5}{x}-\dfrac{6}{x^2}}{1+\dfrac{6}{x}+\dfrac{5}{x^2}}$

12. $\dfrac{1-\dfrac{7}{a}+\dfrac{12}{a^2}}{1+\dfrac{1}{a}-\dfrac{20}{a^2}}$

13. $\dfrac{1-\dfrac{6}{x}+\dfrac{8}{x^2}}{\dfrac{4}{x^2}+\dfrac{3}{x}-1}$

14. $\dfrac{1+\dfrac{3}{x}-\dfrac{18}{x^2}}{\dfrac{21}{x^2}-\dfrac{4}{x}-1}$

15. $\dfrac{x-\dfrac{4}{x+3}}{1+\dfrac{1}{x+3}}$

16. $\dfrac{y+\dfrac{1}{y-2}}{1+\dfrac{1}{y-2}}$

17. $\dfrac{1-\dfrac{x}{2x+1}}{x-\dfrac{1}{2x+1}}$

18. $\dfrac{1-\dfrac{2x-2}{3x-1}}{x-\dfrac{4}{3x-1}}$

19. $\dfrac{x - 5 + \dfrac{14}{x + 4}}{x + 3 - \dfrac{2}{x + 4}}$

20. $\dfrac{a + 4 + \dfrac{5}{a - 2}}{a + 6 + \dfrac{15}{a - 2}}$

21. $\dfrac{x + 3 - \dfrac{10}{x - 6}}{x + 2 - \dfrac{20}{x - 6}}$

22. $\dfrac{x - 7 + \dfrac{5}{x - 1}}{x - 3 + \dfrac{1}{x - 1}}$

23. $\dfrac{y - 6 + \dfrac{22}{2y + 3}}{y - 5 + \dfrac{11}{2y + 3}}$

24. $\dfrac{x + 2 - \dfrac{12}{2x - 1}}{x + 1 - \dfrac{9}{2x - 1}}$

25. $\dfrac{x - \dfrac{2}{2x - 3}}{2x - 1 - \dfrac{8}{2x - 3}}$

26. $\dfrac{x + 3 - \dfrac{18}{2x + 1}}{x - \dfrac{6}{2x + 1}}$

27. $\dfrac{\dfrac{1}{x} - \dfrac{2}{x - 1}}{\dfrac{3}{x} + \dfrac{1}{x - 1}}$

28. $\dfrac{\dfrac{3}{n + 1} + \dfrac{1}{n}}{\dfrac{2}{n + 1} + \dfrac{3}{n}}$

29. $\dfrac{\dfrac{3}{2x - 1} - \dfrac{1}{x}}{\dfrac{4}{x} + \dfrac{2}{2x - 1}}$

30. $\dfrac{\dfrac{4}{3x + 1} + \dfrac{3}{x}}{\dfrac{6}{x} - \dfrac{2}{3x + 1}}$

APPLYING THE CONCEPTS

Simplify.

31. $1 + \dfrac{1}{1 + \dfrac{1}{2}}$

32. $1 + \dfrac{1}{1 + \dfrac{1}{1 + \dfrac{1}{2}}}$

33. $1 - \dfrac{1}{1 - \dfrac{1}{x}}$

34. $\dfrac{a^{-1} - b^{-1}}{a^{-2} - b^{-2}}$

35. $\left(\dfrac{y}{4} - \dfrac{4}{y}\right) \div \left(\dfrac{4}{y} - 3 + \dfrac{y}{2}\right)$

36. $\left(\dfrac{b}{8} - \dfrac{8}{b}\right) \div \left(\dfrac{8}{b} - 5 + \dfrac{b}{2}\right)$

37. $\dfrac{1 + x^{-1}}{1 - x^{-1}}$

38. $\dfrac{x + x^{-1}}{x - x^{-1}}$

39. $\dfrac{x^{-1}}{y^{-1}} + \dfrac{x}{y}$

6.5 Solving Equations Containing Fractions

To solve an equation containing fractions

To solve an equation containing fractions, **clear denominators** by multiplying each side of the equation by the LCM of the denominators. Then solve for the variable.

Solve: $\frac{3x-1}{4}+\frac{2}{3}=\frac{7}{6}$

$$\frac{3x-1}{4}+\frac{2}{3}=\frac{7}{6}$$

$$12\left(\frac{3x-1}{4}+\frac{2}{3}\right)=12\cdot\frac{7}{6}$$

- The LCM is 12. Multiply each side of the equation by the LCM.

$$12\left(\frac{3x-1}{4}\right)+12\cdot\frac{2}{3}=12\cdot\frac{7}{6}$$

- Simplify using the Distributive Property and the Properties of Fractions.

$$\frac{\overset{3}{\cancel{12}}}{1}\left(\frac{3x-1}{\underset{1}{\cancel{4}}}\right)+\frac{\overset{4}{\cancel{12}}}{1}\cdot\frac{2}{\underset{1}{\cancel{3}}}=\frac{\overset{2}{\cancel{12}}}{1}\cdot\frac{7}{\underset{1}{\cancel{6}}}$$

$$\begin{aligned}9x-3+8&=14\\9x+5&=14\\9x&=9\\x&=1\end{aligned}$$

- Solve for x.

1 checks as a solution. The solution is 1.

Occasionally, a value of the variable that appears to be a solution of an equation will make one of the denominators zero. In this case, that value is not a solution of the equation.

Solve: $\frac{2x}{x-2}=1+\frac{4}{x-2}$

$$\frac{2x}{x-2}=1+\frac{4}{x-2}$$

$$(x-2)\frac{2x}{x-2}=(x-2)\left(1+\frac{4}{x-2}\right)$$

- The LCM is $x-2$. Multiply each side of the equation by the LCM.

$$(x-2)\frac{2x}{x-2}=(x-2)\cdot 1+(x-2)\frac{4}{x-2}$$

- Simplify using the Distributive Property and the Properties of Fractions.

$$\frac{\overset{1}{\cancel{(x-2)}}}{1}\cdot\frac{2x}{\underset{1}{\cancel{x-2}}}=(x-2)\cdot 1+\frac{\overset{1}{\cancel{(x-2)}}}{1}\cdot\frac{4}{\underset{1}{\cancel{x-2}}}$$

$$\begin{aligned}2x&=x-2+4\\2x&=x+2\\x&=2\end{aligned}$$

- Solve for x.

When x is replaced by 2, the denominators of $\frac{2x}{x-2}$ and $\frac{4}{x-2}$ are zero. Therefore, the equation has no solution.

Example 1

Solve: $\frac{x}{x+4} = \frac{2}{x}$

Solution

The LCM is $x(x+4)$.

$$\frac{x}{x+4} = \frac{2}{x}$$

$$x(x+4)\left(\frac{x}{x+4}\right) = x(x+4)\left(\frac{2}{x}\right)$$

$$\frac{x\overset{1}{\cancel{(x+4)}}}{1}\cdot\frac{x}{\underset{1}{\cancel{x+4}}} = \frac{\overset{1}{\cancel{x}}(x+4)}{1}\cdot\frac{2}{\underset{1}{\cancel{x}}}$$

$$x^2 = (x+4)2$$

$$x^2 = 2x + 8$$

Solve the quadratic equation by factoring.

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x - 4 = 0 \qquad x + 2 = 0$$

$$x = 4 \qquad x = -2$$

Both 4 and -2 check as solutions.

The solutions are 4 and -2.

You Try It 1

Solve: $\frac{x}{x+6} = \frac{3}{x}$

Your solution

Example 2

Solve: $\frac{3x}{x-4} = 5 + \frac{12}{x-4}$

Solution

The LCM is $x - 4$.

$$\frac{3x}{x-4} = 5 + \frac{12}{x-4}$$

$$(x-4)\left(\frac{3x}{x-4}\right) = (x-4)\left(5 + \frac{12}{x-4}\right)$$

$$\frac{\overset{1}{\cancel{(x-4)}}}{1}\cdot\frac{3x}{\underset{1}{\cancel{x-4}}} = (x-4)5 + \frac{\overset{1}{\cancel{(x-4)}}}{1}\cdot\frac{12}{\underset{1}{\cancel{x-4}}}$$

$$3x = (x-4)5 + 12$$

$$3x = 5x - 20 + 12$$

$$3x = 5x - 8$$

$$-2x = -8$$

$$x = 4$$

4 does not check as a solution.

The equation has no solution.

You Try It 2

Solve: $\frac{5x}{x+2} = 3 - \frac{10}{x+2}$

Your solution

Solutions on p. S15

6.5 Exercises

[illegible]

1. Can 2 be a solution of the equation $\frac{6x}{x+1} - \frac{x}{x-2} = 4$? Explain your answer.

2. After multiplying each side of an equation by a variable expression, why must we check the solution?

Solve.

3. $\frac{2x}{3} - \frac{5}{2} = -\frac{1}{2}$

4. $\frac{x}{3} - \frac{1}{4} = \frac{1}{12}$

5. $\frac{x}{3} - \frac{1}{4} = \frac{x}{4} - \frac{1}{6}$

6. $\frac{2y}{9} - \frac{1}{6} = \frac{y}{9} + \frac{1}{6}$

7. $\frac{2x-5}{8} + \frac{1}{4} = \frac{x}{8} + \frac{3}{4}$

8. $\frac{3x+4}{12} - \frac{1}{3} = \frac{5x+2}{12} - \frac{1}{2}$

9. $\frac{6}{2a+1} = 2$

10. $\frac{12}{3x-2} = 3$

11. $\frac{9}{2x-5} = -2$

12. $\frac{6}{4-3x} = 3$

13. $2 + \frac{5}{x} = 7$

14. $3 + \frac{8}{n} = 5$

15. $1 - \frac{9}{x} = 4$

16. $3 - \frac{12}{x} = 7$

17. $\frac{2}{y} + 5 = 9$

18. $\frac{6}{x} + 3 = 11$

19. $\frac{3}{x-2} = \frac{4}{x}$

20. $\frac{5}{x+3} = \frac{3}{x-1}$

21. $\frac{2}{3x-1} = \frac{3}{4x+1}$

22. $\frac{5}{3x-4} = \frac{-3}{1-2x}$

23. $\frac{-3}{2x+5} = \frac{2}{x-1}$

24. $\dfrac{4}{5y-1}=\dfrac{2}{2y-1}$

25. $\dfrac{4x}{x-4}+5=\dfrac{5x}{x-4}$

26. $\dfrac{2x}{x+2}-5=\dfrac{7x}{x+2}$

27. $2+\dfrac{3}{a-3}=\dfrac{a}{a-3}$

28. $\dfrac{x}{x+4}=3-\dfrac{4}{x+4}$

29. $\dfrac{x}{x-1}=\dfrac{8}{x+2}$

30. $\dfrac{x}{x+12}=\dfrac{1}{x+5}$

31. $\dfrac{2x}{x+4}=\dfrac{3}{x-1}$

32. $\dfrac{5}{3n-8}=\dfrac{n}{n+2}$

33. $x+\dfrac{6}{x-2}=\dfrac{3x}{x-2}$

34. $x-\dfrac{6}{x-3}=\dfrac{2x}{x-3}$

35. $\dfrac{8}{y}=\dfrac{2}{y-2}+1$

APPLYING THE CONCEPTS

36. Explain the procedure for solving an equation containing fractions. Include in your discussion how the LCM of the denominators is used to eliminate fractions in the equation.

Solve.

37. $\dfrac{3}{5}y-\dfrac{1}{3}(1-y)=\dfrac{2y-5}{15}$

38. $\dfrac{3}{4}a=\dfrac{1}{2}(3-a)+\dfrac{a-2}{4}$

39. $\dfrac{b+2}{5}=\dfrac{1}{4}b-\dfrac{3}{10}(b-1)$

40. $\dfrac{x}{2x^2-x-1}=\dfrac{3}{x^2-1}+\dfrac{3}{2x+1}$

41. $\dfrac{x+1}{x^2+x-2}=\dfrac{x+2}{x^2-1}+\dfrac{3}{x+2}$

42. $\dfrac{y+2}{y^2-y-2}+\dfrac{y+1}{y^2-4}=\dfrac{1}{y+1}$

6.6 Ratio and Proportion

To solve a proportion

Quantities such as 4 meters, 15 seconds, and 8 gallons are number quantities written with units. In these examples the units are meters, seconds, and gallons.

A **ratio** is the quotient of two quantities that have the same unit.

The length of a living room is 16 ft and the width is 12 ft. The ratio of the length to the width is written

$\frac{16 \text{ ft}}{12 \text{ ft}} = \frac{16}{12} = \frac{4}{3}$ A ratio is in simplest form when the two numbers do not have a common factor. Note that the units are not written.

A **rate** is the quotient of two quantities that have different units.

There are 2 lb of salt in 8 gal of water. The salt-to-water rate is

$\frac{2 \text{ lb}}{8 \text{ gal}} = \frac{1 \text{ lb}}{4 \text{ gal}}$ A rate is in simplest form when the two numbers do not have a common factor. The units are written as part of the rate.

A **proportion** is an equation that states the equality of two ratios or rates. Examples of proportions are shown at the right.

$$\frac{30 \text{ mi}}{4 \text{ h}} = \frac{15 \text{ mi}}{2 \text{ h}}$$

$$\frac{4}{6} = \frac{8}{12}$$

$$\frac{3}{4} = \frac{x}{8}$$

⇨ Solve the proportion $\frac{4}{x} = \frac{2}{3}$.

$$\frac{4}{x} = \frac{2}{3}$$

$$3x\left(\frac{4}{x}\right) = 3x\left(\frac{2}{3}\right)$$

- The LCM of the denominators is $3x$. Multiply each side of the proportion by the LCM.

$$12 = 2x$$

- Solve the equation.

$$6 = x$$

The solution is 6.

Example 1

Solve the proportion $\frac{8}{x+3} = \frac{4}{x}$.

Solution

$$\begin{aligned} \frac{8}{x+3} &= \frac{4}{x} \\ x(x+3)\frac{8}{x+3} &= x(x+3)\frac{4}{x} \\ 8x &= 4(x+3) \\ 8x &= 4x + 12 \\ 4x &= 12 \\ x &= 3 \end{aligned}$$

The solution is 3.

You Try It 1

Solve the proportion $\frac{2}{x+3} = \frac{6}{5x+5}$.

Your solution

Solution on p. S15

To solve application problems

Example 2

The monthly loan payment for a car is \$28.35 for each \$1000 borrowed. At this rate, find the monthly payment for a \$6000 car loan.

Strategy

To find the monthly payment, write and solve a proportion, using P to represent the monthly car payment.

Solution

$$\frac{28.35}{1000} = \frac{P}{6000}$$

$$6000\left(\frac{28.35}{1000}\right) = 6000\left(\frac{P}{6000}\right)$$

$$170.10 = P$$

The monthly payment is \$170.10.

You Try It 2

Sixteen ceramic tiles are needed to tile a 9-square-foot area. At this rate, how many square feet can be tiled using 256 ceramic tiles?

Your strategy

Your solution

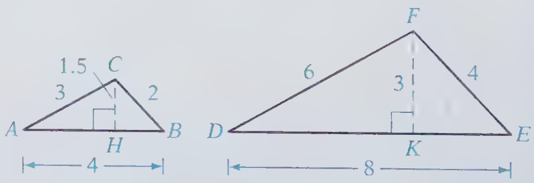
Solution on p. S16

To solve problems involving similar triangles

Similar objects have the same shape but not necessarily the same size. A tennis ball is similar to a basketball. A model ship is similar to an actual ship.

Similar objects have corresponding parts; for example, the rudder on the model ship corresponds to the rudder on the actual ship. The relationship between the sizes of each of the corresponding parts can be written as a ratio, and each ratio will be the same. If the rudder on the model ship is $\frac{1}{100}$ the size of the rudder on the actual ship, then the model wheelhouse is $\frac{1}{100}$ the size of the actual wheelhouse, the width of the model is $\frac{1}{100}$ the width of the actual ship, and so on.

The two triangles ABC and DEF shown at the right are similar. Side AB corresponds to DE, side BC corresponds to EF, and side AC corresponds to DF. The height CH corresponds to the height FK. The ratios of corresponding parts are equal.



$$\frac{AB}{DE} = \frac{4}{8} = \frac{1}{2}, \quad \frac{AC}{DF} = \frac{3}{6} = \frac{1}{2}, \quad \frac{BC}{EF} = \frac{2}{4} = \frac{1}{2}, \quad \text{and} \quad \frac{CH}{FK} = \frac{1.5}{3} = \frac{1}{2}$$

Because the ratios of corresponding parts are equal, three proportions can be formed using the sides of the triangles.

$$\frac{AB}{DE} = \frac{AC}{DF}, \quad \frac{AB}{DE} = \frac{BC}{EF}, \quad \text{and} \quad \frac{AC}{DF} = \frac{BC}{EF}$$

Three proportions can also be formed by using the sides and height of the triangles.

$$\frac{AB}{DE} = \frac{CH}{FK}, \qquad \frac{AC}{DF} = \frac{CH}{FK}, \qquad \text{and} \qquad \frac{BC}{EF} = \frac{CH}{FK}$$

The corresponding angles in similar triangles are equal. Therefore,

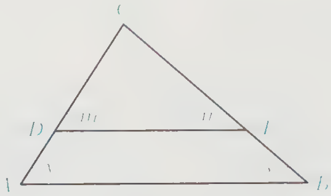
$$\angle A = \angle D, \qquad \angle B = \angle E, \qquad \text{and} \qquad \angle C = \angle F$$

It is also true that if the three angles of one triangle are equal, respectively, to the three angles of another triangle, then the two triangles are similar.

TAKE NOTE

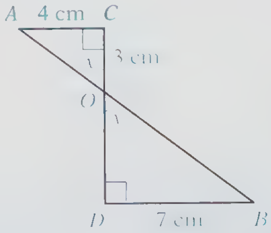
Vertical angles of intersecting lines, parallel lines, and angles of a triangle are discussed in the section on geometry problems.

A line DE is drawn parallel to the base AB in the triangle at the right. $\angle x = \angle m$ and $\angle y = \angle n$ because corresponding angles are equal. Angle C = angle C; thus the three angles of triangle DEC are equal, respectively, to the three angles of triangle ABC. Triangle DEC is similar to triangle ABC.



The sum of the three angles of a triangle is 180°. If two angles of one triangle are equal to two angles of another triangle, then the third angles must be equal. Thus we can say that if two angles of one triangle are equal to two angles of another triangle, then the two triangles are similar.

➡ The line segments AB and CD intersect at point O in the figure at the right. Angles C and D are right angles. Find the length of DO.



First we must determine whether triangle AOC is similar to triangle BOD.

Angle C = angle D because they are right angles.

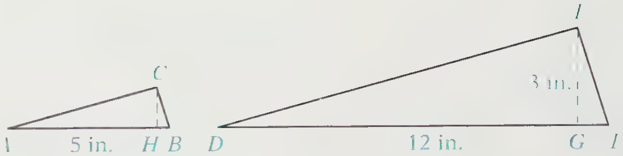
Angle x = angle y because they are vertical angles.

Triangle AOC is similar to triangle BOD because two angles of one triangle are equal to two angles of the other triangle.

$$\begin{aligned} \frac{AC}{DB} &= \frac{CO}{DO} \\ \frac{4}{7} &= \frac{3}{DO} \\ 7(DO)\frac{4}{7} &= 7(DO)\frac{3}{DO} \\ 4(DO) &= 7(3) \\ 4(DO) &= 21 \\ DO &= 5.25 \end{aligned}$$

• **Use a proportion to find the length of the unknown side.**

Triangles ABC and DEF at the right are similar. Find the area of triangle ABC.



$$\frac{AB}{DE} = \frac{CH}{FG}$$

- Solve a proportion to find the height of triangle *ABC*.

$$\frac{5}{12} = \frac{CH}{3}$$

$$12 \cdot \frac{5}{12} = 12 \cdot \frac{CH}{3}$$

$$5 = 4(CH)$$

$$1.25 = CH$$

- The height is 1.25 in. The base is 5 in.

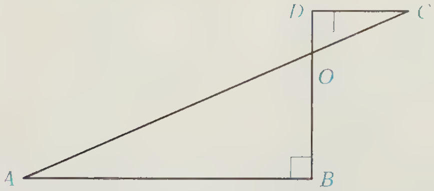
$$A = \frac{1}{2}bh = \frac{1}{2}(5)(1.25) = 3.125$$

- Use the formula for the area of a triangle.

The area of triangle ABC is 3.125 in^2.

Example 3

In the figure below, AB is parallel to DC, and angles B and D are right angles. $AB = 12$ m, $DC = 4$ m, and $AC = 18$ m. Find the length of CO.



Strategy

Triangle AOB is similar to triangle DOC. Solve a proportion to find the length of CO. Let x represent the length of CO and $18 - x$ represent the length of AO.

Solution

$$\frac{DC}{AB} = \frac{CO}{AO}$$

$$\frac{4}{12} = \frac{x}{18 - x}$$

$$12(18 - x) \cdot \frac{4}{12} = 12(18 - x) \cdot \frac{x}{18 - x}$$

$$4(18 - x) = 12x$$

$$72 - 4x = 12x$$

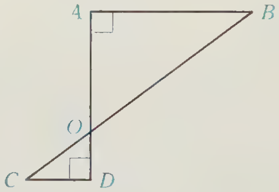
$$72 = 16x$$

$$4.5 = x$$

The length of CO is 4.5 m.

You Try It 3

In the figure below, AB is parallel to DC, and angles A and D are right angles. $AB = 10$ cm, $CD = 4$ cm, and $DO = 3$ cm. Find the area of triangle AOB.



Your strategy

Your solution

Solution on p. S16

6.6 Exercises

[illegible]

1. What is a proportion?

2. Explain a method for solving a proportion.

Solve.

3. $\frac{x}{12} = \frac{3}{4}$

4. $\frac{6}{x} = \frac{2}{3}$

5. $\frac{4}{9} = \frac{x}{27}$

6. $\frac{16}{9} = \frac{64}{x}$

7. $\frac{x + 3}{12} = \frac{5}{6}$

8. $\frac{3}{5} = \frac{x - 4}{10}$

9. $\frac{18}{x + 4} = \frac{9}{5}$

10. $\frac{2}{11} = \frac{20}{x - 3}$

11. $\frac{2}{x} = \frac{4}{x + 1}$

12. $\frac{16}{x - 2} = \frac{8}{x}$

13. $\frac{x + 3}{4} = \frac{x}{8}$

14. $\frac{x - 6}{3} = \frac{x}{5}$

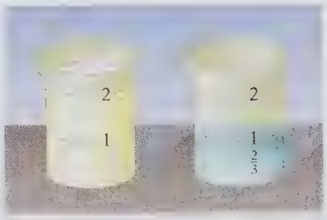
15. $\frac{2}{x - 1} = \frac{6}{2x + 1}$

16. $\frac{9}{x + 2} = \frac{3}{x - 2}$

17. $\frac{2x}{7} = \frac{x - 2}{14}$

Objective B *Application Problems*

18. Simple syrup used in making some desserts requires 2 cups (c) of sugar for every $\frac{2}{3}$ c of boiling water. At this rate, how many cups of sugar are required for 2 c boiling water?



19. An exit poll survey showed that 4 out of every 7 voters cast a ballot in favor of an amendment to a city charter. At this rate, how many voters voted in favor of the amendment if 35,000 people voted?

20. In a city of 25,000 homes, a survey was taken to determine the number with cable television. Of the 300 homes surveyed, 210 had cable television. Estimate the number of homes in the city that have cable television.

21. On a map, two cities are $2\frac{5}{8}$ in. apart. If $\frac{3}{8}$ in. on the map represents 25 mi, find the number of miles between the two cities.

22. A company decides to accept a large shipment of 10,000 computer chips if there are 2 or fewer defects in a sample of 100 randomly chosen chips. Assuming that there are 300 defective chips in the shipment and that the rate of defective chips in the sample is the same as the rate in the shipment, will the shipment be accepted?

23. The sales tax on a car that sold for \$12,000 is \$780. At this rate, how much higher is the sales tax on a car that sells for \$13,500?

24. Leonardo da Vinci measured various distances on the human body in order to make accurate drawings. He determined that generally the ratio of the kneeling height of a person to the standing height of that person was $\frac{3}{4}$. Using this ratio, determine how tall a person is who has a kneeling height of 48 in.

25. In one of Leonardo da Vinci's notebooks, he wrote that "... from the top to the bottom of the chin is the sixth part of a face, and it is the fifty-fourth part of the man." Suppose the distance from the top to the bottom of the chin of a person is 1.25 in. Using da Vinci's measurements, find the height of this person.

26. As part of a conservation effort for a lake, 40 fish are caught, tagged, and then released. Later 80 fish are caught. Four of the 80 fish are found to have tags. Estimate the number of fish in the lake.

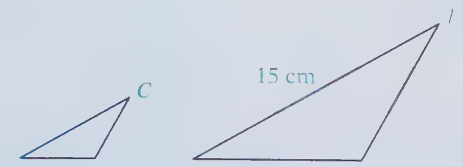
27. In a wildlife preserve, 10 elk are captured, tagged, and then released. Later 15 elk are captured and 2 are found to have tags. Estimate the number of elk in the preserve.

28. The engine of a small rocket burns 170,000 lb of fuel in 1 min. At this rate, how many pounds of fuel does the rocket burn in 45 s?

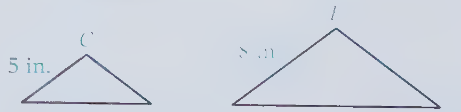
Objective 1

Triangles *ABC* and *DEF* in Exercises 29 to 36 are similar. Round answers to the nearest tenth.

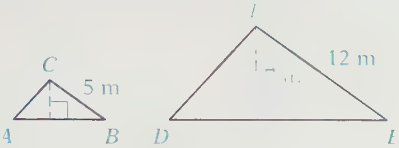
29. Find side *AC*.



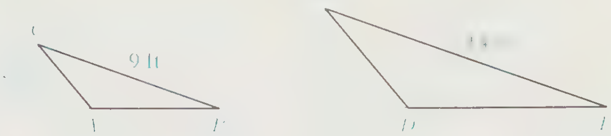
30. Find side *DE*.



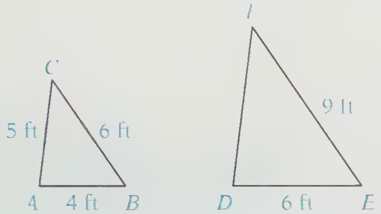
31. Find the height of triangle *ABC*.



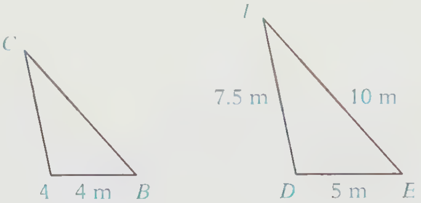
32. Find the height of triangle *DEF*.



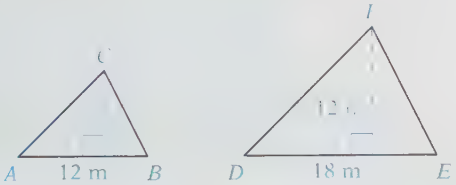
33. Find the perimeter of triangle *DEF*.



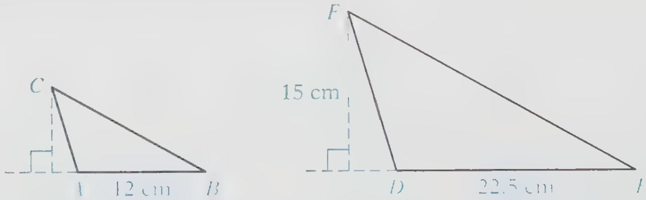
34. Find the perimeter of triangle *ABC*.



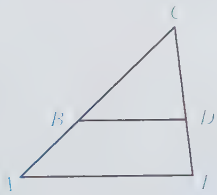
35. Find the area of triangle *ABC*.



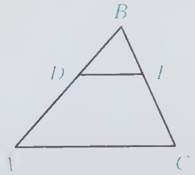
36. Find the area of triangle *ABC*.



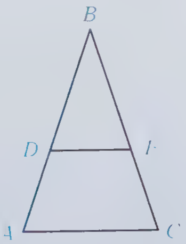
37. Given $BD \parallel AE$, *BD* measures 5 cm, *AE* measures 8 cm, and *AC* measures 10 cm, find the length of *BC*.



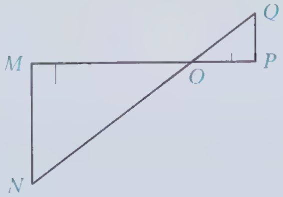
38. Given $AC \parallel DE$, *BD* measures 8 m, *AD* measures 12 m, and *BE* measures 6 m, find the length of *BC*.



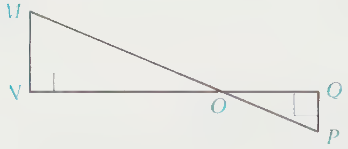
39. Given $DE \parallel AC$, *DE* measures 6 in., *AC* measures 10 in., and *AB* measures 15 in., find the length of *DA*.



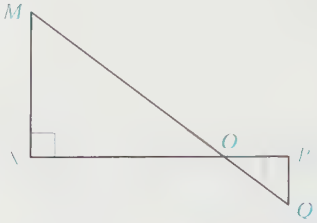
40. Given *MP* and *NQ* intersect at *O*, *NO* measures 25 ft, *MO* measures 20 ft, and *PO* measures 8 ft, find the length of *QO*.



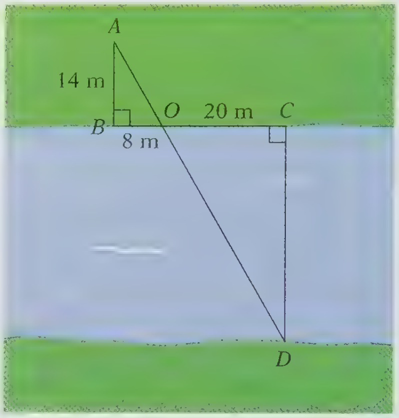
41. Given MP and NQ intersect at O, NO measures 24 cm, MN measures 10 cm, MP measures 39 cm, and QO measures 12 cm, find the length of OP.



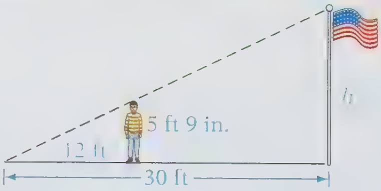
42. Given MQ and NP intersect at O, NO measures 12 m, MN measures 9 m, PQ measures 3 m, and MQ measures 20 m, find the perimeter of triangle OPQ.



43. Similar triangles can be used as an indirect way to measure inaccessible distances. The diagram at the right represents a river of width DC. The triangles AOB and DOC are similar. The distances AB, BO, and OC can be measured. Find the width of the river.



44. The sun's rays cast a shadow as shown in the diagram at the right. Find the height of the flagpole. Write the answer in terms of feet.



APPLYING THE CONCEPTS

45. Three people put their money together to buy lottery tickets. The first person put in \$25, the second person put in \$30, and the third person put in \$35. One of their tickets was a winning ticket. If they won \$4.5 million, what was the first person's share of the winnings?

46. No one belongs to both the Math Club and the Photography Club, but the two clubs join to hold a car wash. Ten members of the Math Club and 6 members of the Photography Club participate. The profits from the car wash are \$120. If each club's profits are proportional to the number of members participating, what share of the profits does the Math Club receive?

47. A basketball player has made 5 out of every 6 foul shots attempted in one year of play. If 42 foul shots were missed that year, how many shots did the basketball player make?

6.7 Literal Equations

OBJECTIVE A

To solve a literal equation for one of the variables

A **literal equation** is an equation that contains more than one variable. Examples of literal equations are shown at the right.

$$2x + 3y = 6$$
$$4w - 2x + z = 0$$

Formulas are used to express a relationship among physical quantities. A **formula** is a literal equation that states rules about measurements. Examples of formulas are shown at the right.

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R} \quad \text{(Physics)}$$
$$s = a + (n - 1)d \quad \text{(Mathematics)}$$
$$A = P + Prt \quad \text{(Business)}$$

The Addition and Multiplication Properties can be used to solve a literal equation for one of the variables. The goal is to rewrite the equation so that the variable being solved for is alone on one side of the equation and all the other numbers and variables are on the other side.

Solve $A = P(1 + i)$ for i.

The goal is to rewrite the equation so that i is on one side of the equation and all other variables are on the other side.

$$A = P(1 + i)$$
$$A = P + Pi$$
- Use the Distributive Property to remove parentheses.

$$A - P = P - P + Pi$$
- Subtract P from each side of the equation.

$$A - P = Pi$$
$$\frac{A - P}{P} = \frac{Pi}{P}$$
- Divide each side of the equation by P.

$$\frac{A - P}{P} = i$$

Example 1

Solve $3x - 4y = 12$ for y.

Solution

$$3x - 4y = 12$$
$$3x - 3x - 4y = -3x + 12$$
$$-4y = -3x + 12$$
$$\frac{-4y}{-4} = \frac{-3x + 12}{-4}$$
$$y = \frac{3}{4}x - 3$$

You Try It 1

Solve $5x - 2y = 10$ for y.

Your solution

Solution on p. S16

Example 2

Solve $I = \dfrac{E}{R + r}$ for R.

Solution

$$\begin{aligned} I &= \frac{E}{R + r} \\ (R + r)I &= (R + r)\frac{E}{R + r} \\ RI + rI &= E \\ RI + rI - rI &= E - rI \\ RI &= E - rI \\ \frac{RI}{I} &= \frac{E - rI}{I} \\ R &= \frac{E - rI}{I} \end{aligned}$$

You Try It 2

Solve $s = \dfrac{A + L}{2}$ for L.

Your solution

Example 3

Solve $L = a(1 + ct)$ for c.

Solution

$$\begin{aligned} L &= a(1 + ct) \\ L &= a + act \\ L - a &= a - a + act \\ L - a &= act \\ \frac{L - a}{at} &= \frac{act}{at} \\ \frac{L - a}{at} &= c \end{aligned}$$

You Try It 3

Solve $S = a + (n - 1)d$ for n.

Your solution

Example 4

Solve $S = C - rC$ for C.

Solution

$$\begin{aligned} S &= C - rC \\ S &= (1 - r)C \\ \frac{S}{1 - r} &= \frac{(1 - r)C}{1 - r} \\ \frac{S}{1 - r} &= C \end{aligned}$$

You Try It 4

Solve $S = rS + C$ for S.

Your solution

Solutions on p. S16

6.7 Exercises

Solve for y.

1. $3x + y = 10$

2. $2x + y = 5$

3. $4x - y = 3$

4. $5x - y = 7$

5. $3x + 2y = 6$

6. $2x + 3y = 9$

7. $2x - 5y = 10$

8. $5x - 2y = 4$

9. $2x + 7y = 14$

10. $6x - 5y = 10$

11. $x + 3y = 6$

12. $x + 2y = 8$

13. $y - 2 = 3(x + 2)$

14. $y + 4 = -2(x - 3)$

15. $y - 1 = -\frac{2}{3}(x + 6)$

Solve for x.

16. $x + 3y = 6$

17. $x + 6y = 10$

18. $3x - y = 3$

19. $2x - y = 6$

20. $2x + 5y = 10$

21. $4x + 3y = 12$

22. $x - 2y + 1 = 0$

23. $x - 4y - 3 = 0$

24. $5x + 4y + 20 = 0$

Solve the formula for the given variable.

25. $d = rt$; t (Physics)

26. $E = IR$; R (Physics)

27. $PV = nRT$; T (Chemistry)

28. $A = bh$; h (Geometry)

29. $P = 2l + 2w; l$ (Geometry)

30. $F = \frac{9}{5}C + 32; C$ (Temperature conversion)

31. $A = \frac{1}{2}h(b_1 + b_2); b_1$ (Geometry)

32. $C = \frac{5}{9}(F - 32); F$ (Temperature conversion)

33. $V = \frac{1}{3}Ah; h$ (Geometry)

34. $P = R - C; C$ (Business)

35. $R = \frac{C - S}{t}; S$ (Business)

36. $P = \frac{R - C}{n}; R$ (Business)

37. $A = P + Prt; P$ (Business)

38. $T = fm - gm; m$ (Engineering)

39. $A = Sw + w; w$ (Physics)

40. $a = S - Sr; S$ (Mathematics)

APPLYING THE CONCEPTS

Breakeven analysis is a method used to determine the sales volume required for a company to break even, or experience neither a profit nor a loss on the sale of a product. The breakeven point represents the number of units that must be made and sold for income from sales to equal the cost of the product. The breakeven point can be calculated using the formula $B = \frac{F}{S - V}$, where F is the fixed costs, S is the selling price per unit, and V is the variable costs per unit.

41. **a.** Solve the formula $B = \frac{F}{S - V}$ for S.

b. Use your answer to part **a** to find the selling price per unit required for a company to break even. The fixed costs are \$20,000, the variable costs per unit are \$80, and the company plans to make and sell 200 desks.

c. Use your answer to part **a** to find the selling price per unit required for a company to break even. The fixed costs are \$15,000, the variable costs per unit are \$50, and the company plans to make and sell 600 cameras.

6.8 Application Problems

[illegible]

To solve work problems

If a painter can paint a room in 4 h, then in 1 h the painter can paint $\frac{1}{4}$ of the room. The painter's rate of work is $\frac{1}{4}$ of the room each hour. The **rate of work** is the part of a task that is completed in 1 unit of time.

A pipe can fill a tank in 30 min. This pipe can fill $\frac{1}{30}$ of the tank in 1 min. The rate of work is $\frac{1}{30}$ of the tank each minute. If a second pipe can fill the tank in x min, the rate of work for the second pipe is $\frac{1}{x}$ of the tank each minute.

In solving a work problem, the goal is to determine the time it takes to complete a task. The basic equation that is used to solve work problems is

Rate of work × time worked = part of task completed

For example, if a faucet can fill a sink in 6 min, then in 5 min the faucet will fill $\frac{1}{6} \times 5 = \frac{5}{6}$ of the sink. In 5 min the faucet completes $\frac{5}{6}$ of the task.

A painter can paint a wall in 20 min. The painter's apprentice can paint the same wall in 30 min. How long will it take them to paint the wall when they work together?

Strategy for Solving a Work Problem

1. For each person or machine, write a numerical or variable expression for the rate of work, the time worked, and the part of the task completed. The results can be recorded in a table.

Unknown time to paint the wall working together: t

	Rate of Work	·	*Time Worked*	=	*Part of Task Completed*
Painter	$\frac{1}{20}$	·	t	=	$\frac{t}{20}$
Apprentice	$\frac{1}{30}$	·	t	=	$\frac{t}{30}$

TAKE NOTE
Use the information given in the problem to fill in the "Rate" and "Time" columns of the table. Fill in the "Part Completed" column by multiplying the two expressions you wrote in each row.

2. Determine how the parts of the task completed are related. Use the fact that the sum of the parts of the task completed must equal 1, the complete task.

The sum of the part of the task completed by the painter and the part of the task completed by the apprentice is 1.

$$\frac{t}{20} + \frac{t}{30} = 1$$

$$60\left(\frac{t}{20} + \frac{t}{30}\right) = 60 \cdot 1$$

- Multiply by the LCM of 20 and 30.

$$3t + 2t = 60$$

$$5t = 60$$

$$t = 12$$

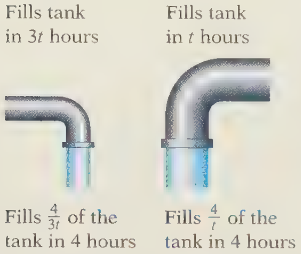
Working together, they will paint the wall in 12 min.

Example 1

A small water pipe takes three times longer to fill a tank than does a large water pipe. With both pipes open it takes 4 h to fill the tank. Find the time it would take the small pipe, working alone, to fill the tank.

Strategy

- Time for large pipe to fill the tank: t
  Time for small pipe to fill the tank: $3t$


Fills tank in $3t$ hours
Fills tank in t hours
Fills $\frac{4}{3t}$ of the tank in 4 hours
Fills $\frac{4}{t}$ of the tank in 4 hours

	Rate	*Time*	*Part*
Small pipe	$\frac{1}{3t}$	4	$\frac{4}{3t}$
Large pipe	$\frac{1}{t}$	4	$\frac{4}{t}$

- The sum of the parts of the task completed by each pipe must equal 1.

Solution

$$\frac{4}{3t} + \frac{4}{t} = 1$$

$$3t\left(\frac{4}{3t} + \frac{4}{t}\right) = 3t \cdot 1$$

$$4 + 12 = 3t$$

$$16 = 3t$$

$$\frac{16}{3} = t$$

$$3t = 3\left(\frac{16}{3}\right) = 16$$

The small pipe working alone takes 16 h to fill the tank.

You Try It 1

Two computer printers that work at the same rate are working together to print the payroll checks for a large corporation. After they work together for 2 h, one of the printers quits. The second requires 3 h more to complete the payroll checks. Find the time it would take one printer, working alone, to print the payroll.

Your strategy

Your solution

Solution on p. S16

To solve uniform motion problems

A car that travels constantly in a straight line at 30 mph is in uniform motion. **Uniform motion** means that the speed or direction of an object does not change.

The basic equation used to solve uniform motion problems is

Distance = rate × time

An alternative form of this equation can be written by solving the equation for time.

$$\frac{\textbf{Distance}}{\textbf{Rate}} = \textbf{time}$$

This form of the equation is useful when the total time of travel for two objects or the time of travel between two points is known.

The speed of a boat in still water is 20 mph. The boat traveled 75 mi down a river in the same amount of time it took to travel 45 mi up the river. Find the rate of the river's current.

Strategy for Solving a Uniform Motion Problem

1. For each object, write a numerical or variable expression for the distance, rate, and time. The results can be recorded in a table.

The unknown rate of the river's current: r

TAKE NOTE
Use the information given in the problem to fill in the "Distance" and "Rate" columns of the table. Fill in the "Time" column by dividing the two expressions you wrote in each row.

	Distance	÷	*Rate*	=	*Time*
Down river	75	÷	$20 + r$	=	$\frac{75}{20 + r}$
Up river	45	÷	$20 - r$	=	$\frac{45}{20 - r}$

2. Determine how the times traveled by each object are related. For example, it may be that the times are equal, or the total time may be known.

$$\frac{75}{20 + r} = \frac{45}{20 - r}$$

- The time down the river is equal to the time up the river.

$$\begin{aligned}(20 + r)(20 - r)\frac{75}{20 + r} &= (20 + r)(20 - r)\frac{45}{20 - r}\\ (20 - r)75 &= (20 + r)45\\ 1500 - 75r &= 900 + 45r\\ -120r &= -600\\ r &= 5\end{aligned}$$

- Multiply by the LCM.

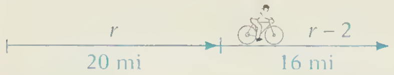
The rate of the river's current is 5 mph.

Example 2

A cyclist rode the first 20 mi of a trip at a constant rate. For the next 16 mi, the cyclist reduced the speed by 2 mph. The total time for the 36 mi was 4 h. Find the rate of the cyclist for each leg of the trip.

Strategy

- Rate for the first 20 mi: r
  Rate for the next 16 mi: $r - 2$



	Distance	*Rate*	*Time*
First 20 mi	20	r	$\frac{20}{r}$
Next 16 mi	16	$r - 2$	$\frac{16}{r-2}$

- The total time for the trip was 4 h.

Solution

$$\frac{20}{r} + \frac{16}{r-2} = 4$$

$$r(r-2)\left[\frac{20}{r} + \frac{16}{r-2}\right] = r(r-2) \cdot 4$$

$$(r-2)20 + 16r = 4r^2 - 8r$$

$$20r - 40 + 16r = 4r^2 - 8r$$

$$36r - 40 = 4r^2 - 8r$$

Solve the quadratic equation by factoring.

$$0 = 4r^2 - 44r + 40$$

$$0 = 4(r^2 - 11r + 10)$$

$$0 = 4(r - 10)(r - 1)$$

$$r - 10 = 0 \qquad r - 1 = 0$$

$$r = 10 \qquad r = 1$$

The solution $r = 1$ mph is not possible, because the rate on the last 16 mi would then be -1 mph.

10 mph was the rate for the first 20 mi.
8 mph was the rate for the next 16 mi.

You Try It 2

The total time it took for a sailboat to sail back and forth across a lake 6 km wide was 2 h. The rate sailing back was three times the rate sailing across. Find the rate sailing out across the lake.

Your strategy

Your solution

Solution on p. S17

6.8 Exercises

Application Problems

1. Explain the meaning of the phrase "rate of work."

2. If $\frac{2}{5}$ of a room can be painted in 1 h, what is the rate of work? At the same rate, how long will it take to paint the entire room?

3. A park has two sprinklers that are used to fill a fountain. One sprinkler can fill the fountain in 3 h, whereas the second sprinkler can fill the fountain in 6 h. How long will it take to fill the fountain with both sprinklers operating?

4. One grocery clerk can stock a shelf in 20 min, whereas a second clerk requires 30 min to stock the same shelf. How long would it take to stock the shelf if the two clerks worked together?

5. One person with a skiploader requires 12 h to remove a large quantity of earth. A second, larger skiploader can remove the same amount of earth in 4 h. How long would it take to remove the earth with both skiploaders working together?

6. An experienced painter can paint a fence twice as fast as an inexperienced painter. Working together, the painters require 4 h to paint the fence. How long would it take the experienced painter, working alone, to paint the fence?

7. One computer can solve a complex prime factorization problem in 75 h. A second computer can solve the same problem in 50 h. How long would it take both computers, working together, to solve the problem?

8. A new machine can make 10,000 aluminum cans three times faster than an older machine. With both machines working, 10,000 cans can be made in 9 h. How long would it take the new machine, working alone, to make the 10,000 cans?

9. A small air conditioner can cool a room 5° in 75 min. A larger air conditioner can cool the room 5° in 50 min. How long would it take to cool the room 5° with both air conditioners working?

10. One printing press can print the first edition of a book in 55 min, whereas a second printing press requires 66 min to print the same number of copies. How long would it take to print the first edition with both presses operating?

11. Two oil pipelines can fill a small tank in 30 min. Using one of the pipelines would require 45 min to fill the tank. How long would it take the second pipeline, working alone, to fill the tank?

12. Working together, two dock workers can load a crate in 6 min. One dock worker, working alone, can load the crate in 15 min. How long would it take the second dock worker, working alone, to load the crate?

13. A mason can construct a retaining wall in 10 h. With the mason's apprentice assisting, the task takes 6 h. How long would it take the apprentice, working alone, to construct the wall?

14. A mechanic requires 2 h to repair a transmission, whereas an apprentice requires 6 h to make the same repairs. The mechanic worked alone for 1 h and then stopped. How long will it take the apprentice, working alone, to complete the repairs?

15. One computer technician can wire a modem in 4 h, whereas it takes 6 h for a second technician to do the same job. After working alone for 2 h, the first technician quit. How long will it take the second technician to complete the wiring?

16. A wallpaper hanger requires 2 h to hang the wallpaper on one wall of a room. A second wallpaper hanger requires 4 h to hang the same amount of paper. The first wallpaper hanger worked alone for 1 h and then quit. How long will it take the second wallpaper hanger, working alone, to complete the wall?

17. Two welders who work at the same rate are welding the girders of a building. After they work together for 10 h, one of the welders quits. The second welder requires 20 more hours to complete the welds. Find the time it would have taken one of the welders, working alone, to complete the welds.

18. A large and a small heating unit are being used to heat the water of a pool. The larger unit, working alone, requires 8 h to heat the pool. After both units have been operating for 2 h, the larger unit is turned off. The small unit requires 9 h more to heat the pool. How long would it take the small unit, working alone, to heat the pool?

19. Two machines that fill cereal boxes work at the same rate. After they work together for 7 h, one machine breaks down. The second machine requires 14 h more to finish filling the boxes. How long would it have taken one of the machines, working alone, to fill the boxes?

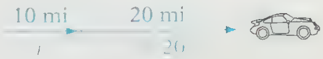
20. A large and a small drain are opened to drain a pool. The large drain can empty the pool in 6 h. After both drains have been open for 1 h, the large drain becomes clogged and is closed. The smaller drain remains open and requires 9 h more to empty the pool. How long would it have taken the small drain, working alone, to empty the pool?

Application Problems

21. Running at a constant speed, a jogger ran 24 mi in 3 h. How far did the jogger run in 2 h?

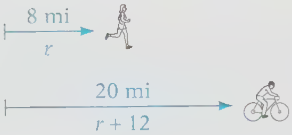
22. For uniform motion, distance = rate · time. How is time related to distance and rate? How is rate related to distance and time?

23. Commuting from work to home, a lab technician traveled 10 mi at a constant rate through congested traffic. On reaching the expressway, the technician increased the speed by 20 mph. An additional 20 mi was traveled at the increased speed. The total time for the trip was 1 h. Find the rate of travel through the congested traffic.



24. The president of a company traveled 1800 mi by jet and 300 mi on a prop plane. The rate of the jet was four times the rate of the prop plane. The entire trip took a total of 5 h. Find the rate of the jet plane.

25. As part of a conditioning program, a jogger ran 8 mi in the same amount of time a cyclist rode 20 mi. The rate of the cyclist was 12 mph faster than the rate of the jogger. Find the rate of the jogger and that of the cyclist.



26. An express train travels 600 mi in the same amount of time it takes a freight train to travel 360 mi. The rate of the express train is 20 mph faster than that of the freight train. Find the rate of each train.

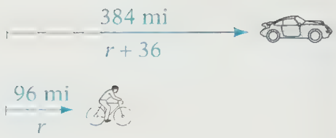
27. To assess the damage done by a fire, a forest ranger traveled 1080 mi by jet and then an additional 180 mi by helicopter. The rate of the jet was four times the rate of the helicopter. The entire trip took a total of 5 h. Find the rate of the jet.

28. A twin-engine plane can fly 800 mi in the same time that it takes a single-engine plane to fly 600 mi. The rate of the twin-engine plane is 50 mph faster than that of the single-engine plane. Find the rate of the twin-engine plane.

29. As part of an exercise plan, Camile Ellison walked for 40 min and then ran for 20 min. If Camile runs 3 mph faster than she walks and covered 5 mi during the 1-hour exercise period, what is her walking speed?

30. A car and a bus leave a town at 1 P.M. and head for a town 300 mi away. The rate of the car is twice the rate of the bus. The car arrives 5 h ahead of the bus. Find the rate of the car.

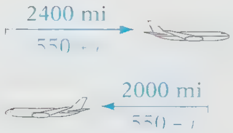
31. A car is traveling at a rate that is 36 mph faster than the rate of a cyclist. The car travels 384 mi in the same time it takes the cyclist to travel 96 mi. Find the rate of the car.



32. A backpacker hiking into a wilderness area walked 9 mi at a constant rate and then reduced this rate by 1 mph. Another 4 mi was hiked at this reduced rate. The time required to hike the 4 mi was 1 h less than the time required to walk the 9 mi. Find the rate at which the hiker walked the first 9 mi.

33. A plane can fly 180 mph in calm air. Flying with the wind, the plane can fly 600 mi in the same amount of time it takes to fly 480 mi against the wind. Find the rate of the wind.

34. A commercial jet can fly 550 mph in calm air. Traveling with the jet stream, the plane flew 2400 mi in the same amount of time it takes to fly 2000 mi against the jet stream. Find the rate of the jet stream.



35. A cruise ship can sail at 28 mph in calm water. Sailing with the gulf current, the ship can sail 170 mi in the same amount of time that it can sail 110 mi against the gulf current. Find the rate of the gulf current.

36. Rowing with the current of a river, a rowing team can row 25 mi in the same amount of time it takes to row 15 mi against the current. The rate of the rowing team in calm water is 20 mph. Find the rate of the current.

37. On a recent trip, a trucker traveled 330 mi at a constant rate. Because of road construction, the trucker then had to reduce the speed by 25 mph. An additional 30 mi was traveled at the reduced rate. The total time for the entire trip was 7 h. Find the rate of the trucker for the first 330 mi.

APPLYING THE CONCEPTS

38. One pipe can fill a tank in 2 h, a second pipe can fill the tank in 4 h, and a third pipe can fill the tank in 5 h. How long will it take to fill the tank with all three pipes working?

39. Because of bad weather, a bus driver reduced the usual speed along a 150-mile bus route by 10 mph. The bus arrived only 30 min later than its usual arrival time. How fast does the bus usually travel?

Focus on Problem Solving

Negations and If ... then Sentences

The sentence "George Washington was the first president of the United States" is a true sentence. The **negation** of that sentence is "George Washington was **not** the first president of the United States." That sentence is false. In general, the negation of a true sentence is a false statement.

The negation of a false sentence is a true sentence. For instance, the sentence "The moon is made of green cheese" is a false statement. The negation of that sentence, "The moon is **not** made of green cheese," is true.

The words *all, no* (or *none*), and *some* are called **quantifiers.** Writing the negation of a sentence that contains these words requires special attention. Consider the sentence "All pets are dogs." This sentence is not true because there are pets that are not dogs; cats, for example, are pets. Because the sentence is false, its negation must be true. You might be tempted to write "All pets are not dogs," but that sentence is not true because some pets are dogs. The correct negation of "All pets are dogs" is "Some pets are not dogs." Note the use of the word *some* in the negation.

Now consider the sentence "Some computers are portable." Because that sentence is true, its negation must be false. Writing "Some computers are not portable" as the negation is not correct, because that sentence is true. The negation of "Some computers are portable" is "No computers are portable."

The sentence "No flowers have red blooms" is false, because there is at least one flower (some roses, for example) that has red blooms. Because the sentence is false, its negation must be true. The negation is "Some flowers have red blooms."

Statement	*Negation*
All *A* are *B*.	Some *A* are not *B*.
No *A* are *B*.	Some *A* are *B*.
Some *A* are *B*.	No *A* are *B*.
Some *A* are not *B*.	All *A* are *B*.

Write the negation of the sentence.

1. All cats like milk.

2. All computers need people.

3. Some trees are tall.

4. No politicians are honest.

5. No houses have kitchens.

6. All police officers are tall.

7. All lakes are not polluted.

8. Some drivers are unsafe.

9. Some speeches are interesting.

10. All laws are good.

11. All businesses are not profitable.

12. All motorcycles are not large.

13. Some vegetables are good for you to eat.

14. Some banks are not open on Sunday.

A **premise** is a known or assumed fact. A premise can be stated using one of the quantifiers (*all, no, none,* or *some*) or using an *If* ... *then* sentence. For instance, the sentence "All triangles have three sides" can be written "*If* a figure is a triangle, *then* it has three sides."

We can write the sentence "No whole numbers are negative numbers" as an *If* ... *then* sentence: If a number is a whole number, then it is not a negative number.

Write the sentence as an *If* ... *then* sentence.

15. All students at Barlock College must take a life science course.

16. All baseballs are round.

17. All computers need people.

18. All cats like milk.

19. No odd number is evenly divisible by 2.

20. No prime number greater than 2 is an even number.

21. No rectangles have five sides.

22. All roads lead to Rome.

23. All dogs have fleas.

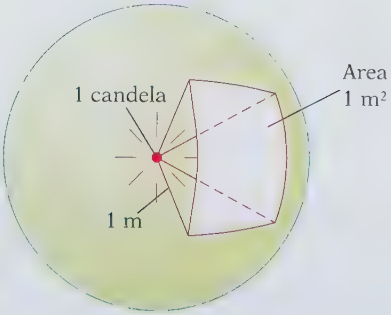
24. No triangle has four angles.

Projects and Group Activities

Intensity of Illumination

You are already aware that the standard unit of length in the metric system is the meter (m) and that the standard unit of mass in the metric system is the gram (g). You may not know that the standard unit of light intensity is the candela (cd).

The rate at which light falls on a 1-square-unit area of surface is called the **intensity of illumination.** Intensity of illumination is measured in **lumens** (lm). A lumen is defined in the following illustration.

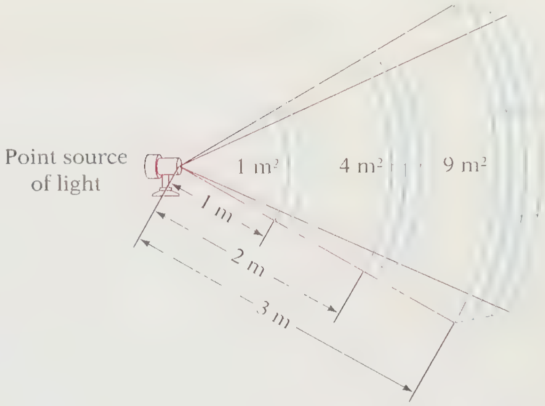


Picture a source of light equal to 1 cd positioned at the center of a hollow sphere that has a radius of 1 m. The rate at which light falls on $1\ m^2$ of the inner surface of the sphere is equal to 1 lm. If a light source equal to 4 cd is positioned at the center of the sphere, each square meter of the inner surface receives four times as much illumination, or 4 lm.

Light rays diverge as they leave a light source. The light that falls on an area of $1\ m^2$ at a distance of 1 m from the source of light spreads out over an area of $4\ m^2$ when it is 2 m from the source. The same light spreads out over an area of $9\ m^2$ when it is 3 m from the light source and over an area of $16\ m^2$ when it is 4 m from the light source. Therefore, as a surface moves farther away from the source of light, the intensity of illumination on the surface decreases from its value at 1 m to $\left(\frac{1}{2}\right)^2$, or $\frac{1}{4}$, that value at 2 m; to $\left(\frac{1}{3}\right)^2$, or $\frac{1}{9}$, that value at 3 m; and to $\left(\frac{1}{4}\right)^2$, or $\frac{1}{16}$, that value at 4 m.

The formula for the intensity of illumination is

$$I = \frac{s}{r^2}$$



where I is the intensity of illumination in lumens, s is the strength of the light source in candelas, and r is the distance in meters between the light source and the illuminated surface.

A 30-candela lamp is 0.5 m above a desk. Find the illumination on the desk.

$$I = \frac{s}{r^2}$$

$$I = \frac{30}{(0.5)^2} = 120$$

The illumination on the desk is 120 lm.

1. A 100-candela light is hanging 5 m above a floor. What is the intensity of the illumination on the floor beneath it?

2. A 25-candela source of light is 2 m above a desk. Find the intensity of illumination on the desk.

3. How strong a light source is needed to cast 20 lm of light on a surface 4 m from the source?

4. How strong a light source is needed to cast 80 lm of light on a surface 5 m from the source?

5. How far from the desk surface must a 40-candela light source be positioned if the desired intensity of illumination is 10 lm?

6. Find the distance between a 36-candela light source and a surface if the intensity of illumination on the surface is 0.01 lm.

7. Two lights cast the same intensity of illumination on a wall. One light is 6 m from the wall and has a rating of 36 candelas. The second light is 8 m from the wall. Find the candela rating of the second light.

8. A 40-candela light source and a 10-candela light source both throw the same intensity of illumination on a wall. The 10-candela light is 6 m from the wall. Find the distance from the 40-candela light to the wall.

Chapter Summary

Key Words

A *rational expression* is a fraction in which the numerator or denominator is a polynomial. [p. 257]

A rational expression is in *simplest form* when the numerator and denominator have no common factors. [p. 257]

The *reciprocal* of a fraction is a fraction with the numerator and denominator interchanged. [p. 260]

The *least common multiple* (LCM) of two or more numbers is the smallest number that contains the prime factorization of each number. [p. 265]

A *complex fraction* is a fraction whose numerator or denominator contains one or more fractions. [p. 277]

A *ratio* is the quotient of two quantities that have the same unit. [p. 285]

A *rate* is the quotient of two quantities that have different units. [p. 285]

A *proportion* is an equation that states the equality of two ratios or rates. [p. 285]

Similar triangles have the same shape but not necessarily the same size. [p. 286]

A *literal equation* is an equation that contains more than one variable. [p. 293]

A *formula* is a literal equation that states rules about measurement. [p. 293]

Essential Rules

To multiply rational numbers, multiply the numerators and multiply the denominators. [p. 254]

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

To divide two fractions, multiply the dividend by the reciprocal of the divisor. [p. 260]

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

To add fractions, write each fraction in terms of a common denominator; then add the numerators. [p. 269]

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

To solve an equation containing fractions, clear denominators by multiplying each side of the equation by the LCM of the denominators. [p. 281]

To determine whether two triangles are similar, show that two angles of one triangle are equal to two angles of the other triangle. [p. 286]

Equation for Work Problems

$$\begin{matrix}\text{Rate of}\\ \text{Work}\end{matrix} \times \begin{matrix}\text{time}\\ \text{worked}\end{matrix} = \begin{matrix}\text{part of task}\\ \text{completed}\end{matrix}$$ [p. 297]

Uniform Motion Equation

$$\frac{\text{Distance}}{\text{Rate}} = \text{time}$$ [p. 299]

Chapter Review

1. Simplify: $\dfrac{6a^2b^7}{25x^3y} \div \dfrac{12a^3b^4}{5x^2y^2}$

2. Simplify: $\dfrac{x+7}{15x} + \dfrac{x-2}{20x}$

3. Simplify: $\dfrac{x - \dfrac{16}{5x-2}}{3x - 4 - \dfrac{88}{5x-2}}$

4. Simplify: $\dfrac{x^2 + x - 30}{15 + 2x - x^2}$

5. Simplify: $\dfrac{16x^5y^3}{24xy^{10}}$

6. Solve: $\dfrac{20}{x+2} = \dfrac{5}{16}$

7. Simplify: $\dfrac{10 - 23y + 12y^2}{6y^2 - y - 5} \div \dfrac{4y^2 - 13y + 10}{18y^2 + 3y - 10}$

8. Simplify: $\dfrac{8ab^2}{15x^3y} \cdot \dfrac{5xy^4}{16a^2b}$

9. Simplify: $\dfrac{1 - \dfrac{1}{x}}{1 - \dfrac{8x-7}{x^2}}$

10. Write each fraction in terms of the LCM of the denominators.

$$\frac{x}{12x^2 + 16x - 3}, \frac{4x^2}{6x^2 + 7x - 3}$$

11. Solve $T = 2(ab + bc + ca)$ for a.

12. Solve: $\dfrac{5}{7} + \dfrac{x}{2} = 2 - \dfrac{x}{7}$

13. Solve $i = \dfrac{100m}{c}$ for c.

14. Solve: $\dfrac{x+8}{x+4} = 1 + \dfrac{5}{x+4}$

15. Simplify: $\dfrac{20x^2 - 45x}{6x^3 + 4x^2} \div \dfrac{40x^3 - 90x^2}{12x^2 + 8x}$

16. Simplify: $\dfrac{2y}{5y-7} + \dfrac{3}{7-5y}$

17. Simplify: $\dfrac{5x+3}{2x^2+5x-3} - \dfrac{3x+4}{2x^2+5x-3}$

18. Find the LCM of $10x^2 - 11x + 3$ and $20x^2 - 17x + 3$.

19. Solve $4x + 9y = 18$ for y.

20. Simplify: $\dfrac{2x^2-5x-3}{3x^2-7x-6} \cdot \dfrac{3x^2+8x+4}{x^2+4x+4}$

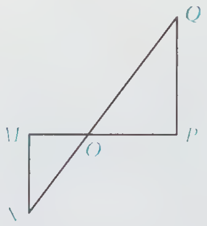
21. Solve: $\dfrac{20}{2x+3} = \dfrac{17x}{2x+3} - 5$

22. Simplify: $\dfrac{x-1}{x+2} + \dfrac{3x-2}{5-x} + \dfrac{5x^2+15x-11}{x^2-3x-10}$

23. Solve: $\dfrac{6}{x-7} = \dfrac{8}{x-6}$

24. Solve: $\dfrac{3}{20} = \dfrac{x}{80}$

25. Given that MP and NQ intersect at O, NQ measures 25 cm, MO measures 6 cm, and PO measures 9 cm, find the length of QO.



26. One hose can fill a pool in 15 h. The second hose can fill the pool in 10 h. How long would it take to fill the pool using both hoses?

27. A car travels 315 mi in the same amount of time in which a bus travels 245 mi. The rate of the car is 10 mph faster than that of the bus. Find the rate of the car.

28. The rate of a jet is 400 mph in calm air. Traveling with the wind, the jet can fly 2100 mi in the same amount of time it takes to fly 1900 mi against the wind. Find the rate of the wind.

29. A pitcher's earned run average (ERA) is the average number of runs allowed in 9 innings of pitching. If a pitcher allows 15 runs in 100 innings, find the pitcher's ERA.

Chapter Test

1. Simplify: $\frac{x}{x+3} - \frac{2x-5}{x^2+x-6}$

2. Solve the proportion: $\frac{3}{x+4} = \frac{5}{x+6}$

3. Simplify: $\frac{x^2+2x-3}{x^2+6x+9} \cdot \frac{2x^2-11x+5}{2x^2+3x-5}$

4. Simplify: $\frac{16x^5y}{24x^2y^4}$

5. Solve $d = s + rt$ for t.

6. Solve: $\frac{6}{x} - 2 = 1$

7. Simplify: $\frac{x^2+4x-5}{1-x^2}$

8. Find the LCM of $6x - 3$ and $2x^2 + x - 1$.

9. Simplify: $\frac{2}{2x-1} - \frac{3}{3x+1}$

10. Simplify: $\frac{x^2+3x+2}{x^2+5x+4} \div \frac{x^2-x-6}{x^2+2x-15}$

11. Simplify: $\dfrac{1 + \frac{1}{x} - \frac{12}{x^2}}{1 + \frac{2}{x} - \frac{8}{x^2}}$

12. Write each fraction in terms of the LCM of the denominators.
$\frac{3}{x^2-2x}, \frac{x}{x^2-4}$

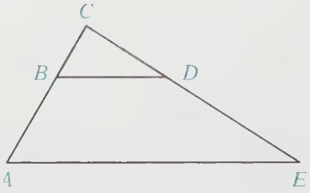
13. Simplify: $\frac{2x}{x^2 + 3x - 10} - \frac{4}{x^2 + 3x - 10}$

14. Solve $3x - 8y = 16$ for y.

15. Solve: $\frac{2x}{x + 1} - 3 = \frac{-2}{x + 1}$

16. Simplify: $\frac{x^3y^4}{x^2 - 4x + 4} \cdot \frac{x^2 - x - 2}{x^6y^4}$

17. Given $AE \parallel BD$, AB measures 5 ft, ED measures 8 ft, and BC measures 3 ft, find the length of CE.



18. A saltwater solution is formed by mixing 4 lb of salt with 10 gal of water. At this rate, how many additional pounds of salt are required for 15 gal of water?

19. A pool can be filled with one pipe in 6 h, whereas a second pipe requires 12 h to fill the pool. How long would it take to fill the pool with both pipes turned on?

20. A small plane can fly at 110 mph in calm air. Flying with the wind, the plane can fly 260 mi in the same amount of time it takes to fly 180 mi against the wind. Find the rate of the wind.

21. A landscape architect uses three sprinklers for each 200 ft^2 of lawn. At this rate, how many sprinklers are needed for a 3600-square-foot lawn?

Cumulative Review

1. Simplify: $\left(\frac{2}{3}\right)^2 \div \left(\frac{3}{2} - \frac{2}{3}\right) + \frac{1}{2}$

2. Evaluate $-a^2 + (a - b)^2$ when $a = -2$ and $b = 3$.

3. Simplify: $-2x - (-3y) + 7x - 5y$

4. Simplify: $2[3x - 7(x - 3) - 8]$

5. Solve: $4 - \frac{2}{3}x = 7$

6. Solve: $3[x - 2(x - 3)] = 2(3 - 2x)$

7. Find $16\frac{2}{3}\%$ of 60.

8. Simplify: $(a^2b^5)(ab^2)$

9. Simplify: $(a - 3b)(a + 4b)$

10. Simplify: $\frac{15b^4 - 5b^2 + 10b}{5b}$

11. Simplify: $(x^3 - 8) \div (x - 2)$

12. Factor: $12x^2 - x - 1$

13. Factor: $y^2 - 7y + 6$

14. Factor: $2a^3 + 7a^2 - 15a$

15. Factor: $4b^2 - 100$

16. Solve: $(x + 3)(2x - 5) = 0$

17. Simplify: $\frac{12x^4y^2}{18xy^7}$

18. Simplify: $\frac{x^2 - 7x + 10}{25 - x^2}$

19. Simplify: $\dfrac{x^2 - x - 56}{x^2 + 8x + 7} \div \dfrac{x^2 - 13x + 40}{x^2 - 4x - 5}$

20. Simplify: $\dfrac{2}{2x - 1} - \dfrac{1}{x + 1}$

21. Simplify: $\dfrac{1 - \dfrac{2}{x} - \dfrac{15}{x^2}}{1 - \dfrac{25}{x^2}}$

22. Solve: $\dfrac{3x}{x - 3} - 2 = \dfrac{10}{x - 3}$

23. Solve the proportion: $\dfrac{2}{x - 2} = \dfrac{12}{x + 3}$

24. Solve $f = v + at$ for t.

25. Translate "the difference between five times a number and thirteen is the opposite of eight" into an equation and solve.

26. A silversmith mixes 60 g of an alloy that is 40% silver with 120 g of another silver alloy. The resulting alloy is 60% silver. Find the percent of silver in the 120-gram alloy.

27. The length of the base of a triangle is 2 in. less than twice the height. The area of the triangle is 30 in^2. Find the base and height of the triangle.

28. A life insurance policy costs $16 for every $1000 of coverage. At this rate, how much money would a policy of $5000 cost?

29. One water pipe can fill a tank in 9 min, whereas a second pipe requires 18 min to fill the tank. How long would it take both pipes, working together, to fill the tank?

30. The rower of a boat can row at a rate of 5 mph in calm water. Rowing with the current, the boat travels 14 mi in the same amount of time it takes to travel 6 mi against the current. Find the rate of the current.

Chapter 7

Linear Equations in Two Variables

Real data does not provide just the past and present conditions of a situation; it can also give a possible glimpse into the future. Tables and graphs are used to analyze data over many years and reveal certain trends. If graphed data points fall in a straight line, then a linear model, like the one shown in **Exercise 15 on page 364**, can be determined and used to estimate possible future data points. For example, annual college tuition may approximate a linear model; this information can be used to plan for future college tuition and to save accordingly.

Objectives

Section 7.1

- A To graph points in a rectangular coordinate system
- B To determine ordered-pair solutions of an equation in two variables
- C To determine whether a set of ordered pairs is a function
- D To evaluate a function written in functional notation

Section 7.2

- A To graph an equation of the form $y = mx + b$
- B To graph an equation of the form $Ax + By = C$
- C To solve application problems

Section 7.3

- A To find the x- and y-intercepts of a straight line
- B To find the slope of a straight line
- C To graph a line using the slope and the y-intercept

Section 7.4

- A To find the equation of a line given a point and the slope
- B To find the equation of a line given two points
- C To solve application problems

Need help? For online student resources, such as section quizzes, visit this textbook's website at **math.college.hmco.com/students.**

Prep Test

1. Simplify: $-\dfrac{5-(-7)}{4-8}$

2. Evaluate $\dfrac{a-b}{c-d}$ when $a = 3$, $b = -2$, $c = -3$, and $d = 2$.

3. Simplify: $-3(x-4)$

4. Solve: $3x + 6 = 0$

5. Solve $4x + 5y = 20$ when $y = 0$.

6. Solve $3x - 7y = 11$ when $x = -1$.

7. Divide: $\dfrac{12x - 15}{-3}$

8. Solve: $\dfrac{2x+1}{3} = \dfrac{3x}{4}$

9. Solve $3x - 5y = 15$ for y.

10. Solve $y + 3 = -\dfrac{1}{2}(x+4)$ for y.

Go Figure

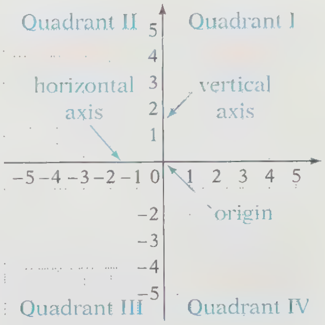
Points A, B, C, and D lie on the same line and in that order. The ratio of AB to AC is $\dfrac{1}{4}$ and the ratio of BC to CD is $\dfrac{1}{2}$. Find the ratio of AB to CD.

7.1 The Rectangular Coordinate System

To graph points in a rectangular coordinate system

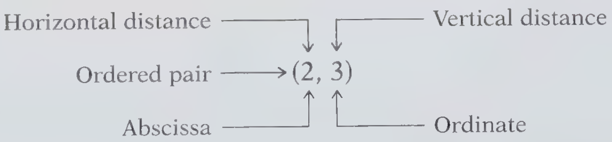
Before the 15th century, geometry and algebra were considered separate branches of mathematics. That all changed when René Descartes, a French mathematician who lived from 1596 to 1650, founded **analytic geometry.** In this geometry, a *coordinate system* is used to study relationships between variables.

A **rectangular coordinate system** is formed by two number lines, one horizontal and one vertical, that intersect at the zero point of each line. The point of intersection is called the **origin.** The two lines are called **coordinate axes,** or simply **axes.**

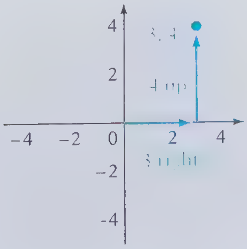


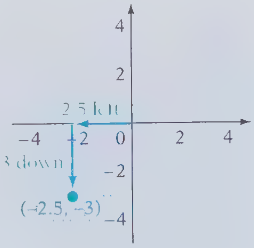
The axes determine a **plane,** which can be thought of as a large, flat sheet of paper. The two axes divide the plane into four regions called **quadrants,** which are numbered counterclockwise from I to IV.

Each point in the plane can be identified by a pair of numbers called an **ordered pair.** The first number of the pair measures a horizontal distance and is called the **abscissa.** The second number of the pair measures a vertical distance and is called the **ordinate.** The **coordinates** of the point are the numbers in the ordered pair associated with the point. The abscissa is also called the **first coordinate** of the ordered pair, and the ordinate is also called the **second coordinate** of the ordered pair.



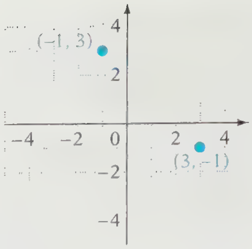
To **graph** or **plot** a point in the plane, place a dot at the location given by the ordered pair. The **graph of an ordered pair** is the dot drawn at the coordinates of the point in the plane. The points whose coordinates are (3, 4) and (−2.5, −3) are graphed in the figures below.



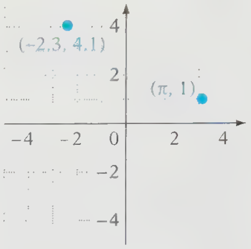


TAKE NOTE
This is very important. An **ordered pair** is a *pair* of coordinates, and the *order* in which the coordinates appear is crucial.

The points whose coordinates are (3, −1) and (−1, 3) are graphed at the right. Note that the graphed points are in different locations. *The order of the coordinates of an ordered pair is important.*

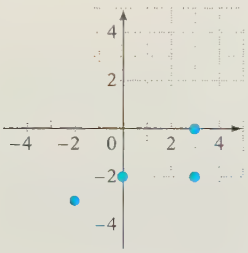


Each point in the plane is associated with an ordered pair, and each ordered pair is associated with a point in the plane. Although only the labels for integers are given on a coordinate grid, the graph of any ordered pair can be approximated. For example, the points whose coordinates are (−2.3, 4.1) and (π, 1) are shown on the graph at the right.



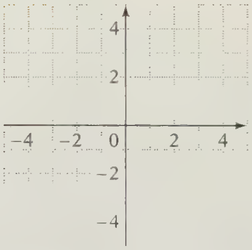
Example 1 Graph the ordered pairs (−2, −3), (3, −2), (0, −2), and (3, 0).

Solution

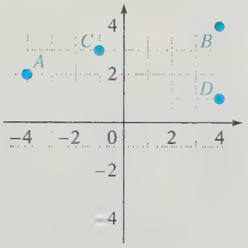


You Try It 1 Graph the ordered pairs (−4, 1), (3, −3), (0, 4), and (−3, 0).

Your solution

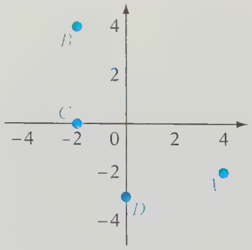


Example 2 Give the coordinates of the points labeled *A* and *B*. Give the abscissa of point *C* and the ordinate of point *D*.



Solution The coordinates of *A* are (−4, 2).
The coordinates of *B* are (4, 4).
The abscissa of *C* is −1.
The ordinate of *D* is 1.

You Try It 2 Give the coordinates of the points labeled *A* and *B*. Give the abscissa of point *D* and the ordinate of point *C*.



Your solution

Solutions on p. S17

Objective B

To determine ordered-pair solutions of an equation in two variables

When drawing a rectangular coordinate system, we often label the horizontal axis x and the vertical axis y. In this case, the coordinate system is called an ***xy*-coordinate system.** The coordinates of the points are given by ordered pairs (x, y), where the abscissa is called the ***x*-coordinate** and the ordinate is called the ***y*-coordinate.**

A coordinate system is used to study the relationship between two variables. Frequently this relationship is given by an equation. Examples of equations in two variables include

$$y = 2x - 3 \qquad 3x + 2y = 6 \qquad x^2 - y = 0$$

A **solution of an equation in two variables** is an ordered pair (x, y) whose coordinates make the equation a true statement.

⇨ Is $(-3, 7)$ a solution of $y = -2x + 1$?

$$\begin{array}{c|l} \multicolumn{2}{c}{y = -2x + 1} \\ \hline 7 & -2(-3) + 1 \\ & 6 + 1 \end{array}$$

• Replace x by -3; replace y by 7.

$7 = 7$ • The results are equal.

$(-3, 7)$ is a solution of the equation $y = -2x + 1$.

Besides $(-3, 7)$, there are many other ordered-pair solutions of $y = -2x + 1$. For example, $(0, 1)$, $\left(-\frac{3}{2}, 4\right)$, and $(4, -7)$ are also solutions. In general, an equation in two variables has an infinite number of solutions. By choosing any value of x and substituting that value into the equation, we can calculate a corresponding value of y.

⇨ Find the ordered-pair solution of $y = \frac{2}{3}x - 3$ that corresponds to $x = 6$.

$$y = \frac{2}{3}x - 3$$

$$= \frac{2}{3}(6) - 3$$ • Replace x by 6.

$$= 4 - 3 = 1$$ • Simplify.

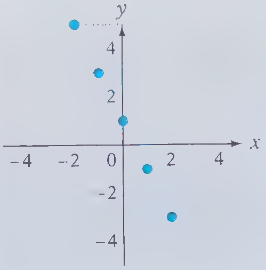
The ordered-pair solution is $(6, 1)$.

The solutions of an equation in two variables can be graphed in an xy-coordinate system.

⇨ Graph the ordered-pair solutions of $y = -2x + 1$ when $x = -2, -1, 0, 1$, and 2.

Use the values of x to determine ordered-pair solutions of the equation. It is convenient to record these in a table.

x	$y = -2x + 1$	y	(x, y)
-2	$-2(-2) + 1$	5	$(-2, 5)$
-1	$-2(-1) + 1$	3	$(-1, 3)$
0	$-2(0) + 1$	1	$(0, 1)$
1	$-2(1) + 1$	-1	$(1, -1)$
2	$-2(2) + 1$	-3	$(2, -3)$



Example 3

Is $(3, -2)$ a solution of $3x - 4y = 15$?

Solution

$$\begin{array}{r|l} \multicolumn{2}{c}{3x - 4y = 15} \\ \hline 3(3) - 4(-2) & 15 \\ 9 + 8 & \\ \multicolumn{2}{c}{17 \neq 15} \end{array}$$

• Replace x by 3 and y by -2.

No. $(3, -2)$ is not a solution of $3x - 4y = 15$.

You Try It 3

Is $(-2, 4)$ a solution of $x - 3y = -14$?

Your solution

Example 4

Graph the ordered-pair solutions of $2x - 3y = 6$ when $x = -3$, 0, 3, and 6.

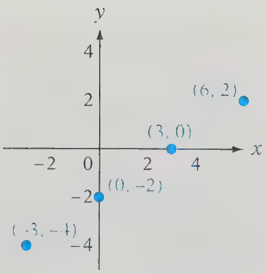
Solution

$$\begin{aligned} 2x - 3y &= 6 \\ -3y &= -2x + 6 \\ y &= \frac{2}{3}x - 2 \end{aligned}$$

• Solve $2x - 3y = 6$ for y.

Replace x in $y = \frac{2}{3}x - 2$ by -3, 0, 3, and 6. For each value of x, determine the value of y.

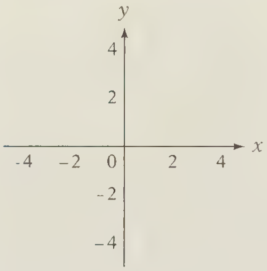
x	$y = \frac{2}{3}x - 2$	y	(x, y)
-3	$\frac{2}{3}(-3) - 2$	-4	$(-3, -4)$
0	$\frac{2}{3}(0) - 2$	-2	$(0, -2)$
3	$\frac{2}{3}(3) - 2$	0	$(3, 0)$
6	$\frac{2}{3}(6) - 2$	2	$(6, 2)$



You Try It 4

Graph the ordered-pair solutions of $x + 2y = 4$ when $x = -4$, -2, 0, and 2.

Your solution



Solutions on p. S17

[illegible]

To determine whether a set of ordered pairs is a function

Discovering a relationship between two variables is an important task in the application of mathematics. Here are some examples.

- Botanists study the relationship between the number of bushels of wheat yielded per acre and the amount of watering per acre.
- Environmental scientists study the relationship between the incidents of skin cancer and the amount of ozone in the atmosphere.
- Business analysts study the relationship between the price of a product and the number of products that are sold at that price.

Each of these relationships can be described by a set of ordered pairs.

Definition of a Relation

A **relation** is any set of ordered pairs.

The following table shows the number of hours that each of 9 students spent studying for a midterm exam and the grade that each of these 9 students received.

Hours	3	3.5	2.75	2	4	4.5	3	2.5	5
Grade	78	75	70	65	85	85	80	75	90

This information can be written as the relation

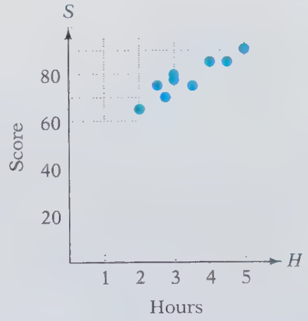
$$\{(3, 78), (3.5, 75), (2.75, 70), (2, 65), (4, 85), (4.5, 85), (3, 80), (2.5, 75), (5, 90)\}$$

where the first coordinate of the ordered pair is the hours spent studying and the second coordinate is the score on the midterm.

The **domain** of a relation is the set of first coordinates of the ordered pairs; the **range** is the set of second coordinates. For the relation above,

$$\text{Domain} = \{2, 2.5, 2.75, 3, 3.5, 4, 4.5, 5\} \qquad \text{Range} = \{65, 70, 75, 78, 80, 85, 90\}$$

The **graph of a relation** is the graph of the ordered pairs that belong to the relation. The graph of the relation given above is shown at the right. The horizontal axis represents the hours spent studying (the domain); the vertical axis represents the test score (the range). The axes could be labeled H for hours studied and S for test score.



A *function* is a special type of relation in which no two ordered pairs have the same first coordinate.

Definition of a Function

A **function** is a relation in which no two ordered pairs have the same first coordinate.

The table at the right is the grading scale for a 100-point test. This table defines a relationship between the *score* on the test and a *letter grade*. Some of the ordered pairs of this function are (78, C), (97, A), (84, B), and (82, B).

Score	*Grade*
90–100	A
80–89	B
70–79	C
60–69	D
0–59	F

The grading-scale table defines a function, because no two ordered pairs can have the *same* first coordinate and *different* second coordinates. For instance, it is not possible to have the ordered pairs (72, C), and (72, B)—same first coordinate (test score) but different second coordinates (test grade). The domain of this function is $\{0, 1, 2, \ldots, 99, 100\}$. The range is $\{A, B, C, D, F\}$.

The example of hours spent studying and test score given earlier is *not* a function, because (3, 78) and (3, 80) are ordered pairs of the relation that have the *same* first coordinate but *different* second coordinates.

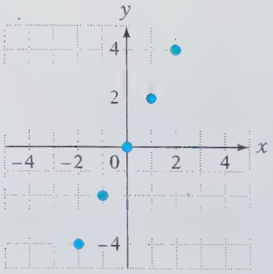
Consider, again, the grading-scale example. Note that (84, B) and (82, B) are ordered pairs of the function. Ordered pairs of a function may have the same *second* coordinates but not the same first coordinates.

Although relations and functions can be given by tables, they are frequently given by an equation in two variables.

The equation $y = 2x$ expresses the relationship between a number, x, and twice the number, y. For instance, if $x = 3$, then $y = 6$, which is twice 3. To indicate exactly which ordered pairs are determined by the equation, the domain (values of x) is specified. If $x \in \{-2, -1, 0, 1, 2\}$, then the ordered pairs determined by the equation are $\{(-2, -4), (-1, -2), (0, 0), (1, 2), (2, 4)\}$. This relation is a function because no two ordered pairs have the same first coordinate.

The graph of the function $y = 2x$ with domain $\{-2, -1, 0, 1, 2\}$ is shown at the right. The horizontal axis (domain) is labeled x; the vertical axis (range) is labeled y.

The domain $\{-2, -1, 0, 1, 2\}$ was chosen arbitrarily. Other domains could have been selected. The type of application usually influences the choice of the domain.



For the equation $y = 2x$, we say that "y is a function of x" because the set of ordered pairs is a function.

Not all equations, however, define a function. For instance, the equation $|y| = x + 2$ does not define y as a function of x. The ordered pairs (2, 4) and (2, −4) both satisfy the equation. Thus there are two ordered pairs with the same first coordinate but different second coordinates.

Example 5

The numbers of gold medals earned by several countries during the 1996 and 2000 Olympic Summer Games are recorded in the following table.

Country	*1996*	*2000*
United States	44	40
Canada	3	3
China	16	28
Italy	13	13
France	15	13
Germany	20	17

Write a relation wherein the first coordinate is the number of gold medals earned in 1996 and the second coordinate is the number earned in 2000. Is the relation a function?

Solution

$\{(44, 40), (3, 3), (16, 28), (13, 13), (15, 13), (20, 17)\}$

Yes. The relation is a function. No two ordered pairs have the same first coordinate.

You Try It 5

Six students decided to go on a diet and fitness program over the summer. Their weights (in pounds) at the beginning and end of the program are given in the table below.

Beginning	*End*
145	140
140	125
150	130
165	150
140	130
165	160

Write a relation wherein the first coordinate is the weight at the beginning of the summer and the second coordinate is the weight at the end of the summer. Is the relation a function?

Your solution

Example 6

Does $y = x^2 + 3$, where $x \in \{-2, -1, 1, 3\}$, define y as a function of x?

Solution

Determine the ordered pairs defined by the equation. Replace x in $y = x^2 + 3$ by the given values and solve for y.

$\{(-2, 7), (-1, 4), (1, 4), (3, 12)\}$

No two ordered pairs have the same first coordinate. Therefore, the relation is a function and the equation $y = x^2 + 3$ defines y as a function of x.

Note that $(-1, 4)$ and $(1, 4)$ are ordered pairs that belong to this function. Ordered pairs of a function may have the same *second* coordinate but not the same *first* coordinate.

You Try It 6

Does $y = \frac{1}{2}x + 1$, where $x \in \{-4, 0, 2\}$, define y as a function of x?

Your solution

Solutions on p. S17

To evaluate a function written in functional notation 7

When an equation defines y as a function of x, **functional notation** is frequently used to emphasize that the relation is a function. In this case, it is common to use the notation $f(x)$, where

$f(x)$ is read "f of x" or "the value of f at x."

For instance, the equation $y = x^2 + 3$ from Example 6 defined y as a function of x. The equation can also be written in functional notation as

$$f(x) = x^2 + 3$$

where y has been replaced by $f(x)$.

The symbol $f(x)$ is called the **value of the function** at x because it is the result of evaluating a variable expression. For instance, $f(4)$ means to replace x by 4 and then simplify the resulting numerical expression.

$$\begin{aligned} f(x) &= x^2 + 3 \\ f(4) &= 4^2 + 3 && \bullet \text{ Replace } x \text{ by } 4. \\ &= 16 + 3 = 19 \end{aligned}$$

This process is called **evaluating the function.**

Given $f(x) = x^2 + x - 3$, find $f(-2)$.

$$\begin{aligned} f(x) &= x^2 + x - 3 \\ f(-2) &= (-2)^2 + (-2) - 3 && \bullet \text{ Replace } x \text{ by } -2. \\ &= 4 - 2 - 3 = -1 \end{aligned}$$

$f(-2) = -1$

In this example, $f(-2)$ is the second coordinate of an ordered pair of the function; the first coordinate is -2. Therefore, an ordered pair of this function is $(-2, f(-2))$, or, because $f(-2) = -1$, $(-2, -1)$.

For the function given by $y = f(x) = x^2 + x - 3$, y is called the **dependent variable** because its value depends on the value of x. The **independent variable** is x.

Functions can be written using other letters or even combinations of letters. For instance, some calculators use $ABS(x)$ for the absolute-value function. Thus the equation $y = |x|$ would be written $ABS(x) = |x|$, where $ABS(x)$ replaces y.

Example 7

Given $G(t) = \frac{3t}{t+4}$, find $G(1)$.

Solution

$$\begin{aligned} G(t) &= \frac{3t}{t+4} \\ G(1) &= \frac{3(1)}{1+4} && \bullet \text{ Replace } t \text{ by 1. Then simplify.} \\ G(1) &= \frac{3}{5} \end{aligned}$$

You Try It 7

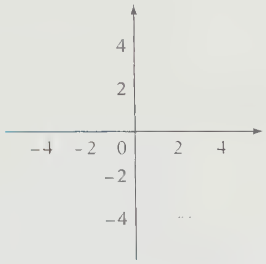
Given $H(x) = \frac{x}{x-4}$, find $H(8)$.

Your solution

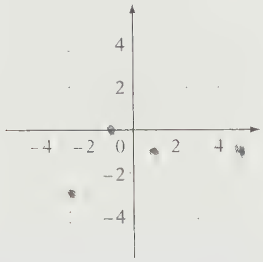
Solution on p. S17

7.1 Exercises

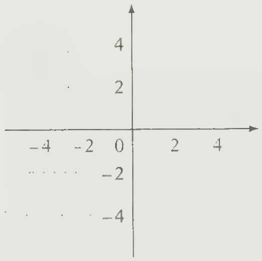
1. Graph (−2, 1), (3, −5), (−2, 4), and (0, 3).



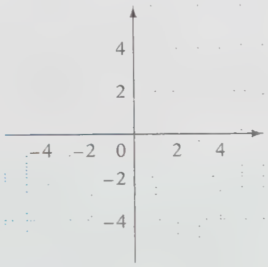
2. Graph (5, −1), (−3, −3), (−1, 0), and (1, −1).



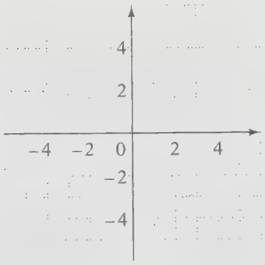
3. Graph (0, 0), (0, −5), (−3, 0), and (0, 2).



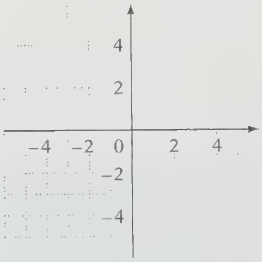
4. Graph (−4, 5), (−3, 1), (3, −4), and (5, 0).



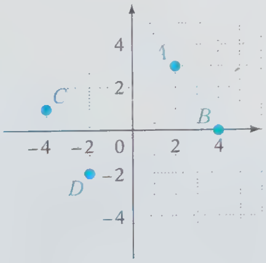
5. Graph (−1, 4), (−2, −3), (0, 2), and (4, 0).



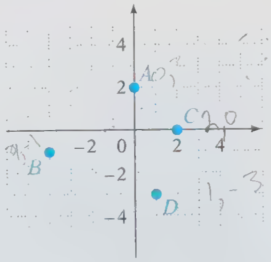
6. Graph (5, 2), (−4, −1), (0, 0), and (0, 3).



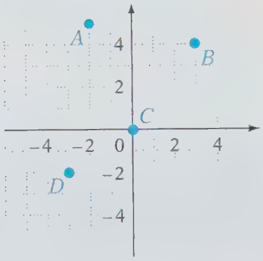
7. Find the coordinates of each of the points.



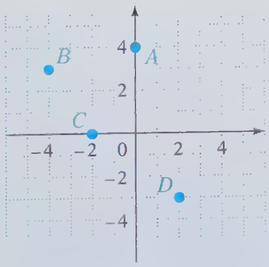
8. Find the coordinates of each of the points.



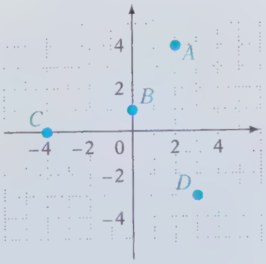
9. Find the coordinates of each of the points.



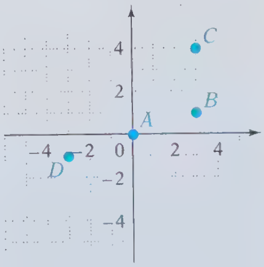
10. Find the coordinates of each of the points.

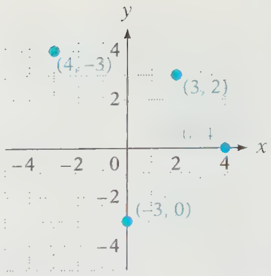


11. **a.** Name the abscissas of points *A* and *C*.
b. Name the ordinates of points *B* and *D*.



12. **a.** Name the abscissas of points *A* and *C*.
b. Name the ordinates of points *B* and *D*.





13. Suppose you are helping a student who is having trouble graphing ordered pairs. The work of the student is at the right. What can you say to this student to correct the error that is being made?

14. **a.** What are the signs of the coordinates of a point in the third quadrant?

b. What are the signs of the coordinates of a point in the fourth quadrant?

c. On an *xy*-coordinate system, what is the name of the axis for which all the *x*-coordinates are zero?

d. On an *xy*-coordinate system, what is the name of the axis for which all the *y*-coordinates are zero?

[illegible]

15. Is (3, 4) a solution of $y = -x + 7$?

16. Is (2, −3) a solution of $y = x + 5$?

17. Is (−1, 2) a solution of $y = \frac{1}{2}x - 1$?

18. Is (1, −3) a solution of $y = -2x - 1$?

19. Is (4, 1) a solution of $2x - 5y = 4$?

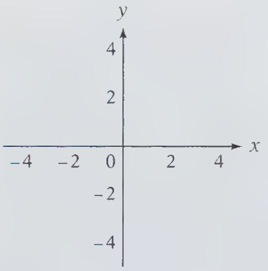
20. Is (−5, 3) a solution of $3x - 2y = 9$?

21. Is (0, 4) a solution of $3x - 4y = -4$?

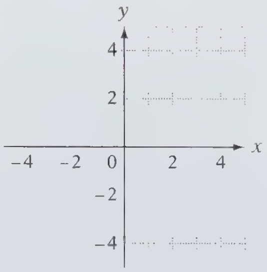
22. Is (−2, 0) a solution of $x + 2y = -1$?

Graph the ordered-pair solutions of each equation for the given values of *x*.

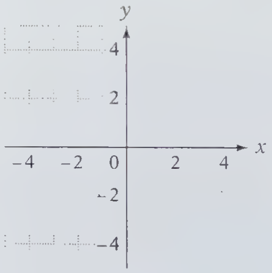
23. $y = 2x$; $x = -2, -1, 0, 2$



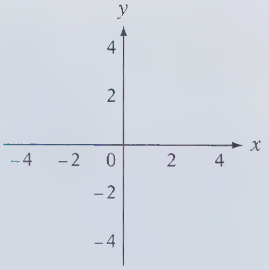
24. $y = -2x$; $x = -2, -1, 0, 2$



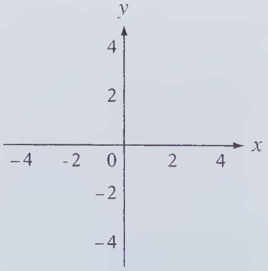
25. $y = \frac{2}{3}x + 1$; $x = -3, 0, 3$



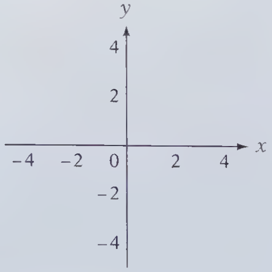
26. $y = -\frac{1}{3}x - 2$; $x = -3, 0, 3$



27. $2x + 3y = 6$; $x = -3, 0, 3$



28. $x - 2y = 4$; $x = -2, 0, 2$



29. The table below shows the length, in centimeters, of the humerus (long bone of the forelimb, from shoulder to elbow) and the total wingspan, in centimeters, of several pterosaurs, which are extinct flying reptiles of the order Pterosauria. Write a relation where the first coordinate is the length of the humerus and the second is the wingspan. Is the relation a function?

Humerus (centimeters)	24	32	22	15	4.4	17	15	4.4
Wingspan (centimeters)	600	750	430	300	68	370	310	55

30. The table below, based in part on data from the National Oceanic and Atmospheric Administration, shows the average annual concentration of atmospheric carbon dioxide (in parts per million) and the average sea surface temperature (in degrees Celsius) for eight consecutive years. Write a relation wherein the first coordinate is the carbon dioxide concentration and the second coordinate is the average sea surface temperature. Is the relation a function?

Carbon dioxide concentration (parts per million)	352	353	354	355	356	358	360	361
Surface sea temperature (degrees Celsius)	15.4	15.4	15.1	15.1	15.2	15.4	15.3	15.5

31. The ratings (each rating point is 1,008,000 households) and share (the percentage of television sets in use tuned to a specific program) for selected television programs for a week in September 2000 are shown in the table at the right. Write a relation wherein the first coordinate is the ratings and the second coordinate is the share. Is the relation a function?

Television Show	*Rating*	*Share*
Who Wants to Be a Millionaire	15.7	26
NFL Monday Night Football	15.3	27
Law and Order	9.0	15
Big Brother	7.9	14
Everybody Loves Raymond	7.6	12
60 Minutes II	7.1	12
Friends	7.1	13

Source: *Nielsen Media Research, September 2000.* From **ytv.yahoo.com/nielsen**, *January 21, 2001.*

32. The average time it takes to commute to work in six cities and the average number of days with some precipitation in those cities are given in the table at the right. Write a relation wherein the first coordinate is the average commute time and the second coordinate is the average number of days with some precipitation. Is the relation a function?

City	*Commute time (minutes)*	*Rain/snow days*
Portland, OR	20.7	152
Providence, RI	19.5	134
Chicago, IL	28	123
Raleigh-Durham, NC	20	112
Salt Lake City, UT	19.5	88
Sarasota, FL	18.4	107

Source: *Money Magazine website, June 22, 2001.* From **www.money.com/money/depts/real_estate**.

33. Does $y = -2x - 3$, where $x \in \{-2, -1, 0, 3\}$, define y as a function of x?

34. Does $y = 2x + 3$, where $x \in \{-2, -1, 1, 4\}$, define y as a function of x?

35. Does $|y| = x - 1$, where $x \in \{1, 2, 3, 4\}$, define y as a function of x?

36. Does $|y| = x + 2$, where $x \in \{-2, -1, 0, 3\}$, define y as a function of x?

37. Does $y = x^2$, where $x \in \{-2, -1, 0, 1, 2\}$, define y as a function of x?

38. Does $y = x^2 - 1$, where $x \in \{-2, -1, 0, 1, 2\}$, define y as a function of x?

39. Given $f(x) = 3x - 4$, find $f(4)$.

40. Given $f(x) = 5x + 1$, find $f(2)$.

41. Given $f(x) = x^2$, find $f(3)$.

42. Given $f(x) = x^2 - 1$, find $f(1)$.

43. Given $G(x) = x^2 + x$, find $G(-2)$.

44. Given $H(x) = x^2 - x$, find $H(-2)$.

45. Given $s(t) = \frac{3}{t - 1}$, find $s(-2)$.

46. Given $P(x) = \frac{4}{2x + 1}$, find $P(-2)$.

47. Given $h(x) = 3x^2 - 2x + 1$, find $h(3)$.

48. Given $Q(r) = 4r^2 - r - 3$, find $Q(2)$.

49. Given $f(x) = \frac{x}{x + 5}$, find $f(-3)$.

50. Given $v(t) = \frac{2t}{2t + 1}$, find $v(3)$.

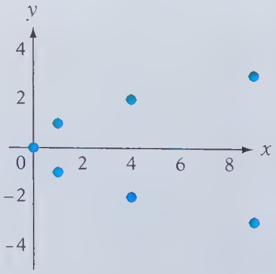
51. Given $g(x) = x^3 - x^2 + 2x - 7$, find $g(0)$.

52. Given $F(z) = \frac{z}{z^2 + 1}$, find $F(0)$.

APPLYING THE CONCEPTS

53. Write a few sentences that describe the similarities and differences between relations and functions.

54. The graph of $y^2 = x$, where $x \in \{0, 1, 4, 9\}$, is shown at the right. Is this the graph of a function? Explain your answer.



55. Is it possible to evaluate $f(x) = \frac{5}{x - 1}$ when $x = 1$? If so, what is $f(1)$? If not, explain why not.

7.2 Linear Equations in Two Variables

Objective A

To graph an equation of the form $y = mx + b$ 7

The **graph of an equation in two variables** is a graph of the ordered-pair solutions of the equation.

Consider $y = 2x + 1$. Choosing $x = -2, -1, 0, 1$, and 2 and determining the corresponding values of y produces some of the ordered pairs of the equation. These are recorded in the table at the right. See the graph of the ordered pairs in Figure 1.

x	$y = 2x + 1$	y	(x, y)
-2	$2(-2) + 1$	-3	$(-2, -3)$
-1	$2(-1) + 1$	-1	$(-1, -1)$
0	$2(0) + 1$	1	$(0, 1)$
1	$2(1) + 1$	3	$(1, 3)$
2	$2(2) + 1$	5	$(2, 5)$

Choosing values of x that are not integers produces more ordered pairs to graph, such as $\left(-\frac{5}{2}, -4\right)$ and $\left(\frac{3}{2}, 4\right)$, as shown in Figure 2. Choosing still other values of x would result in more and more ordered pairs being graphed. The result would be so many dots that the graph would appear as the straight line shown in Figure 3, which is the graph of $y = 2x + 1$.

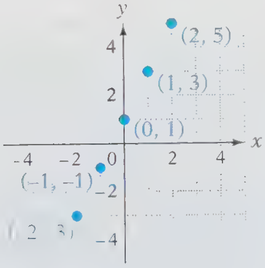


Figure 1

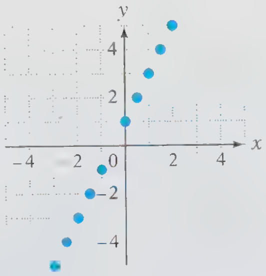


Figure 2

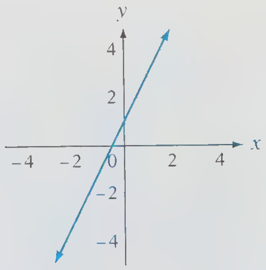


Figure 3

Equations in two variables have characteristic graphs. The equation $y = 2x + 1$ is an example of a *linear equation*, or *linear function*, because its graph is a straight line. It is also called a *first-degree equation* in two variables because the exponent on each variable is the first power.

Linear Equation in Two Variables

Any equation of the form $y = mx + b$, where m and b are constants, is a **linear equation in two variables,** or a **first-degree equation in two variables.** The graph of a linear equation in two variables is a straight line.

Examples of linear equations are shown at the right. These equations represent linear functions because there is only one possible y for each x. Note that for $y = 3 - 2x$, m is the coefficient of x and b is the constant.

$y = 2x + 1$	$(m = 2, b = 1)$
$y = x - 4$	$(m = 1, b = -4)$
$y = -\frac{3}{4}x$	$\left(m = -\frac{3}{4}, b = 0\right)$
$y = 3 - 2x$	$(m = -2, b = 3)$

The equation $y = x^2 + 4x + 3$ is not a linear equation in two variables because there is a term with a variable squared. The equation $y = \frac{3}{x - 4}$ is not a linear equation because a variable occurs in the denominator of a fraction.

The Projects and Group Activities at the end of this chapter contain information on using calculators to graph an equation.

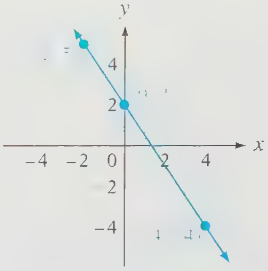
To graph a linear equation, choose some values of x and then find the corresponding values of y. Because a straight line is determined by two points, it is sufficient to find only two ordered-pair solutions. However, it is recommended that at least three ordered-pair solutions be used to ensure accuracy.

Graph $y = -\frac{3}{2}x + 2$.

This is a linear equation with $m = -\frac{3}{2}$ and $b = 2$. Find at least three solutions.

Because m is a fraction, choose values of x that will simplify the calculations. We have chosen -2, 0, and 4 for x. (Any values of x could have been selected.)

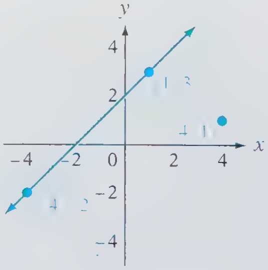
x	$y = -\frac{3}{2}x + 2$	y	(x, y)
-2	$-\frac{3}{2}(-2) + 2$	5	$(-2, 5)$
0	$-\frac{3}{2}(0) + 2$	2	$(0, 2)$
4	$-\frac{3}{2}(4) + 2$	-4	$(4, -4)$



The graph of $y = -\frac{3}{2}x + 2$ is shown at the right.

Remember that a graph is a drawing of the ordered-pair solutions of the equation. Therefore, every point on the graph is a solution of the equation, and every solution of the equation is a point on the graph.

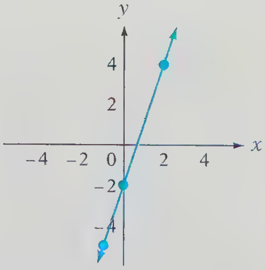
The graph at the right is the graph of $y = x + 2$. Note that $(-4, -2)$ and $(1, 3)$ are points on the graph and that these points are solutions of $y = x + 2$. The point whose coordinates are $(4, 1)$ is not a point on the graph and is not a solution of the equation.



Example 1 Graph $y = 3x - 2$.

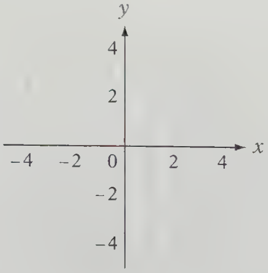
Solution

x	y
0	−2
−1	−5
2	4



You Try It 1 Graph $y = 3x + 1$.

Your solution

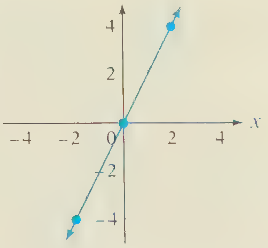


Solution on p. S17

Example 2 Graph $y = 2x$.

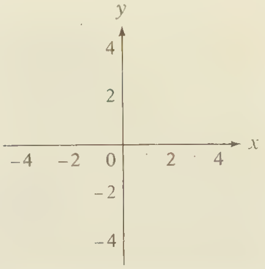
Solution

x	y
0	0
2	4
−2	4



You Try It 2 Graph $y = -2x$.

Your solution

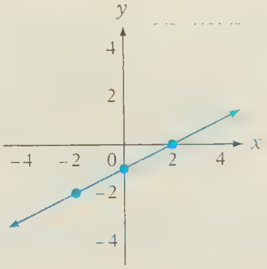


Example 3

Graph $y = \frac{1}{2}x - 1$.

Solution

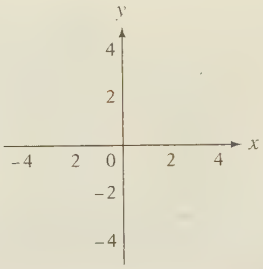
x	y
0	−1
2	0
−2	−2



You Try It 3

Graph $y = \frac{1}{3}x - 3$.

Your solution



Solutions on p. S17

To graph an equation of the form $Ax + By = C$

7

The equation $Ax + By = C$, where A, B, and C are constants, is also a linear equation. Examples of these equations are shown at the right.

$2x + 3y = 6$ $(A = 2, B = 3, C = 6)$
$x - 2y = -4$ $(A = 1, B = -2, C = -4)$
$2x + y = 0$ $(A = 2, B = 1, C = 0)$
$4x - 5y = 2$ $(A = 4, B = -5, C = 2)$

To graph an equation of the form $Ax + By = C$, first solve the equation for y. Then follow the same procedure used for graphing $y = mx + b$.

⇨ Graph $3x + 4y = 12$.

$$3x + 4y = 12$$

- Solve for y.

$$4y = -3x + 12$$

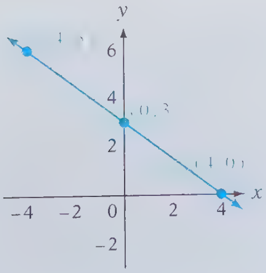
- Subtract $3x$ from each side of the equation.

$$y = -\frac{3}{4}x + 3$$

- Divide each side of the equation by 4.

x	y
0	3
4	0
−4	6

- Find three ordered-pair solutions of the equation.



- Graph the ordered pairs and then draw a line through the points.

The graph of a linear equation with one of the variables missing is either a horizontal or a vertical line.

The equation $y = 2$ could be written $0 \cdot x + y = 2$. Because $0 \cdot x = 0$ for any value of x, the value of y is always 2 no matter what value of x is chosen. For instance, replace x by -4, by -1, by 0, and by 3. In each case, $y = 2$.

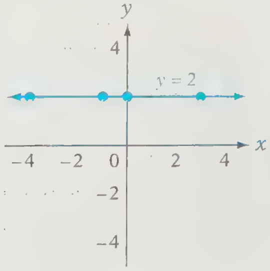
$0x + y = 2$

$0(-4) + y = 2$ $\quad$ $(-4, 2)$ is a solution.

$0(-1) + y = 2$ $\quad$ $(-1, 2)$ is a solution.

$0(0) + y = 2$ $\quad$ $(0, 2)$ is a solution.

$0(3) + y = 2$ $\quad$ $(3, 2)$ is a solution.



The solutions are plotted in the graph at the right, and a line is drawn through the plotted points. Note that the line is horizontal.

Graph of a Horizontal Line

The graph of $y = b$ is a horizontal line passing through $(0, b)$.

The equation $x = -2$ could be written $x + 0 \cdot y = -2$. Because $0 \cdot y = 0$ for any value of y, the value of x is always -2 no matter what value of y is chosen. For instance, replace y by -2, by 0, by 2, and by 3. In each case, $x = -2$.

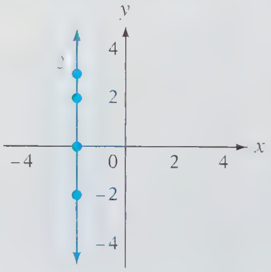
$x + 0y = -2$

$x + 0(-2) = -2$ $\quad$ $(-2, -2)$ is a solution.

$x + 0(0) = -2$ $\quad$ $(-2, 0)$ is a solution.

$x + 0(2) = -2$ $\quad$ $(-2, 2)$ is a solution.

$x + 0(3) = -2$ $\quad$ $(-2, 3)$ is a solution.

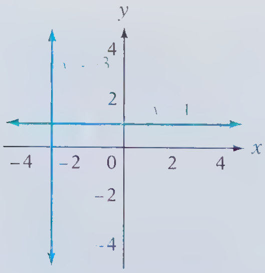


The solutions are plotted in the graph at the right, and a line is drawn through the plotted points. Note that the line is vertical.

Graph of a Vertical Line

The graph of $x = a$ is a vertical line passing through $(a, 0)$.

Graph $x = -3$ and $y = 1$ on the same coordinate grid.



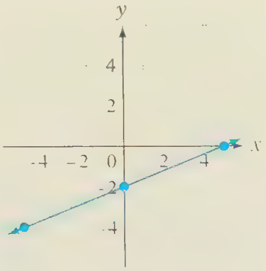
- The graph of $x = -3$ is a vertical line passing through $(-3, 0)$.
- The graph of $y = 1$ is a horizontal line passing through $(0, 1)$.

Example 4 Graph $2x - 5y = 10$.

Solution

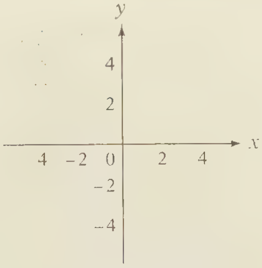
$$2x - 5y = 10$$
$$-5y = -2x + 10$$
$$y = \frac{2}{5}x - 2$$

x	y
0	−2
5	0
−5	−4



You Try It 4 Graph $5x - 2y = 10$.

Your solution

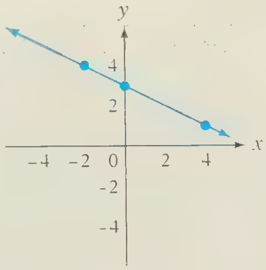


Example 5 Graph $x + 2y = 6$.

Solution

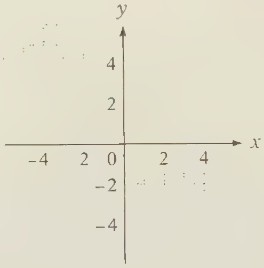
$$x + 2y = 6$$
$$2y = -x + 6$$
$$y = -\frac{1}{2}x + 3$$

x	y
0	3
−2	4
4	1



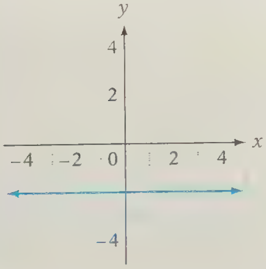
You Try It 5 Graph $x - 3y = 9$.

Your solution



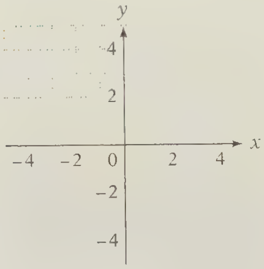
Example 6 Graph $y = -2$.

Solution
The graph of an equation of the form $y = b$ is a horizontal line passing through the point $(0, b)$.



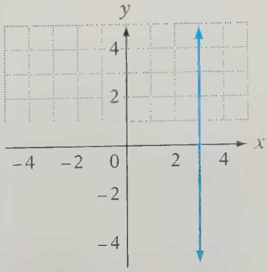
You Try It 6 Graph $y = 3$.

Your solution



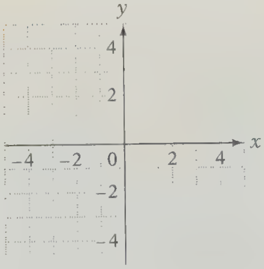
Example 7 Graph $x = 3$.

Solution
The graph of an equation of the form $x = a$ is a vertical line passing through the point $(a, 0)$.



You Try It 7 Graph $x = -4$.

Your solution



Solutions on p. S18

Objective C To solve application problems

There are a variety of applications of linear functions.

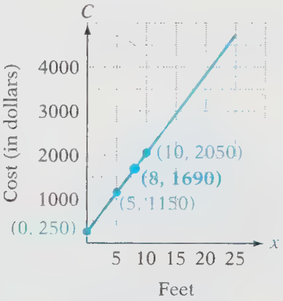
Solve: An installer of marble kitchen countertops charges \$250 plus \$180 per foot of countertop. The equation that describes the total cost, C, to have x feet of countertop installed is $C = 180x + 250$.

a. Graph this equation for $0 \le x \le 25$. (*Note:* In many applications, the domain of the variable is given so that the equation makes sense. For instance, it would not be sensible to have values of x that are less than 0. This would mean negative countertop! The choice of 25 is somewhat arbitrary, but most kitchens have less than 25 ft of counter space.)

b. The point whose coordinates are (8, 1690) is on the graph. Write a sentence that describes this ordered pair.

Solution

a.



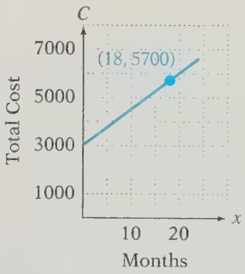
- Choosing $x = 0$, 5, and 10, you find that the corresponding ordered pairs are (0, 250), (5, 1150), and (10, 2050). Plot these points and draw a line through them.

b. The point whose coordinates are (8, 1690) means that 8 ft of countertop costs \$1690 to install.

Example 8

A local car dealer is advertising a 2-year lease for a new Honda Civic. The upfront cost is \$3000 with a monthly lease payment of \$150. The total cost, C, after x months of the lease is given by $C = 150x + 3000$. Graph this equation for $0 \le x \le 24$. The point whose coordinates are (18, 5700) is on the graph. Write a sentence that describes this ordered pair.

Solution

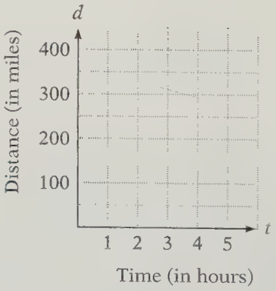


The ordered pair (18, 5700) means that the total cost of the lease payments for 18 months is \$5700.

You Try It 8

A car is traveling at a uniform speed of 40 mph. The distance, d, the car travels in t hours is given by $d = 40t$. Graph this equation for $0 \le t \le 5$. The point whose coordinates are (3, 120) is on the graph. Write a sentence that describes this ordered pair.

Your solution



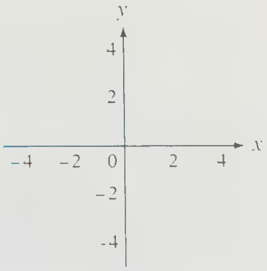
Solution on p. S18

7.2 Exercises

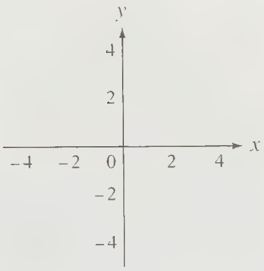
[illegible]

Graph.

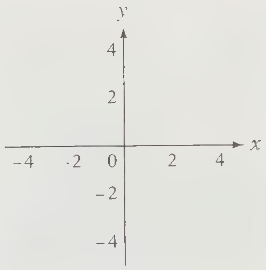
1. $y = 2x - 3$



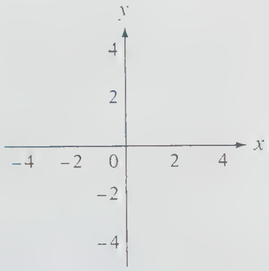
2. $y = -2x + 2$



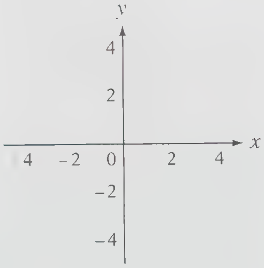
3. $y = \frac{1}{3}x$



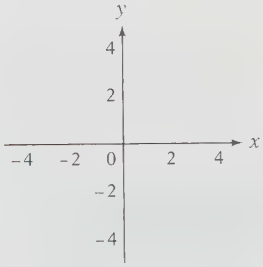
4. $y = -3x$



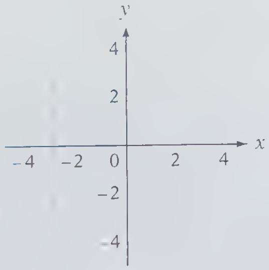
5. $y = \frac{2}{3}x - 1$



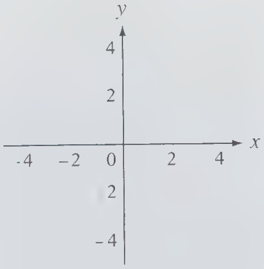
6. $y = \frac{3}{4}x + 2$



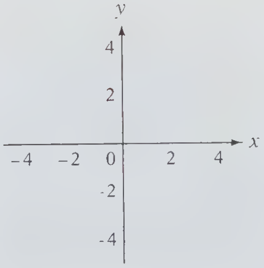
7. $y = -\frac{1}{4}x + 2$



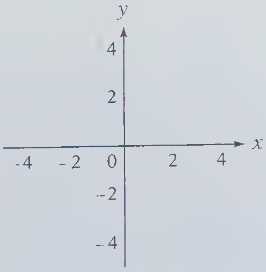
8. $y = -\frac{1}{3}x + 1$



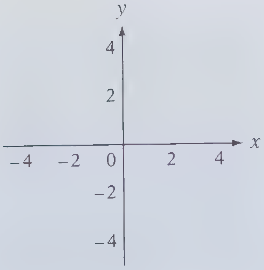
9. $y = -\frac{2}{5}x + 1$



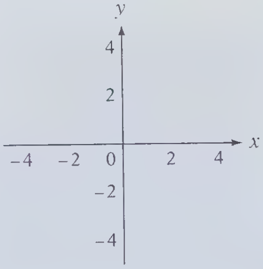
10. $y = -\frac{1}{2}x + 3$



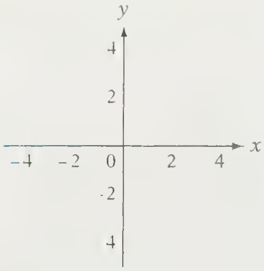
11. $y = 2x - 4$



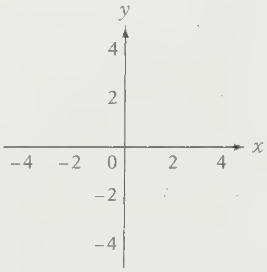
12. $y = 3x - 4$



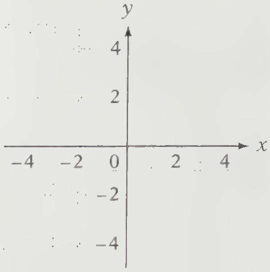
13. $y = x - 3$



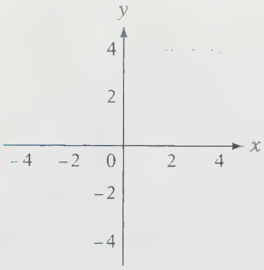
14. $y = x + 2$



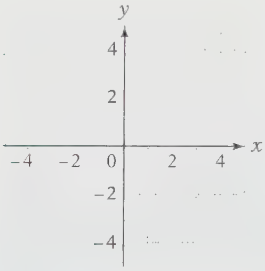
15. $y = -x + 2$



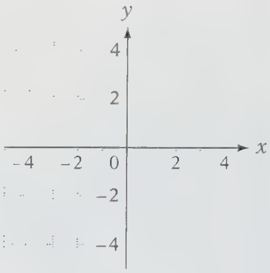
16. $y = -x - 1$



17. $y = -\frac{2}{3}x + 1$



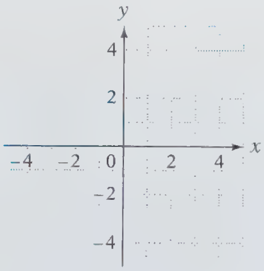
18. $y = 5x - 4$



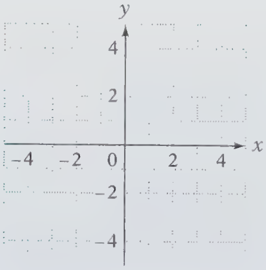
Objective B

Graph.

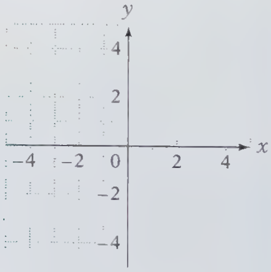
19. $3x + y = 3$



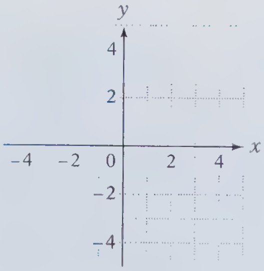
20. $2x + y = 4$



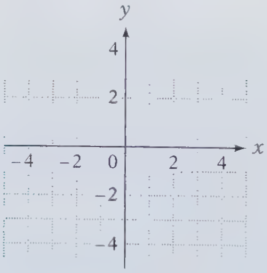
21. $2x + 3y = 6$



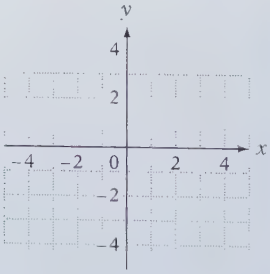
22. $3x + 2y = 4$



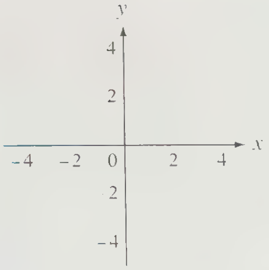
23. $x - 2y = 4$



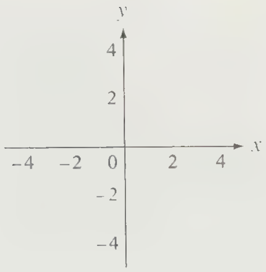
24. $x - 3y = 6$



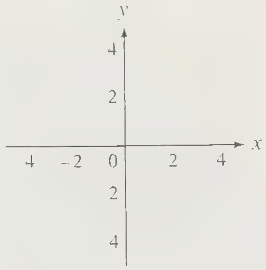
25. $2x - 3y = 6$



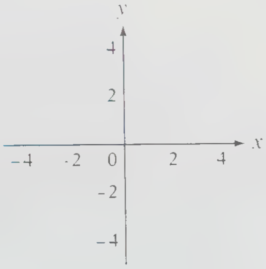
26. $3x - 2y = 8$



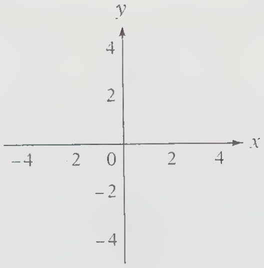
27. $2x + 5y = 10$



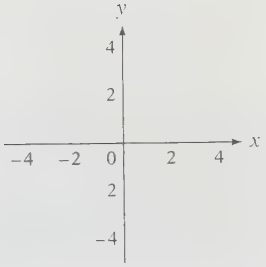
28. $3x + 4y = 12$



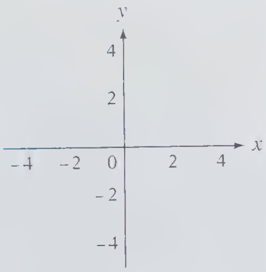
29. $x = 3$



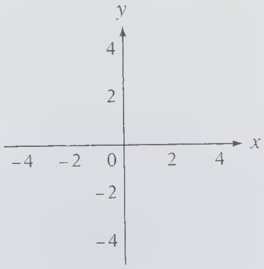
30. $y = -4$



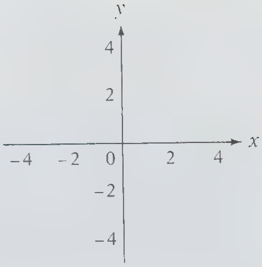
31. $x + 4y = 4$



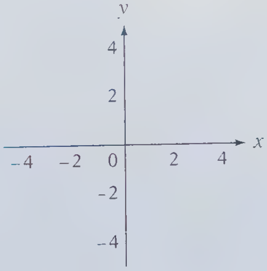
32. $4x - 3y = 12$



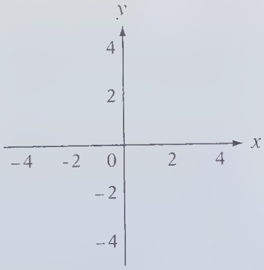
33. $y = 4$



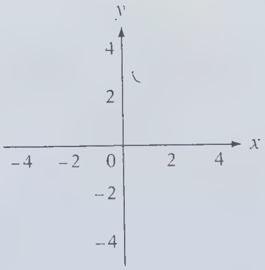
34. $x = -2$



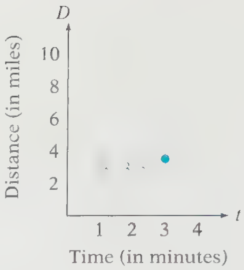
35. $\frac{x}{5} + \frac{y}{4} = 1$



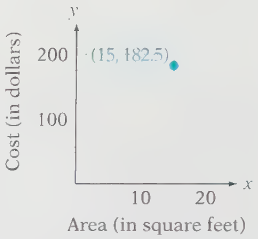
36. $\frac{x}{4} - \frac{y}{3} = 1$



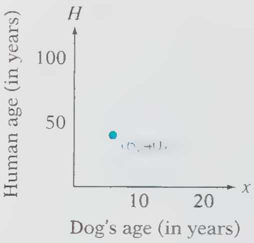
Application Problems



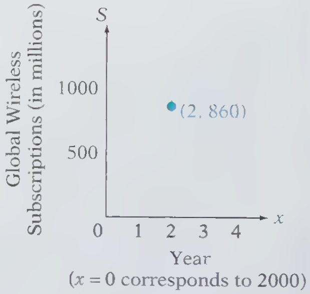
37. A rescue helicopter is rushing at a constant speed of 150 mph to reach several people stranded in the ocean 11 mi away after their boat sank. The rescuers can determine how far they are from the victims using the equation $D = 11 - 2.5t$, where D is the distance in miles and t is the time elapsed in minutes. Graph this equation for $0 \le t \le 4$. The point (3, 3.5) is on the graph. Write a sentence that describes the meaning of this ordered pair.



38. A custom-illustrated sign or banner can be commissioned for a cost of \$25 for the material and \$10.50 per square foot for the artwork. The equation that represents this cost is given by $y = 10.50x + 25$, where y is the cost and x is the number of square feet in the sign. Graph this equation for $0 \le x \le 20$. The point (15, 182.5) is on the graph. Write a sentence that describes the meaning of this ordered pair.



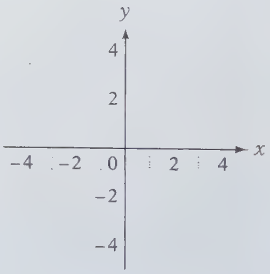
39. According to some veterinarians, the age, x, of a dog can be translated to "human years" by using the equation $H = 4x + 16$, where H is the human equivalent age for the dog. Graph this equation for $2 \le x \le 21$. The point whose coordinates are (6, 40) is on this graph. Write a sentence that explains the meaning of this ordered pair.



40. Judging on the basis of data from Cahners In-Stat Group, the projected number of global wireless subscriptions for the years 2000 to 2004 can be approximated by the equation $S = 280x + 300$, where $0 \le x \le 4$ and 0 corresponds to the year 2000. Graph the equation. The point whose coordinates are (2, 860) is a point on the graph. Write a sentence that explains the meaning of this ordered pair.

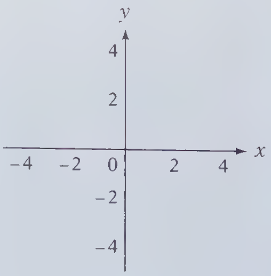
APPLYING THE CONCEPTS

41. Graph $y = 2x - 2$, $y = 2x$, and $y = 2x + 3$ in the first coordinate system at the right. What observation can you make about the graphs?



42. For the equation $y = 3x + 2$, when the value of x changes from 1 to 2, does the value of y increase or decrease? What is the change in y? Suppose that the value of x changes from 13 to 14. What is the change in y?

43. Graph $y = x + 3$, $y = 2x + 3$, and $y = -\frac{1}{2}x + 3$ in the second coordinate system at the right. What observation can you make about the graphs?



44. For the equation $y = -2x + 1$, when the value of x changes from 1 to 2, does the value of y increase or decrease? What is the change in y? Suppose the value of x changes from 13 to 14. What is the change in y?

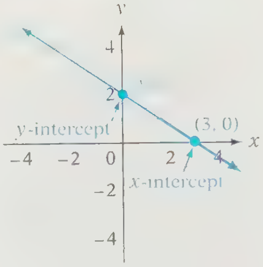
7.3 Intercepts and Slopes of Straight Lines

Objective A

To find the x- and y-intercepts of a straight line

TAKE NOTE
To find the x-intercept, let $y = 0$. To find the y-intercept, let $x = 0$.

The graph of the equation $2x + 3y = 6$ is shown at the right. The graph crosses the x-axis at the point (3, 0). This point is called the **x-intercept.** The graph also crosses the y-axis at the point (0, 2). This point is called the **y-intercept.**



Find the x-intercept and the y-intercept of the graph of the equation $2x - 3y = 12$.

To find the x-intercept, let $y = 0$. (Any point on the x-axis has y-coordinate 0.)

$$\begin{aligned} 2x - 3y &= 12 \\ 2x - 3(0) &= 12 \\ 2x &= 12 \\ x &= 6 \end{aligned}$$

The x-intercept is (6, 0).

To find the y-intercept, let $x = 0$. (Any point on the y-axis has x-coordinate 0.)

$$\begin{aligned} 2x - 3y &= 12 \\ 2(0) - 3y &= 12 \\ -3y &= 12 \\ y &= -4 \end{aligned}$$

The y-intercept is (0, −4).

Find the y-intercept of $y = 3x + 4$.

$y = 3x + 4 = 3(0) + 4 = 4$ • **Let $x = 0$.**

The y-intercept is (0, 4).

For any equation of the form $y = mx + b$, the y-intercept is $(0, b)$.

Some linear equations can be graphed by finding the x- and y-intercepts and then drawing a line through these two points.

Example 1 Find the x- and y-intercepts for $x - 2y = 4$. Graph the line.

Solution

x-intercept:

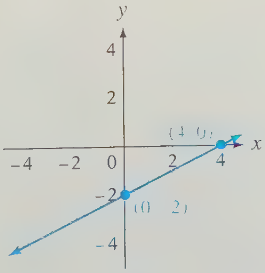
$$\begin{aligned} x - 2y &= 4 \\ x - 2(0) &= 4 \\ x &= 4 \end{aligned}$$

(4, 0)

y-intercept:

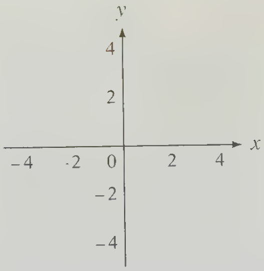
$$\begin{aligned} x - 2y &= 4 \\ 0 - 2y &= 4 \\ -2y &= 4 \\ y &= -2 \end{aligned}$$

(0, −2)



You Try It 1 Find the x- and y-intercepts for $y = 2x - 4$. Graph the line.

Your solution



Solution on p. S18

To find the slope of a straight line

The graphs of $y = \frac{2}{3}x + 1$ and $y = 2x + 1$ are shown in Figure 1. Each graph crosses the y-axis at the point (0, 1), but the graphs have different slants. The **slope** of a line is a measure of the slant of the line. The symbol for slope is m.

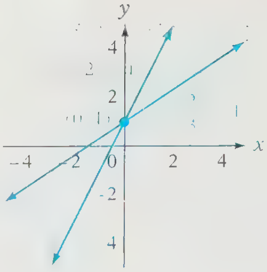
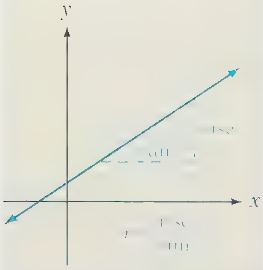


Figure 1

TAKE NOTE

The change in the y values can be thought of as the *rise* of the line, and the change in the x values can be thought of as the *run*. Then

$$\text{Slope} = m = \frac{\text{rise}}{\text{run}}$$



The slope of a line containing two points is the ratio of the change in the y values of the two points to the change in the x values. The line containing the points $(-2, -3)$ and $(6, 1)$ is graphed in Figure 2. The change in the y values is the difference between the two ordinates.

$$\text{Change in } y = 1 - (-3) = 4$$

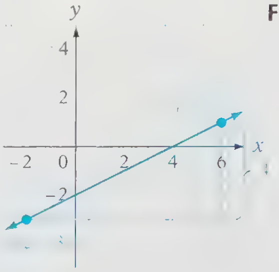


Figure 2

The change in the x values is the difference between the two abscissas (Figure 3).

$$\text{Change in } x = 6 - (-2) = 8$$

$$\text{Slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{4}{8} = \frac{1}{2}$$

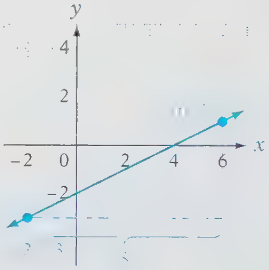


Figure 3

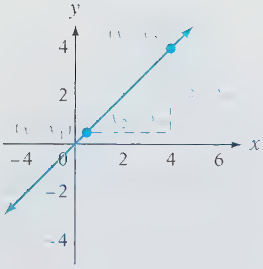


Figure 4

Slope Formula

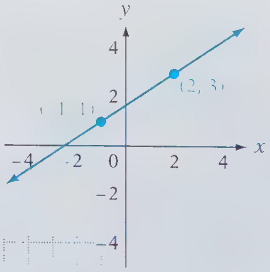
If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are two points on a line and $x_1 \neq x_2$, then $m = \frac{y_2 - y_1}{x_2 - x_1}$ (Figure 4). If $x_1 = x_2$, the slope is undefined.

⇒ Find the slope of the line containing the points (−1, 1) and (2, 3).

Let P_1 be $(-1, 1)$ and P_2 be $(2, 3)$. Then $x_1 = -1$, $y_1 = 1$, $x_2 = 2$, and $y_2 = 3$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{2 - (-1)} = \frac{2}{3}$$

The slope is $\frac{2}{3}$.



Positive slope

A line that slants upward to the right always has a **positive slope.**

TAKE NOTE

Positive slope means that the value of y increases as the value of x increases.

Note that you obtain the same results if the points are named oppositely. Let P_1 be (2, 3) and P_2 be (−1, 1). Then $x_1 = 2$, $y_1 = 3$, $x_2 = -1$, and $y_2 = 1$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{-1 - 2} = \frac{-2}{-3} = \frac{2}{3}$$

The slope is $\frac{2}{3}$.

Therefore, it does not matter which point is named P_1 and which is named P_2; the slope remains the same.

TAKE NOTE

Negative slope means that the value of y decreases as x increases. Compare this to positive slope.

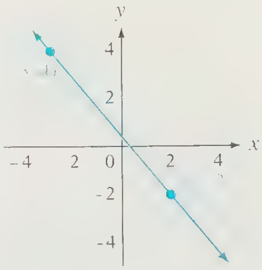
Find the slope of the line containing the points $(-3, 4)$ and $(2, -2)$.

Let P_1 be $(-3, 4)$ and P_2 be $(2, -2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{2 - (-3)} = \frac{-6}{5} = -\frac{6}{5}$$

The slope is [illegible]

A line that slants downward to the right always has a **negative slope.**



Negative slope

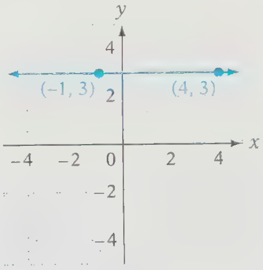
Find the slope of the line containing the points $(-1, 3)$ and $(4, 3)$.

Let P_1 be $(-1, 3)$ and P_2 be $(4, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 3}{4 - (-1)} = \frac{0}{5} = 0$$

The slope is 0.

A horizontal line has **zero slope.**



Zero slope

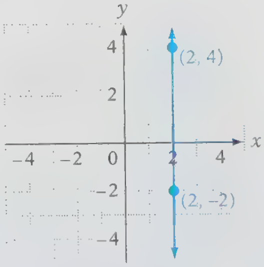
Find the slope of the line containing the points $(2, -2)$ and $(2, 4)$.

Let P_1 be $(2, -2)$ and P_2 be $(2, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{2 - 2} = \frac{6}{0}$$

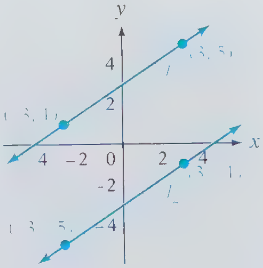
Division by zero is not defined.

The slope of a vertical line is undefined.



Undefined

Two lines in the plane that never intersect are called **parallel** lines. The lines l_1 and l_2 in the figure at the right are parallel. Calculating the slope of each line, we have



$$\text{Slope of } l_1\text{: } m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}$$

$$\text{Slope of } l_2\text{: } m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-5)}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}$$

Note that these parallel lines have the same slope. This is always true.

TAKE NOTE

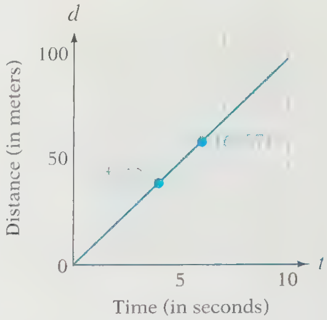
We must separate the description of parallel lines at the right because vertical lines in the plane are parallel but their slopes are undefined.

Parallel Lines

Two nonvertical lines in the plane are parallel if and only if they have the same slope. Vertical lines in the plane are parallel.

There are many applications of the concept of slope. Here is a possibility.

When Florence Griffith-Joyner set the world record for the 100-meter dash, her average rate of speed was approximately 9.5 meters per second. The graph at the right shows the distance she ran during her record-setting run. From the graph, note that after 4 s she had traveled 38 m and that after 6 s she had traveled 57 m. The slope of the line between these two points is



$$m = \frac{57 - 38}{6 - 4} = \frac{19}{2} = 9.5$$

Note that the slope of the line is the same as the rate she was running, 9.5 meters per second. The average speed of an object is related to slope.

Example 2

Find the slope of the line containing the points with coordinates $(-2, -3)$ and $(3, 4)$.

Solution

Let $P_1 = (-2, -3)$ and $P_2 = (3, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-3)}{3 - (-2)} = \frac{7}{5}$$

The slope is $\frac{7}{5}$.

You Try It 2

Find the slope of the line containing the points with coordinates $(-3, 8)$ and $(1, 4)$.

Your solution

Example 3

Find the slope of the line containing the points with coordinates $(-1, 4)$ and $(-1, 0)$.

Solution

Let $P_1 = (-1, 4)$ and $P_2 = (-1, 0)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{-1 - (-1)} = \frac{-4}{0}$$

The slope is undefined.

You Try It 3

Find the slope of the line containing the points with coordinates $(-1, 2)$ and $(4, 2)$.

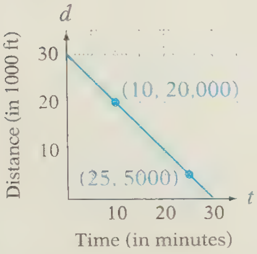
Your solution

Solutions on p. S18

Example 4

The graph below shows the height of a plane above an airport during its 30-minute descent from cruising altitude to landing. Find the slope of the line. Write a sentence that explains the meaning of the slope.

Solution



$$m = \frac{5000 - 20{,}000}{25 - 10} = \frac{-15{,}000}{15}$$

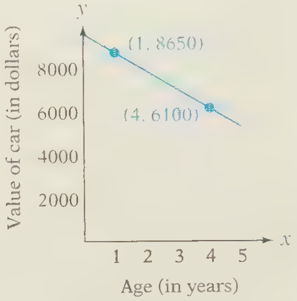
$$= -1000$$

A slope of -1000 means that the height of the plane is *decreasing* at the rate of 1000 ft/min.

You Try It 4

The graph below shows the approximate decline in the value of a used car over a 5-year period. Find the slope of the line. Write a sentence that states the meaning of the slope.

Your solution



Solution on p. S18

Objective C

To graph a line using the slope and the y-intercept

TAKE NOTE

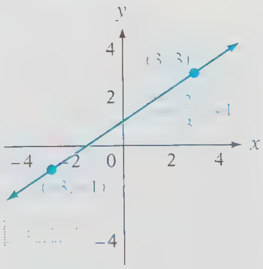
Here are some equations in slope-intercept form.

$y = 2x - 3$: Slope is 2; y-intercept is $(0, -3)$.

$y = -x + 2$: Slope is -1 (recall that $-x = -1x$); y-intercept is $(0, 2)$.

$y = \frac{x}{2}$: Because $\frac{x}{2} = \frac{1}{2}x$, slope is $\frac{1}{2}$; y-intercept is $(0, 0)$.

The graph of the equation $y = \frac{2}{3}x + 1$ is shown at the right. The points $(-3, -1)$ and $(3, 3)$ are on the graph. The slope of the line between the two points is



$$m = \frac{3 - (-1)}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}$$

Observe that the slope of the line is the coefficient of x in the equation $y = \frac{2}{3}x + 1$. Also recall that the y-intercept is $(0, 1)$, where 1 is the constant term of the equation.

Slope-Intercept Equation of a Line

An equation of the form $y = mx + b$ is called the **slope-intercept** form of a straight line. The slope of the line is m, the coefficient of x. The y-intercept is $(0, b)$, where b is the constant term of the equation.

If a linear equation is not in slope-intercept form, solve the equation for y.

⇒ Find the slope and y-intercept for the graph of $3x + 2y = 12$.

Write the equation in slope-intercept form by solving for y.

$$3x + 2y = 12$$
$$2y = -3x + 12$$
$$y = -\frac{3}{2}x + 6$$

The slope is $-\frac{3}{2}$; the y-intercept is $(0, 6)$.

When an equation of a straight line is written in slope-intercept form, the graph can be drawn using the slope and the y-intercept. First locate the y-intercept. Use the slope to find a second point on the line. Then draw a line through the two points.

Graph $y = 2x - 3$.

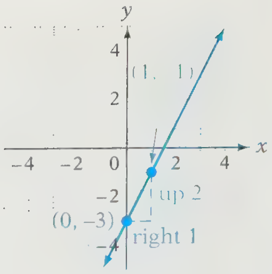
y-intercept $= (0, b) = (0, -3)$

$$m = 2 = \frac{2}{1} = \frac{\text{change in } y}{\text{change in } x}$$

Beginning at the y-intercept, move right 1 unit (change in x) and then up 2 units (change in y).

$(1, -1)$ is a second point on the graph.

Draw a line through the two points $(0, -3)$ and $(1, -1)$.

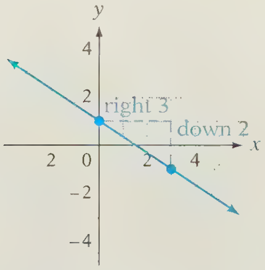


Example 5

Graph $y = -\frac{2}{3}x + 1$ by using the slope and y-intercept.

Solution y-intercept $= (0, b) = (0, 1)$

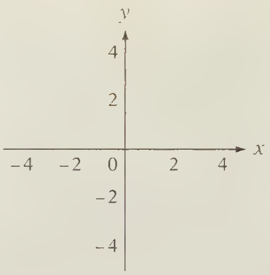
$$m = -\frac{2}{3} = \frac{-2}{3} = \frac{\text{change in } y}{\text{change in } x}$$



You Try It 5

Graph $y = -\frac{1}{4}x - 1$ by using the slope and y-intercept.

Your solution

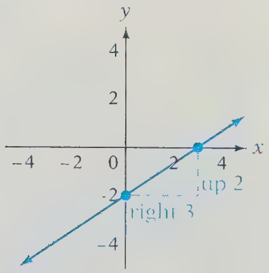


Example 6 Graph $2x - 3y = 6$ by using the slope and y-intercept.

Solution Solve the equation for y.

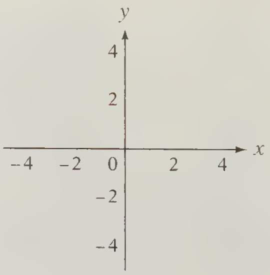
$$2x - 3y = 6$$
$$-3y = -2x + 6$$
$$y = \frac{2}{3}x - 2$$

y-intercept $= (0, -2)$; $m = \frac{2}{3}$



You Try It 6 Graph $x - 2y = 4$ by using the slope and y-intercept.

Your solution



Solutions on pp. S18–S19

7.3 Exercises

Objective A

Find the x- and y-intercepts.

1. $x - y = 3$

2. $3x + 4y = 12$

3. $y = 3x - 6$

4. $y = 2x + 10$

5. $x - 5y = 10$

6. $3x + 2y = 12$

7. $y = 3x + 12$

8. $y = 5x + 10$

9. $2x - 3y = 0$

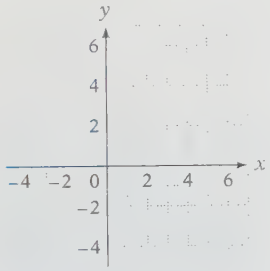
10. $3x + 4y = 0$

11. $y = -\frac{1}{2}x + 3$

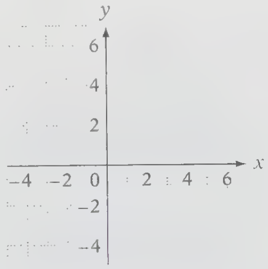
12. $y = \frac{2}{3}x - 4$

Find the x- and y-intercepts and then graph.

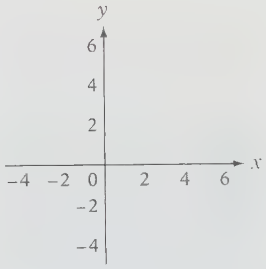
13. $5x + 2y = 10$



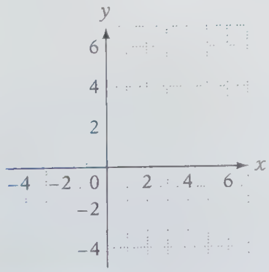
14. $x - 3y = 6$



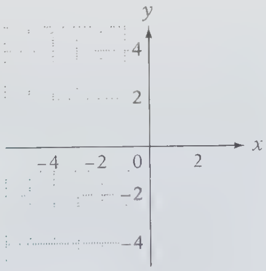
15. $y = \frac{3}{4}x - 3$



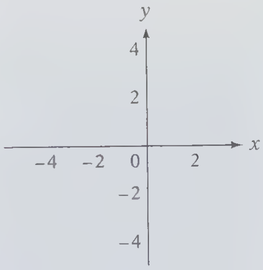
16. $y = \frac{2}{5}x - 2$



17. $5y - 3x = 15$



18. $9y - 4x = 18$



Objective B

19. Explain how to find the slope of a line given two points on the line.

20. What is the difference between a line that has zero slope and one that has undefined slope?

Find the slope of the line containing the given points.

21. $P_1(4, 2), P_2(3, 4)$

22. $P_1(2, 1), P_2(3, 4)$

23. $P_1(-1, 3), P_2(2, 4)$

24. $P_1(-2, 1), P_2(2, 2)$

25. $P_1(2, 4), P_2(4, -1)$

26. $P_1(1, 3), P_2(5, -3)$

27. $P_1(3, -4), P_2(3, 5)$

28. $P_1(-1, 2), P_2(-1, 3)$

29. $P_1(4, -2), P_2(3, -2)$

30. $P_1(5, 1), P_2(-2, 1)$

31. $P_1(0, -1), P_2(3, -2)$

32. $P_1(3, 0), P_2(2, -1)$

Determine whether the line through P_1 and P_2 is parallel to the line through Q_1 and Q_2.

33. $P_1(-3, 4), P_2(2, -5); Q_1(1, -2), Q_2(6, 7)$

34. $P_1(4, -5), P_2(6, -9); Q_1(5, -4), Q_2(1, 4)$

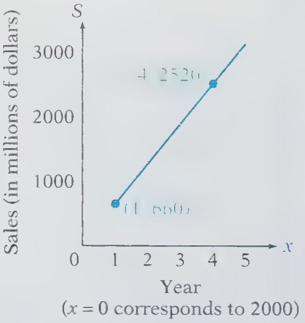
35. $P_1(0, 1), P_2(2, 5); Q_1(-4, -7), Q_2(2, 5)$

36. $P_1(3, 7), P_2(-2, 4); Q_1(1, 6), Q_2(11, 1)$

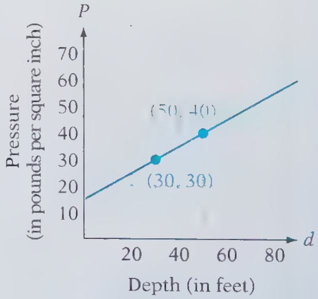
37. $P_1(-2, 4), P_2(2, 4); Q_1(-3, 6), Q_2(4, 6)$

38. $P_1(-1, 2), P_2(-1, -6); Q_1(1, -2), Q_2(1, 3)$

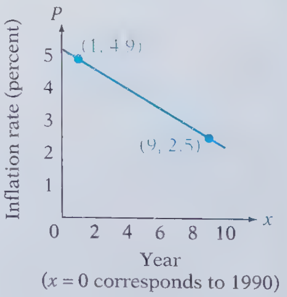
39. The graph at the right is based on data from Andersen Consulting. It shows the projected ebooks retail sales through the year 2005. Find the slope of the line. Write a sentence that states the meaning of the slope.



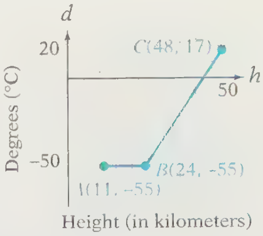
40. The pressure, in pounds per square inch (psi), on a diver is shown in the graph at the right. Find the slope of the line. Write a sentence that explains the meaning of the slope.



41. The graph at the right is based on data from the Bureau of Labor Statistics. It shows the approximate inflation rate for the years 1990 to 2000. Find the slope of the line. Write a sentence that states the meaning of the slope.



42. The stratosphere extends from approximately 11 km to 50 km above Earth. The graph at the right shows how the temperature, in degrees Celsius, changes in the stratosphere. Explain the meaning of the horizontal line segment from A to B. Find the slope of the line from B to C and explain its meaning.



Find the slope and y-intercept for the graph of each equation.

43. $2x - 3y = 6$

44. $4x + 3y = 12$

45. $2x + 5y = 10$

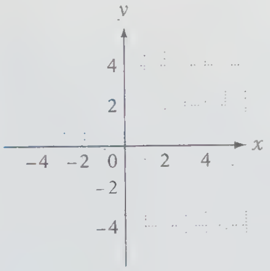
46. $2x + y = 0$

47. $x - 4y = 0$

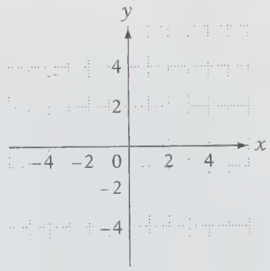
48. $2x + 3y = 8$

Graph by using the slope and y-intercept.

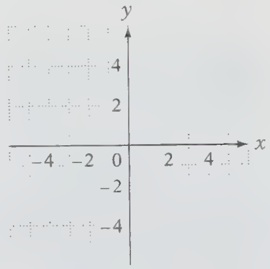
49. $y = 3x + 1$



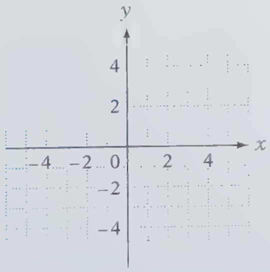
50. $y = -2x - 1$



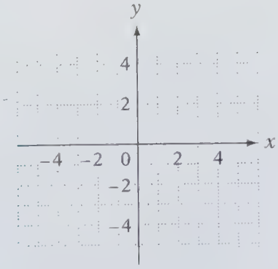
51. $y = \frac{2}{5}x - 2$



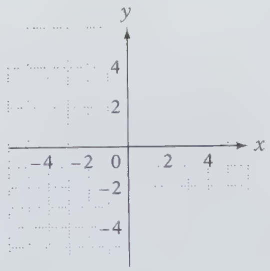
52. $y = \frac{3}{4}x + 1$



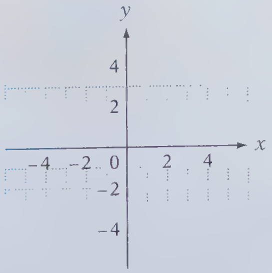
53. $2x + y = 3$



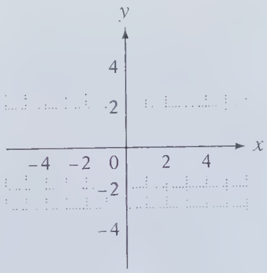
54. $3x - y = 1$



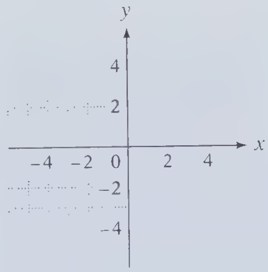
55. $x - 2y = 4$



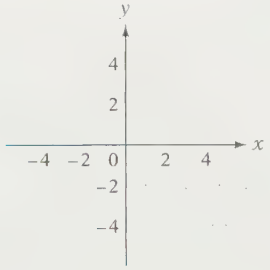
56. $x + 3y = 6$



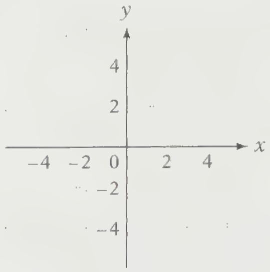
57. $y = \frac{2}{3}x$



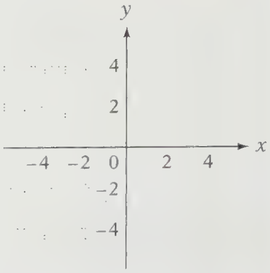
58. $y = \frac{1}{2}x$



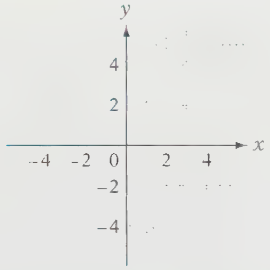
59. $y = -x + 1$



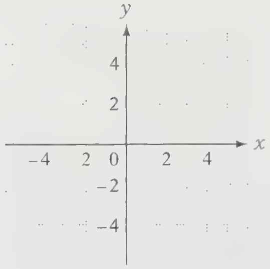
60. $y = -x - 3$



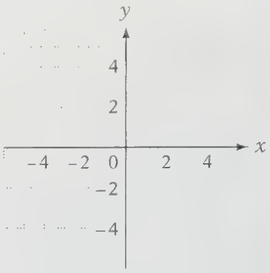
61. $3x - 4y = 12$



62. $5x - 2y = 10$



63. $y = -4x + 2$



APPLYING THE CONCEPTS

64. Do all straight lines have a y-intercept? If not, give an example of one that does not.

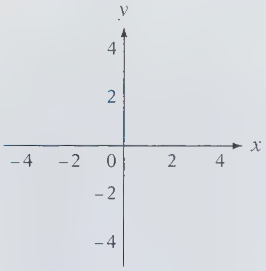
65. If two lines have the same slope and the same y-intercept, must the graphs of the lines be the same? If not, give an example.

66. Draw the graphs of **a.** $\frac{x}{3} + \frac{y}{4} = 1$ **b.** $\frac{x}{2} - \frac{y}{3} = 1$ **c.** $-\frac{x}{4} + \frac{y}{2} = 1$

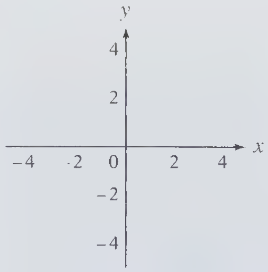
What observation can you make about the x- and y-intercepts of these graphs and the coefficients of x and y?

d. Use this observation to draw the graph of $\frac{x}{4} - \frac{y}{3} = 1$.

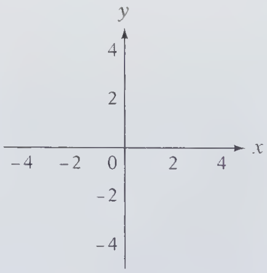
a.



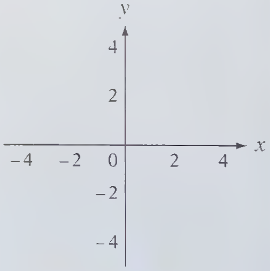
b.



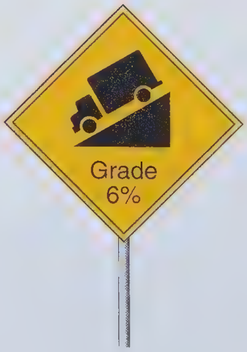
c.



d.



67. What does the highway sign at the right have to do with slope?



7.4 Equations of Straight Lines

To find the equation of a line given a point and the slope

In earlier sections, the equation of a line was given and you were asked to determine some properties of the line, such as its intercepts and slope. Here, the process is reversed. Given properties of a line, you will determine its equation.

If the slope and y-intercept of a line are known, the equation of the line can be determined by using the slope-intercept form of a straight line.

Find the equation of the line with slope $-\frac{1}{2}$ and y-intercept $(0, 3)$.

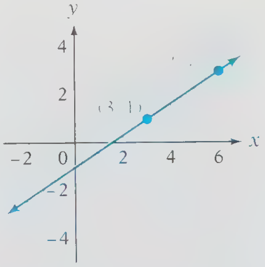
$y = mx + b$ • **Use the slope-intercept formula.**

$y = -\frac{1}{2}x + 3$ • **$m = -\frac{1}{2}$; $(0, b) = (0, 3)$, so $b = 3$.**

The equation of the line is $y = -\frac{1}{2}x + 3$.

When the slope and the coordinates of a point other than the y-intercept are known, the equation of the line can be found by using the formula for slope.

Suppose a line passes through the point $(3, 1)$ and has a slope of $\frac{2}{3}$. The equation of the line with these properties is determined by letting (x, y) be the coordinates of an unknown point on the line. Because the slope of the line is known, use the slope formula to write an equation. Then solve for y.



$\frac{y - 1}{x - 3} = \frac{2}{3}$ • **$\frac{y_2 - y_1}{x_2 - x_1} = m$; $m = \frac{2}{3}$; $(x_2, y_2) = (x, y)$; $(x_1, y_1) = (3, 1)$**

$\frac{y - 1}{x - 3}(x - 3) = \frac{2}{3}(x - 3)$ • **Multiply each side by $(x - 3)$.**

$y - 1 = \frac{2}{3}x - 2$ • **Simplify.**

$y = \frac{2}{3}x - 1$ • **Solve for y.**

The equation of the line is $y = \frac{2}{3}x - 1$.

The same procedure that was used above is used to derive the *point-slope formula*. We use this formula to determine the equation of a line when we are given the coordinates of a point on the line and the slope of the line.

Let (x_1, y_1) be the given coordinates of a point on a line, m the given slope of the line, and (x, y) the coordinates of an unknown point on the line. Then

$\frac{y - y_1}{x - x_1} = m$ • **Formula for slope.**

$\frac{y - y_1}{x - x_1}(x - x_1) = m(x - x_1)$ • **Multiply each side by $x - x_1$.**

$y - y_1 = m(x - x_1)$ • **Simplify.**

Point-Slope Formula

If (x_1, y_1) is a point on a line with slope m, then $y - y_1 = m(x - x_1)$.

Find the equation of the line that passes through the point (2, 3) and has slope −2.

$y - y_1 = m(x - x_1)$ • Use the point-slope formula.

$y - 3 = -2(x - 2)$ • $m = -2$; $(x_1, y_1) = (2, 3)$

$y - 3 = -2x + 4$ • Solve for y.

$y = -2x + 7$

The equation of the line is $y = -2x + 7$.

Example 1

Find the equation of the line whose slope is $-\frac{2}{3}$ and whose y-intercept is (0, −1).

Solution

Because the slope and y-intercept are known, use the slope-intercept formula, $y = mx + b$.

$y = -\frac{2}{3}x - 1$ • $m = -\frac{2}{3}$; $b = -1$

You Try It 1

Find the equation of the line whose slope is $\frac{5}{3}$ and whose y-intercept is (0, 2).

Your solution

Example 2

Use the point-slope formula to find the equation of the line that passes through the point (−2, −1) and has slope $\frac{3}{2}$.

Solution

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = \frac{3}{2}[x - (-2)]$$ • $m = \frac{3}{2}$; $(x_1, y_1) = (-2, -1)$

$$y + 1 = \frac{3}{2}(x + 2)$$

$$y + 1 = \frac{3}{2}x + 3$$

$$y = \frac{3}{2}x + 2$$

The equation of the line is $y = \frac{3}{2}x + 2$.

You Try It 2

Use the point-slope formula to find the equation of the line that passes through the point (4, −2) and has slope $\frac{3}{4}$.

Your solution

Solutions on p. S19

Objective B To find the equation of a line given two points

The point-slope formula is used to find the equation of a line when a point on the line and the slope of the line are known. But this formula can also be used to find the equation of a line given two points on the line. In this case,

1. Use the slope formula to determine the slope of the line between the points.
2. Use the point-slope formula, the slope you just calculated, and one of the given points to find the equation of the line.

Find the equation of the line that passes through the points whose coordinates are $(-3, -1)$ and $(3, 3)$.

Use the slope formula to determine the slope of the line between the points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-1)}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}$$

• $(x_1, y_1) = (-3, -1)$; $(x_2, y_2) = (3, 3)$

Use the point-slope formula, the slope you just calculated, and one of the given points to find the equation of the line.

$$\begin{aligned} y - y_1 &= m(x - x_1) && \bullet \text{ Point-slope formula} \\ y - (-1) &= \frac{2}{3}[x - (-3)] && \bullet\ m = \frac{2}{3};\ (x_1, y_1) = (-3, -1) \\ y + 1 &= \frac{2}{3}(x + 3) \\ y + 1 &= \frac{2}{3}x + 2 \\ y &= \frac{2}{3}x + 1 \end{aligned}$$

Check:

$$\begin{array}{c|c} \multicolumn{2}{c}{y = \frac{2}{3}x + 1} \\ \hline -1 & \frac{2}{3}(-3) + 1 \\ -1 & -2 + 1 \\ \multicolumn{2}{l}{-1 = -1} \end{array}$$

• $(x, y) = (-3, -1)$

$$\begin{array}{c|c} \multicolumn{2}{c}{y = \frac{2}{3}x + 1} \\ \hline 3 & \frac{2}{3}(3) + 1 \\ 3 & 2 + 1 \\ \multicolumn{2}{l}{3 = 3} \end{array}$$

• $(x, y) = (3, 3)$

The equation of the line that passes through the two points is $y = \frac{2}{3}x + 1$.

TAKE NOTE
You can verify that the equation $y = \frac{2}{3}x + 1$ passes through the points $(-3, -1)$ and $(3, 3)$ by substituting the coordinates of these points into the equation.

Example 3
Find the equation of the line that passes through the points $(-4, 0)$ and $(2, -3)$.

Solution
Find the slope of the line between the two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 0}{2 - (-4)} = \frac{-3}{6} = -\frac{1}{2}$$

Use the point-slope formula.

$$\begin{aligned} y - y_1 &= m(x - x_1) && \bullet \text{ Point-slope formula} \\ y - 0 &= -\frac{1}{2}[x - (-4)] && \bullet\ m = -\frac{1}{2};\ (x_1, y_1) = (-4, 0) \\ y &= -\frac{1}{2}(x + 4) \\ y &= -\frac{1}{2}x - 2 \end{aligned}$$

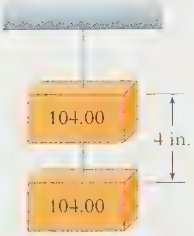
The equation of the line is $y = -\frac{1}{2}x - 2$.

You Try It 3
Find the equation of the line that passes through the points $(-6, -1)$ and $(3, 1)$.

Your solution

Solution on p. S19

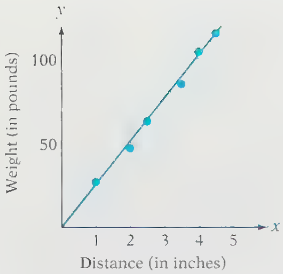
To solve application problems



A **linear model** is a first-degree equation that is used to describe a relationship between quantities. In many cases, a linear model is used to approximate collected data. The data are graphed as points in a coordinate system, and then a line is drawn that approximates the data. The graph of the points is called a **scatter diagram;** the line is called a **line of best fit.**

Consider an experiment to determine the weight required to stretch a spring a certain distance. Data from such an experiment are shown in the table below.

Distance (in inches)	2.5	4	2	3.5	1	4.5
Weight (in pounds)	63	104	47	85	27	115



The accompanying graph shows the scatter diagram, which is the plotted points, and the line of best fit, which is the line that approximately goes through the plotted points. The equation of the line of best fit is $y = 25.6x - 1.3$, where x is the number of inches the spring is stretched and y is the weight in pounds.

The table below shows the values that the model would predict to the nearest tenth. Good linear models should predict values that are close to the actual values. A more thorough analysis of lines of best fit is undertaken in statistics courses.

Distance, x	2.5	4	2	3.5	1	4.5
Weight predicted using $y = 25.6x - 1.3$	63.0	101.1	49.9	88.3	24.3	113.9

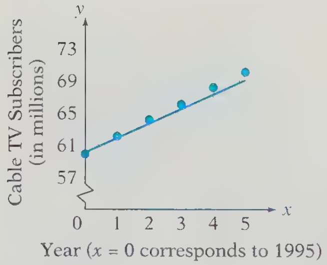
Example 4

The data in the table below show the number of television cable subscribers in the United States for the years 1995 to 2000. The line of best fit is $y = 1.8x + 60.8$, where x is the year (with 1995 corresponding to $x = 0$) and y is the number of cable subscribers in millions.

Year	0	1	2	3	4	5
Subscribers (in millions)	60.9	62.8	64.2	66.1	68.2	70.1

Graph the data and the line of best fit in the coordinate system below. Write a sentence that describes the meaning of the slope of the line.

Solution



The slope of the line means that the number of cable subscribers is increasing by 1.8 million per year.

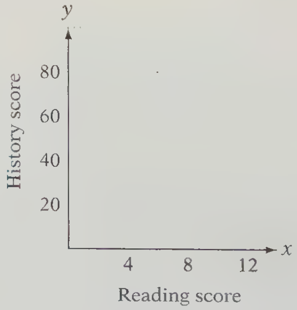
You Try It 4

The data in the table below show a reading test grade and the final exam grade in a history class. The line of best fit is $y = 8.3x - 7.8$, where x is the reading test score and y is the history test score.

Reading	8.5	9.4	10.0	11.4	12.0
History	64	68	76	87	92

Graph the data and the line of best fit in the coordinate system below. Write a sentence that describes the meaning of the slope of the line of best fit.

Your solution



Solution on p. S19

7.4 Exercises

Objective A

1. What is the point-slope formula and how is it used?

3. Find the equation of the line that contains the point (0, 2) and has slope 2.

5. Find the equation of the line that contains the point (−1, 2) and has slope −3.

7. Find the equation of the line that contains the point (3, 1) and has slope $\frac{1}{3}$.

9. Find the equation of the line that contains the point (4, −2) and has slope $\frac{3}{4}$.

11. Find the equation of the line that contains the point (5, −3) and has slope $-\frac{3}{5}$.

13. Find the equation of the line that contains the point (2, 3) and has slope $\frac{1}{4}$.

2. Can the point-slope formula be used to find the equation of any line? If not, equations for which types of lines cannot be found using this formula?

4. Find the equation of the line that contains the point (0, −1) and has slope −2.

6. Find the equation of the line that contains the point (2, −3) and has slope 3.

8. Find the equation of the line that contains the point (−2, 3) and has slope $\frac{1}{2}$.

10. Find the equation of the line that contains the point (2, 3) and has slope $-\frac{1}{2}$.

12. Find the equation of the line that contains the point (5, −1) and has slope $\frac{1}{5}$.

14. Find the equation of the line that contains the point (−1, 2) and has slope $-\frac{1}{2}$.

15. Find the equation of the line that passes through the points $(1, -1)$ and $(-2, -7)$.

16. Find the equation of the line that passes through the points $(2, 3)$ and $(3, 2)$.

17. Find the equation of the line that passes through the points $(-2, 1)$ and $(1, -5)$.

18. Find the equation of the line that passes through the points $(-1, -3)$ and $(2, -12)$.

19. Find the equation of the line that passes through the points $(0, 0)$ and $(-3, -2)$.

20. Find the equation of the line that passes through the points $(0, 0)$ and $(-5, 1)$.

21. Find the equation of the line that passes through the points $(2, 3)$ and $(-4, 0)$.

22. Find the equation of the line that passes through the points $(3, -1)$ and $(0, -3)$.

23. Find the equation of the line that passes through the points $(-4, 1)$ and $(4, -5)$.

24. Find the equation of the line that passes through the points $(-5, 0)$ and $(10, -3)$.

25. Find the equation of the line that passes through the points $(-2, 1)$ and $(2, 4)$.

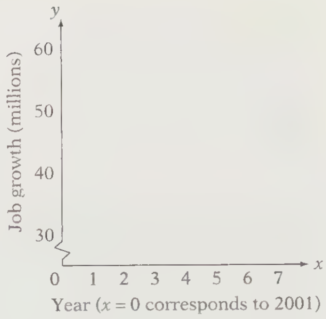
26. Find the equation of the line that passes through the points $(3, -2)$ and $(-3, -3)$.

Application Problems

27. The data in the table below are estimates of the projected annual numbers of job openings for graduates with a 4-year degree. The line of best fit is $y = 4.8x + 29$, where x is the year (with $x = 0$ corresponding to 2001) and y is the number of job openings (in millions) available that year.

Year, x	0	1	2	3	4	5	6	7
Jobs, y (in millions)	30.1	34.1	38.5	42.8	46.5	51.2	59.2	63.6

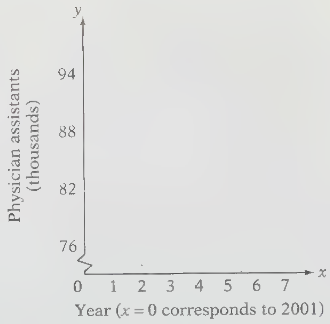
Graph the data and the line of best fit in the coordinate system at the right. Write a sentence that describes the meaning of the slope of the line of best fit.



28. The data in the table below are estimates of the projected annual numbers of job openings for physician assistants. The line of best fit is $y = 3.2x + 76$, where x is the year (with $x = 0$ corresponding to 2001) and y is the number of job openings (in thousands) available that year.

Year, x	0	1	2	3	4	5	6	7
Jobs, y (in thousands)	75.6	78.5	82.1	85.2	88.5	91.7	94.9	98

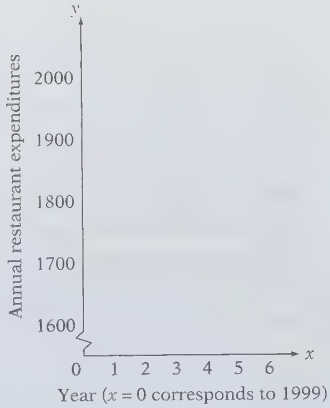
Graph the data and the line of best fit in the coordinate system at the right. Write a sentence that describes the meaning of the slope of the line of best fit.


y
94
88
82
76
Physician assistants (thousands)
0 1 2 3 4 5 6 7
x
Year (x = 0 corresponds to 2001)

29. The data in the table below are estimates of the projected annual amount spent by a family on eating in restaurants. The line of best fit is $y = 80x + 1614$, where x is the year (with $x = 0$ corresponding to 1999) and y is the number of dollars spent on eating in restaurants.

Year, x	0	1	2	3	4	5	6
Spent, y (in dollars)	1615	1690	1772	1855	1938	2015	2085

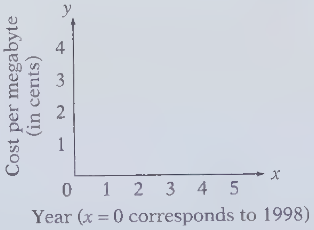
Graph the data and the line of best fit in the coordinate system at the right. Write a sentence that describes the meaning of the slope of the line of best fit.


y
2000
1900
1800
1700
1600
Annual restaurant expenditures
0 1 2 3 4 5 6
x
Year (x = 0 corresponds to 1999)

30. The data in the table below are estimates of the projected cost, in cents, for one megabyte (1,000,000 bytes) of hard-disk storage space. The line of best fit is $y = -0.8x + 4.3$, where x is the year (with $x = 0$ corresponding to 1998) and y is the cost per megabyte for that year.

Year, x	0	1	2	3	4	5
Cost, y (cents per megabyte)	4.3	3.4	2.8	2.0	1.2	0.3

Graph the data and the line of best fit in the coordinate system at the right. Write a sentence that describes the meaning of the slope of the line of best fit.


y
4
3
2
1
Cost per megabyte (in cents)
0 1 2 3 4 5
x
Year (x = 0 corresponds to 1998)

APPLYING THE CONCEPTS

In Exercises 31 to 34, the first two given points are on a line. Determine whether the third point is on the line.

31. $(-3, 2), (4, 1); (-1, 0)$

32. $(2, -2), (3, 4); (-1, 5)$

33. $(-3, -5), (1, 3); (4, 9)$

34. $(-3, 7), (0, -2); (1, -5)$

35. If $(-2, 4)$ are the coordinates of a point on the line whose equation is $y = mx + 1$, what is the slope of the line?

36. If $(3, 1)$ are the coordinates of a point on the line whose equation is $y = mx - 3$, what is the slope of the line?

37. If $(0, -3)$, $(6, -7)$, and $(3, n)$ are coordinates of points on the same line, determine n.

38. If $(-4, 11)$, $(2, -4)$, and $(6, n)$ are coordinates of points on the same line, determine n.

The formula $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$, where $x_1 \neq x_2$, is called the **two-point formula** for a straight line. This formula can be used to find the equation of a line given two points. Use this formula for Exercises 39 and 40.

39. Find the equation of the line passing through $(-2, 3)$ and $(4, -1)$.

40. Find the equation of the line passing through $(3, -1)$ and $(4, -3)$.

41. Explain why the condition $x_1 \neq x_2$ is placed on the two-point formula given above.

42. Explain how the two-point formula given above can be derived from the point-slope formula.

Focus on Problem Solving

Counterexamples

Some of the exercises in this text ask you to determine whether a statement is true or false. For instance, the statement "Every real number has a reciprocal" is false because 0 is a real number and 0 does not have a reciprocal.

Finding an example, such as "0 has no reciprocal," to show that a statement is not always true is called finding a counterexample. A **counterexample** is an example that shows that a statement is not always true.

Consider the statement "The product of two numbers is greater than either factor." A counterexample to this statement is the factors $\frac{2}{3}$ and $\frac{3}{4}$. The product of these numbers is $\frac{1}{2}$, and $\frac{1}{2}$ is smaller than $\frac{2}{3}$ or $\frac{3}{4}$. There are many other counterexamples to the given statement.

Here are some counterexamples to the statement "The square of a number is always larger than the number."

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4} \quad \text{but} \quad \frac{1}{4} < \frac{1}{2} \qquad\qquad 1^2 = 1 \quad \text{but} \quad 1 = 1$$

For each of the next five statements, find at least one counterexample to show that the statement, or conjecture, is false.

1. The product of two integers is always a positive number.
2. The sum of two prime numbers is never a prime number.
3. For all real numbers, $|x + y| = |x| + |y|$.
4. If x and y are nonzero real numbers and $x > y$, then $x^2 > y^2$.
5. The quotient of any two nonzero real numbers is less than either one of the numbers.

When a problem is posed, it may not be known whether the statement is true or false. For instance, Christian Goldbach (1690–1764) stated that every even integer greater than 2 can be written as the sum of two prime numbers. No one has been able to find a counterexample to this statement, but neither has anyone been able to prove that it is always true.

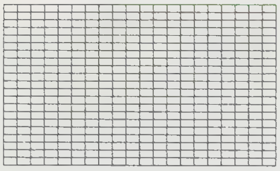
In the next four exercises, answer true if the statement is always true. If there is an instance when the statement is false, give a counterexample.

6. The reciprocal of a positive number is always smaller than the number.
7. If $x < 0$, then $|x| = -x$.
8. For any two real numbers x and y, $x + y > x - y$.
9. The list of numbers 1, 11, 111, 1111, 11111, ... contains infinitely many composite numbers. (*Hint:* A number is divisible by 3 if the sum of the digits of the number is divisible by 3.)

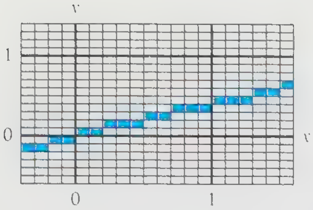
Projects and Group Activities

Graphing Linear Equations with a Graphing Utility

A computer or graphing calculator screen is divided into *pixels*. There are approximately 6000 to 790,000 pixels available on the screen (depending on the computer or calculator). The greater the number of pixels, the smoother a graph will appear. A portion of a screen is shown at the left. Each little rectangle represents one pixel.



The graphing utilities that are used by computers or calculators to graph an equation do basically what we have shown in the text: They choose values of x and, for each, calculate the corresponding value of y. The pixel corresponding to the ordered pair is then turned on. The graph is jagged because pixels are much larger than the dots we draw on paper.



The graph of $y = 0.45x$ is shown at the left as the calculator drew it (jagged). The x- and y-axes have been chosen so that each pixel represents $\frac{1}{10}$ of a unit. Consider the region of the graph where $x = 1$, 1.1, and 1.2.

The corresponding values of y are 0.45, 0.495, and 0.54. Because the y-axis is in tenths, the numbers 0.45, 0.495, and 0.54 are rounded to the nearest tenth before plotting. Rounding 0.45, 0.495, and 0.54 to the nearest tenth results in 0.5 for each number. Thus the ordered pairs (1, 0.45), (1.1, 0.495), and (1.2, 0.54) are graphed as (1, 0.5), (1.1, 0.5), and (1.2, 0.5). These points appear as three illuminated horizontal pixels. The graph of the line appears horizontal. However, if you use the TRACE feature of the calculator (see the Appendix), the actual y-coordinate for each value of x is displayed.

TAKE NOTE

Xmin and Xmax are the smallest and largest values of x that will be shown on the screen. Ymin and Ymax are the smallest and largest values of y that will be shown on the screen.

Here are the keystrokes for a TI-83 calculator to graph $y = \frac{2}{3}x + 1$. First the equation is entered. Then the domain (Xmin to Xmax) and the range (Ymin to Ymax) are entered. This is called the **viewing window.**

$\boxed{\text{Y=}}$ $\boxed{\text{CLEAR}}$ 2 $\boxed{X,T,\Theta,n}$ $\boxed{\div}$ 3 $\boxed{+}$ 1 $\boxed{\text{WINDOW}}$ $\boxed{(-)}$ 10 $\boxed{\text{ENTER}}$ 10 $\boxed{\text{ENTER}}$ 1 $\boxed{\text{ENTER}}$ $\boxed{(-)}$ 10 $\boxed{\text{ENTER}}$ 10 $\boxed{\text{ENTER}}$ 1 $\boxed{\text{ENTER}}$ $\boxed{\text{GRAPH}}$

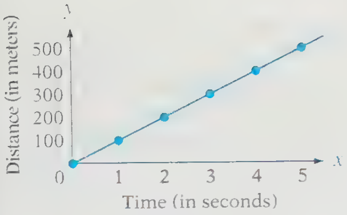
By changing the keystrokes 2 $\boxed{X,T,\Theta,n}$ $\boxed{\div}$ 3 $\boxed{+}$ 1, you can graph different equations.

1. $y = 2x + 1$	For $2x$, you may enter $2 \times x$ or just $2x$. The times sign × is not necessary on many graphing calculators.
2. $y = -\frac{1}{2}x - 2$	Use the $\boxed{(-)}$ key to enter a negative sign.
3. $3x + 2y = 6$	Solve for y. Then enter the equation.
4. $4x + 3y = 75$	You must adjust the viewing window. *Suggestion:* Xmin = −25, Xmax = 25, Xscl = 5, Ymin = −35, Ymax = 35, Yscl = 5. See the Appendix for assistance.

Graphs of Motion

A graph can be useful in analyzing the motion of a body. For example, consider an airplane in uniform motion traveling at 100 m/s. The table at the right shows the distance, in meters, traveled by the plane at the end of each of five one-second intervals.

Time (in seconds)	*Distance (in meters)*
0	0
1	100
2	200
3	300
4	400
5	500



These data can be graphed on a rectangular coordinate system and a straight line drawn through the points plotted. The travel time is shown along the horizontal axis, and the distance traveled by the plane is shown along the vertical axis. (Note that the units along the two axes are not the same length.)

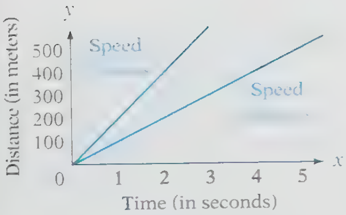
To write the equation for the line just graphed, use the coordinates of any two points on the line to find the slope. The y-intercept is $(0, 0)$.

Let $(x_1, y_1) = (1, 100)$ and $(x_2, y_2) = (2, 200)$.

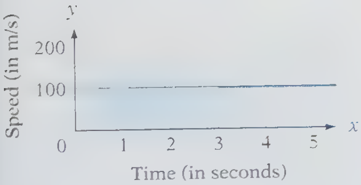
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{200 - 100}{2 - 1} = 100$$

$$y = mx + b$$
$$y = 100x + 0$$
$$y = 100x$$

Note that the slope of the line, 100, is equal to the speed, 100 m/s. *The slope of a distance-time graph represents the speed of the object.*



The distance-time graphs for two planes are shown at the left. One plane is traveling at 100 m/s, and the other is traveling at 200 m/s. The slope of the line representing the faster plane is greater than the slope of the line representing the slower plane.



In the speed-time graph at the left, the time a plane has been flying at 100 m/s is shown along the horizontal axis, and its speed is shown along the vertical axis. Because the speed is constant, the graph is a horizontal line.

The area between the horizontal line graphed and the horizontal axis is equal to the distance traveled by the plane up to that time. For example, the area of the shaded region on the graph is

$$\text{Length} \cdot \text{width} = (3\text{ s})(100\text{ m/s}) = 300\text{ m}$$

The distance traveled by the plane in 3 s is equal to 300 m.

1. A car in uniform motion is traveling at 20 m/s.
   - **a.** Prepare a distance-time graph for the car for 0 s to 5 s.
   - **b.** Find the slope of the line.
   - **c.** Find the equation of the line.
   - **d.** Prepare a speed-time graph for the car for 0 s to 5 s.
   - **e.** Find the distance traveled by the car after 3 s.

2. One car in uniform motion is traveling at 10 m/s. A second car in uniform motion is traveling at 15 m/s.
   - **a.** Prepare one distance-time graph for both cars for 0 s to 5 s.
   - **b.** Find the slope of each line.
   - **c.** Find the equation of each line graphed.
   - **d.** Assuming that the cars started at the same point at 0 s, find the distance between the cars at the end of 5 s.

3.
   - **a.** In a distance-time graph, is it possible for the graph to be a horizontal line?
   - **b.** What does a horizontal line reveal about the motion of the object during that time period?

Chapter Summary

Key Words

A *rectangular coordinate system* is formed by two number lines, one horizontal and one vertical, that intersect at the zero point of each line. [p. 317]

The number lines that make up a rectangular coordinate system are called the *coordinate axes* or simply *axes*. [p. 317]

The *origin* is the point of intersection of the two coordinate axes. [p. 317]

A rectangular coordinate system divides the plane into four regions called *quadrants*. [p. 317]

An *ordered pair* (x, y) is used to locate a point in a plane. [p. 317]

The first number in an ordered pair is called the *abscissa* or x-coordinate. [p. 317, 319]

The second number in an ordered pair is called the *ordinate* or y-coordinate. [p. 317, 319]

The *coordinates* of a point are the numbers in the ordered pair that is associated with the point. [p. 317]

A *relation* is any set of ordered pairs. [p. 321]

A *function* is a relation in which no two ordered pairs have the same first coordinate. [p. 321]

A function designated by $f(x)$ is written in *functional notation*. The *value of the function* at x is $f(x)$. [p. 324]

An equation of the form $y = mx + b$, where m and b are constants, is a *linear equation in two variables* or a *linear function*. [p. 329]

The point at which a graph crosses the x-axis is called the *x-intercept*. [p. 339]

The point at which a graph crosses the y-axis is called the *y-intercept*. [p. 339]

The *slope* of a line is the measure of the slant of a line. The symbol for slope is m. [p. 340]

A line that slants upward to the right has a *positive slope*. [p. 340]

A line that slants downward to the right has a *negative slope*. [p. 341]

A horizontal line has *zero slope*. [p. 341]

The slope of a vertical line is *undefined*. [p. 341]

Two nonvertical lines in the plane are parallel if and only if they have the same slope. Vertical lines in the plane are parallel. [p. 341]

Essential Rules

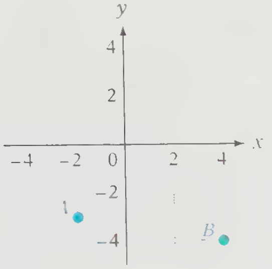
Slope of a straight line Slope $= m = \dfrac{y_2 - y_1}{x_2 - x_1}$ [p. 340]

Slope-intercept form of a straight line $y = mx + b$ [p. 343]

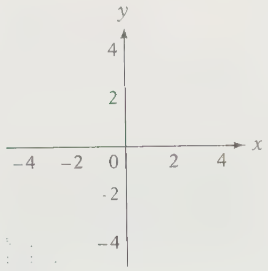
Point-slope formula $y - y_1 = m(x - x_1)$ [p. 350]

Chapter Review

1. **a.** Graph the ordered pairs $(-2, 4)$ and $(3, -2)$.
b. Name the abscissa of point A.
c. Name the ordinate of point B.



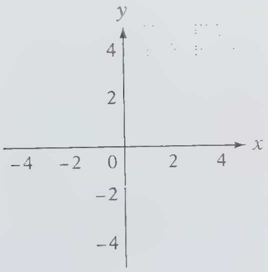
2. Graph the ordered-pair solutions of $y = -\frac{1}{2}x - 2$ when $x \in \{-4, -2, 0, 2\}$.



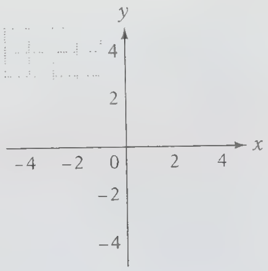
3. Determine the equation of the line that passes through the points $(-1, 3)$ and $(2, -5)$.

4. Determine the equation of the line that passes through the point $(6, 1)$ and has slope $-\frac{5}{2}$.

5. Graph $y = \frac{1}{4}x + 3$.



6. Graph $5x + 3y = 15$.



7. Is the line that passes through $(-2, 3)$ and $(3, 7)$ parallel to the line that passes through $(3, -4)$ and $(5, 0)$?

8. Given $f(x) = x^2 - 2$, find $f(-1)$.

9. Determine the equation of the line that passes through the points $(-2, 5)$ and $(4, 1)$.

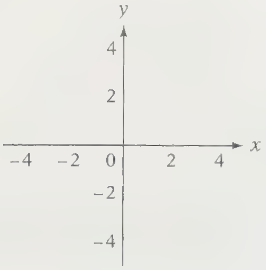
10. Does $y = -x + 3$, where $x \in \{-2, 0, 3, 5\}$, define y as a function of x?

11. Find the slope of the line containing the points $(9, 8)$ and $(-2, 1)$.

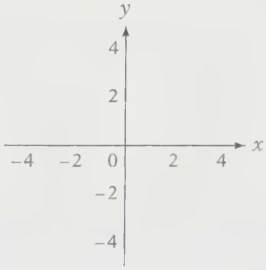
12. Find the x- and y-intercepts of $3x - 2y = 24$.

13. Find the slope of the line containing the points $(-2, -3)$ and $(4, -3)$.

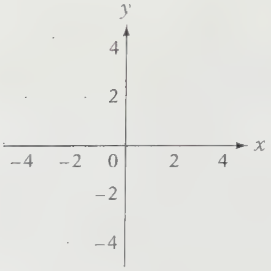
14. Graph the line that has slope $\frac{1}{2}$ and y-intercept $(0, -1)$.



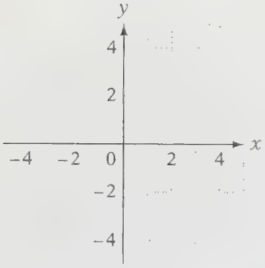
15. Graph $x = -3$.



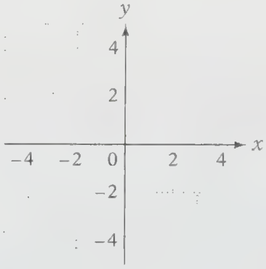
16. Graph the line that has slope $-\frac{2}{3}$ and y-intercept $(0, 2)$.



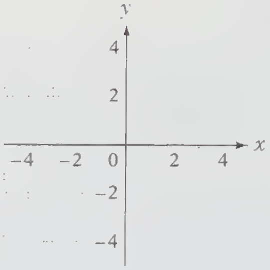
17. Graph $y = -2x - 1$.



18. Graph the line that has slope 2 and y-intercept $(0, -4)$.



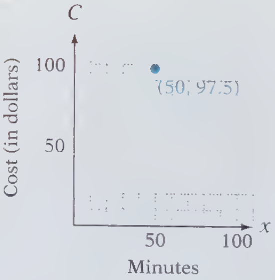
19. Graph $3x - 2y = -6$.



20. The height and weight of 8 seventh-grade students are shown in the following table. Write a relation where the first coordinate is height, in inches, and the second coordinate is weight, in pounds. Is the relation a function?

Height (in inches)	55	57	53	57	60	61	58	54
Weight (in pounds)	95	101	94	98	100	105	97	95

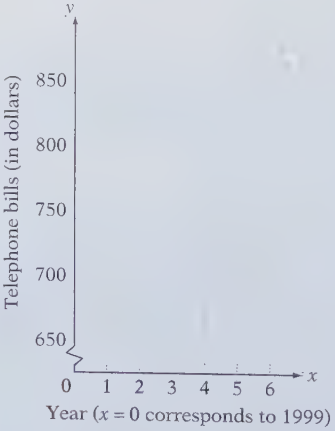
21. An online research service charges a monthly access fee of \$75 plus \$.45 per minute to use the service. An equation that represents the monthly cost to use this service is $C = 0.45x + 75$, where C is the monthly cost and x is the number of minutes. Graph this equation for $0 \leq x \leq 100$. The point $(50, 97.5)$ is on the graph. Write a sentence that describes the meaning of this ordered pair.



22. The data in the table below are estimates of the projected annual increase in average telephone bills for a family. The line of best fit is $y = 34x + 657$, where x is the year (with $x = 0$ corresponding to 1999) and y is the annual cost, in dollars, of telephone bills.

Year, x	0	1	2	3	4	5	6
Telephone bills, y (in dollars)	658	690	708	772	809	830	849

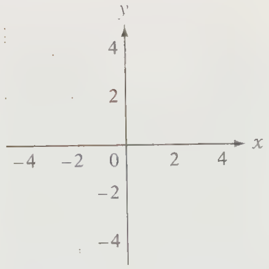
Graph the data and the line of best fit in the coordinate system at the right. Write a sentence that describes the meaning of the slope of the line of best fit.



Chapter Test

1. Find the ordered-pair solution of $2x - 3y = 15$ corresponding to $x = 3$.

2. Graph the ordered-pair solutions of $y = -\frac{3}{2}x + 1$ for $x \in \{-2, 0, 4\}$.



3. Does $y = \frac{1}{2}x - 3$ define y as a function of x for $x \in \{-2, 0, 4\}$?

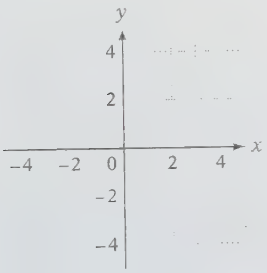
4. Given $f(t) = t^2 + t$, find $f(2)$.

5. Given $f(x) = x^2 - 2x$, find $f(-1)$.

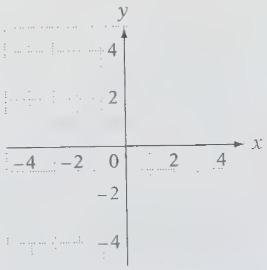
6. The distance a house is from a fire station and the amount of damage that the house sustained in a fire are given in the following table. Write a relation wherein the first coordinate of the ordered pair is the distance, in miles, from the fire station and the second coordinate is the amount of damage in thousands of dollars. Is the relation a function?

Distance (in miles)	3.5	4.0	5.2	5.0	4.0	6.3	5.4
Damage (in thousands of dollars)	25	30	45	38	42	12	34

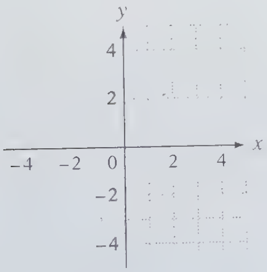
7. Graph $y = 3x + 1$.



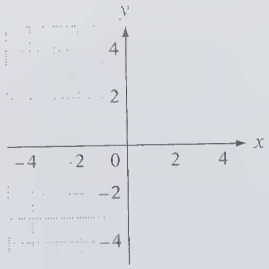
8. Graph $y = -\frac{3}{4}x + 3$.



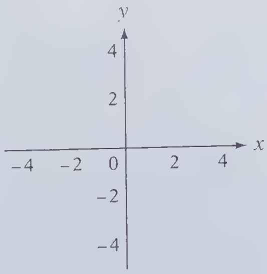
9. Graph $3x - 2y = 6$.



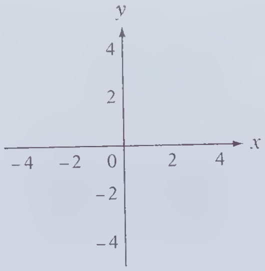
10. Graph $x + 3 = 0$.



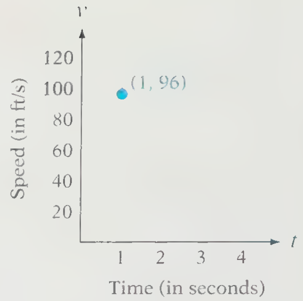
11. Graph the line that has slope $-\frac{2}{3}$ and y-intercept $(0, 4)$.



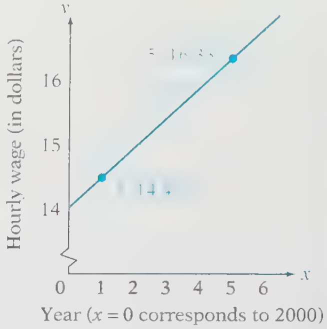
12. Graph the line that has slope 2 and y-intercept -2.



13. The equation for the speed of a ball that is thrown straight up with an initial speed of 128 ft/s is $v = 128 - 32t$, where v is the speed of the ball after t seconds. Graph this equation for $0 \le t \le 4$. The point whose coordinates are (1, 96) is on the graph. Write a sentence that describes this ordered pair.



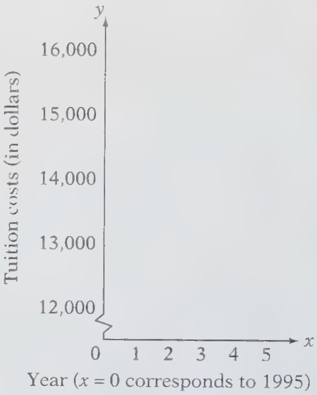
14. The graph at the right shows the projected increase in the average hourly wage of a U.S. worker for the years 2000 through 2006 (with $x = 0$ corresponding to 2000). Find the slope of the line. Write a sentence that states the meaning of the slope.



15. The data in the table below are the average annual tuition costs for 4-year private colleges in the United States. The line of best fit is $y = 809x + 12{,}195$, where x is the year (with $x = 0$ corresponding to 1995) and y is the annual tuition cost in dollars rounded to the nearest 100.

Year, x	0	1	2	3	4	5
Tuition Costs, y (in dollars)	12,400	12,800	13,700	14,700	15,400	16,300

Graph the data and the line of best fit in the coordinate system at the right. Write a sentence that describes the meaning of the slope of the line of best fit.



16. Find the x- and y-intercepts for $6x - 4y = 12$.

17. Find the x- and y-intercepts for $y = \frac{1}{2}x + 1$.

18. Find the slope of the line containing the points $(2, -3)$ and $(4, 1)$.

19. Is the line that passes through (2, 5) and $(-1, 1)$ parallel to the line that passes through $(-2, 3)$ and (4, 11)?

20. Find the slope of the line containing the points $(-5, 2)$ and $(-5, 7)$.

21. Find the slope of the line whose equation is $2x + 3y = 6$.

22. Find the equation of the line that contains the point $(0, -1)$ and has slope 3.

23. Find the equation of the line that contains the point $(-3, 1)$ and has slope $\frac{2}{3}$.

24. Find the equation of the line that passes through the points $(5, -4)$ and $(-3, 1)$.

25. Find the equation of the line that passes through the points $(-2, 0)$ and $(5, -2)$.

Cumulative Review

1. Simplify: $12 - 18 \div 3 \cdot (-2)^2$

2. Evaluate $\dfrac{a-b}{a^2-c}$ when $a = -2$, $b = 3$, and $c = -4$.

3. Given $f(x) = \dfrac{2}{x-1}$, find $f(-2)$.

4. Solve: $2x - \dfrac{2}{3} = \dfrac{7}{3}$

5. Solve: $3x - 2[x - 3(2 - 3x)] = x - 7$

6. Write $6\frac{2}{3}\%$ as a fraction.

7. Simplify: $(-2x^2y)^3(2xy^2)^2$

8. Simplify: $\dfrac{-15x^7}{5x^5}$

9. Divide: $(x^2 - 4x - 21) \div (x - 7)$

10. Factor: $5x^2 + 15x + 10$

11. Factor: $x(a + 2) + y(a + 2)$

12. Solve: $x(x - 2) = 8$

13. Simplify: $\dfrac{x^5y^3}{x^2 - x - 6} \cdot \dfrac{x^2 - 9}{x^2y^4}$

14. Simplify: $\dfrac{3x}{x^2 + 5x - 24} - \dfrac{9}{x^2 + 5x - 24}$

15. Solve: $3 - \dfrac{1}{x} = \dfrac{5}{x}$

16. Solve $4x - 5y = 15$ for y.

17. Find the ordered-pair solution of $y = 2x - 1$ corresponding to $x = -2$.

18. Find the slope of the line that contains the points (2, 3) and (−2, 3).

19. Find the equation of the line that contains the point (2, −1) and has slope $\frac{1}{2}$.

20. Find the equation of the line that contains the point (0, 2) and has slope −3.

21. Find the equation of the line that contains the point (−1, 0) and has slope 2.

22. Find the equation of the line that contains the point (6, 1) and has slope $\frac{2}{3}$.

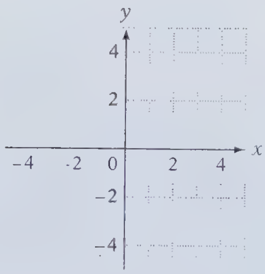
23. A suit that regularly sells for $89 is on sale for 30% off the regular price. Find the sale price.

24. The first angle of a triangle is 3° more than the measure of the second angle. The third angle is 5° more than twice the measure of the second angle. Find the measure of each angle.

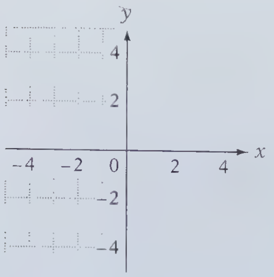
25. The real estate tax for a home that costs $50,000 is $625. At this rate, what is the value of a home for which the real estate tax is $1375?

26. An electrician requires 6 h to wire a garage. An apprentice can do the same job in 10 h. How long would it take to wire the garage if both the electrician and the apprentice were working?

27. Graph $y = \frac{1}{2}x - 1$.



28. Graph the line that has slope $-\frac{2}{3}$ and y-intercept 2.



Chapter 8

Systems of Linear Equations

Objectives

Section 8.1

A To solve a system of linear equations by graphing

Section 8.2

A To solve a system of linear equations by the substitution method

B To solve investment problems

Section 8.3

A To solve a system of linear equations by the addition method

Section 8.4

A To solve rate-of-wind or rate-of-current problems

B To solve application problems using two variables

Financial managers recommend that a client's investment portfolio be diversified in an effort to manage risk while (hopefully!) earning a high return. This results in investing money in different types of accounts with varying interest rates. A system of equations can be used to help determine how best to distribute money over various investment choices, as shown in the **Example on page 381**.

Need help? For online student resources, such as section quizzes, visit this textbook's website at **math.college.hmco.com/students.**

Prep Test

1. Solve $3x - 4y = 24$ for y.

2. Solve: $50 + 0.07x = 0.05(x + 1400)$

3. Simplify: $-3(2x - 7y) + 3(2x + 4y)$

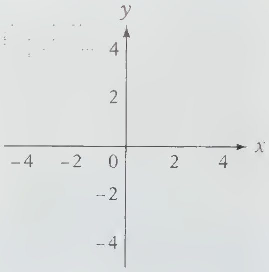
4. Simplify: $4x + 2(3x - 5)$

5. Is $(-4, 2)$ a solution of $3x - 5y = -22$?

6. Find the x- and y-intercepts for $3x - 4y = 12$.

7. Are the graphs of $3x + y = 6$ and $y = -3x - 4$ parallel?

8. Graph: $y = \frac{5}{4}x - 2$



9. A pharmacist has 20 ml of an 80% acetic acid solution. How many milliliters of a 55% acetic acid solution should be mixed with the 20-milliliter solution to produce a solution that is 75% acetic acid?

10. One hiker starts along a trail walking at 3 mph. One-half hour later, another hiker starts on the same walking trail at a speed of 4 mph. How long after the second hiker started will the two hikers be side-by-side?

Go Figure

Two children are running from school to home. Carla runs half the time and walks half the time. James runs half the distance and walks half the distance. Assuming the children run and walk at the same rate, which child arrives home first?

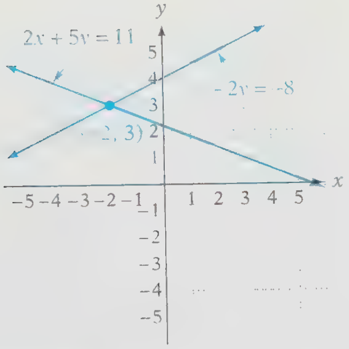
8.1 Solving Systems of Linear Equations by Graphing

[illegible] To solve a system of linear equations by graphing 8

Two or more equations considered together are called a **system of equations.** Three examples of *linear* systems of equations in *two* variables are shown below, along with the graphs of the equations of each system.

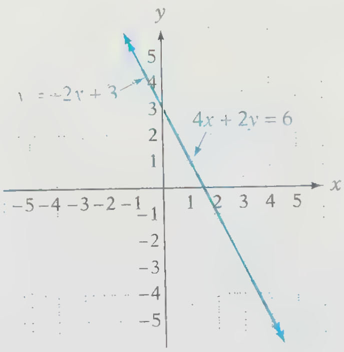
System I

$$x - 2y = -8$$
$$2x + 5y = 11$$



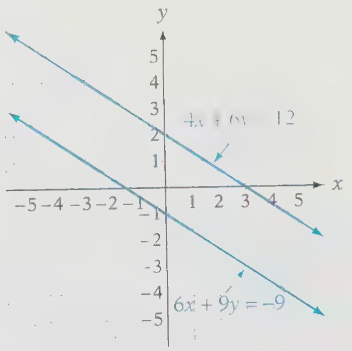
System II

$$4x + 2y = 6$$
$$y = -2x + 3$$



System III

$$4x + 6y = 12$$
$$6x + 9y = -9$$



TAKE NOTE
The systems of equations above are *linear systems of equations* because each of the equations in the system has a graph that is a line. Also, each equation has two variables. In future math courses, you will study equations that contain more than two variables.

For system I, the two lines intersect at a single point whose coordinates are (−2, 3). Because this point lies on both lines, it is a solution of each equation of the system of equations. We can check this by replacing x by −2 and y by 3. The check is shown below.

$$\begin{array}{r|l} \multicolumn{2}{c}{x - 2y = -8} \\ \hline -2 - 2(3) & -8 \\ -2 - 6 & -8 \\ \multicolumn{2}{r}{-8 = -8 \ \surd} \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{2x + 5y = 11} \\ \hline 2(-2) + 5(3) & 11 \\ -4 + 15 & 11 \\ \multicolumn{2}{r}{11 = 11 \ \surd} \end{array}$$

- Replace x by −2 and replace y by 3.

A **solution of a system of equations in two variables** is an ordered pair that is a solution of each equation of the system. The ordered pair (−2, 3) is a solution of system I.

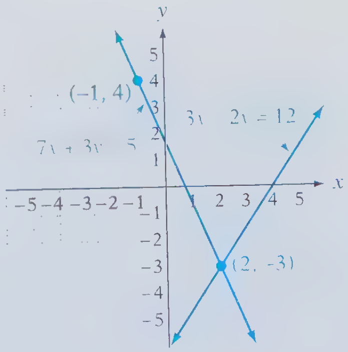
Is (−1, 4) a solution of the system of equations?

$$7x + 3y = 5$$
$$3x - 2y = 12$$

$$\begin{array}{r|l} \multicolumn{2}{c}{7x + 3y = 5} \\ \hline 7(-1) + 3(4) & 5 \\ -7 + 12 & 5 \\ \multicolumn{2}{r}{5 = 5 \ \surd} \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{3x - 2y = 12} \\ \hline 3(-1) - 2(4) & 12 \\ -3 - 8 & 12 \\ \multicolumn{2}{r}{-11 \neq 12} \end{array} \quad \text{Does not check}$$

- Replace x by −1 and replace y by 4.

Because (−1, 4) is not a solution of both equations, (−1, 4) is not a solution of the system of equations.



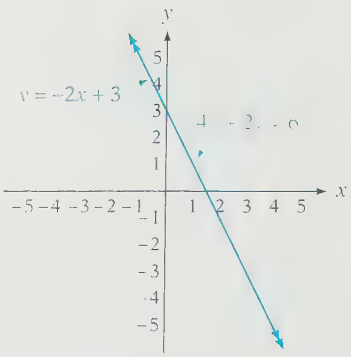
Using the system of equations above and the graph at the right, note that the graph of the ordered pair (−1, 4) lies on the graph of $7x + 3y = 5$ but not on *both* lines. The ordered pair (−1, 4) is *not* a solution of the system of equations. The graph of the ordered pair (2, −3) does lie on both lines and therefore the ordered pair (2, −3) is a solution of the system of equations.

TAKE NOTE

The fact that there are an infinite number of ordered pairs that are solutions of the system at the right does not mean *every* ordered pair is a solution. For instance, (0, 3), (−2, 7), and (2, −1) are solutions. However, (3, 1), (−1, 4), and (1, 6) are not solutions. You should verify these statements.

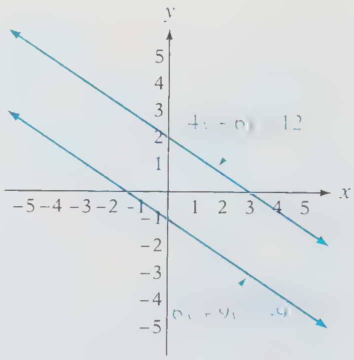
System II from the preceding page and the graph of the equations of that system are shown again at the right. Note that the graph of $y = -2x + 3$ lies directly on top of the graph of $4x + 2y = 6$. Thus the two lines intersect at an infinite number of points. The graphs intersect at an infinite number of points, so there are an infinite number of solutions of this system of equations. Because each equation represents the same set of points, the solutions of the system of equations can be stated by using the ordered pairs of either one of the equations. Therefore, we can say, "The solutions are the ordered pairs that satisfy $4x + 2y = 6$," or we can say "The solutions are the ordered pairs that satisfy $y = -2x + 3$."

$$\begin{aligned} 4x + 2y &= 6 \\ y &= -2x + 3 \end{aligned}$$

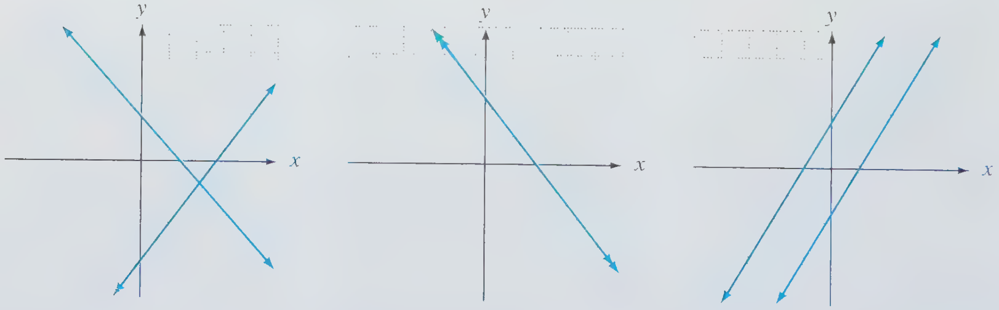


System III from the preceding page and the graph of the equations of that system are shown again at the right. Note that in this case, the graphs of the lines are parallel and do not intersect. Because the graphs do not intersect, there is no point that is on both lines. Therefore, the system of equations has no solution.

$$\begin{aligned} 4x + 6y &= 12 \\ 6x + 9y &= -9 \end{aligned}$$



For a system of linear equations in two variables, the graphs can intersect at one point, the graphs can intersect at infinitely many points (the graphs are the same line), or the lines can be parallel and never intersect. Such systems are called **independent, dependent,** and **inconsistent,** respectively.



Independent:
one solution

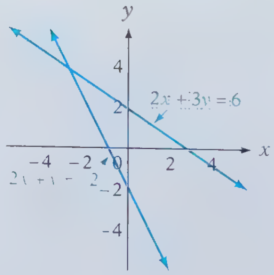
Dependent:
infinitely many solutions

Inconsistent:
no solutions

The graphs of the equations for the system of equations below are shown at the right. What is the solution of the system of equations?

$$\begin{aligned} 2x + 3y &= 6 \\ 2x + y &= -2 \end{aligned}$$

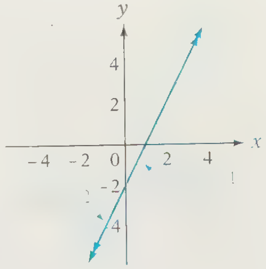
The graphs intersect at $(-3, 4)$. Therefore, the solution of the system of equations is $(-3, 4)$. This is an *independent* system of equations.



TAKE NOTE
Because both equations represent the same ordered pairs, we can also say that the solutions of the system of equations are the ordered pairs that satisfy $x = \frac{1}{2}y + 1$. Either answer is correct.

The graphs of the equations for the system of equations below are shown at the right. What is the solution of the system of equations?

$$y = 2x - 2$$
$$x = \frac{1}{2}y + 1$$



The two graphs lie directly on top of one another. Thus the two lines intersect at an infinite number of points, and the system of equations has an infinite number of solutions. The solutions of the system of equations are the ordered pairs that satisfy $y = 2x - 2$. This is a *dependent* system of equations.

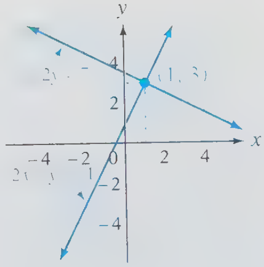
TAKE NOTE
The Projects and Group Activities at the end of this chapter discusses using a calculator to approximate the solution of an independent system of equations.

Solving a system of equations means finding the ordered-pair solutions of the system. One way to do this is to draw the graphs of the equations in the system of equations and determine where the graphs intersect.

To solve a system of equations by graphing, draw the graph of each equation in the system of equations, and then determine the points of intersection.

Solve by graphing: $2x - y = -1$
$x + 2y = 7$

Graph each line.



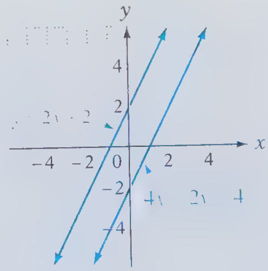
The point of intersection of the two graphs lies on both lines and is therefore the solution of the system of equations.

(1, 3) is a solution of each equation. The system of equations is *independent*.

The solution is (1, 3).

Solve by graphing: $y = 2x + 2$
$4x - 2y = 4$

Graph each line.



The graphs do not intersect. The system of equations is *inconsistent*.

The system of equations has no solution.

Example 1

Is $(1, -3)$ a solution of the following system?

$$\begin{aligned} 3x + 2y &= -3 \\ x - 3y &= 6 \end{aligned}$$

Solution

$$\begin{array}{r|l} \multicolumn{2}{c}{3x + 2y = -3} \\ \hline 3 \cdot 1 + 2(-3) & -3 \\ 3 + (-6) & -3 \\ \multicolumn{2}{r}{-3 = -3} \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{x - 3y = 6} \\ \hline 1 - 3(-3) & 6 \\ 1 - (-9) & 6 \\ \multicolumn{2}{r}{10 \neq 6} \end{array}$$

No, $(1, -3)$ is not a solution of the system of equations.

You Try It 1

Is $(-1, -2)$ a solution of the following system?

$$\begin{aligned} 2x - 5y &= 8 \\ -x + 3y &= -5 \end{aligned}$$

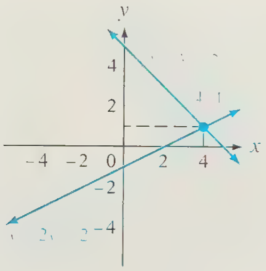
Your solution

Example 2

Solve by graphing:

$$\begin{aligned} x - 2y &= 2 \\ x + y &= 5 \end{aligned}$$

Solution



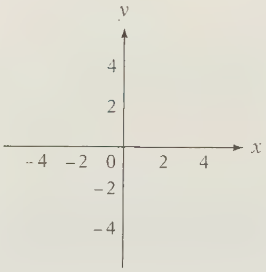
The solution is $(4, 1)$.

You Try It 2

Solve by graphing:

$$\begin{aligned} x + 3y &= 3 \\ -x + y &= 5 \end{aligned}$$

Your solution

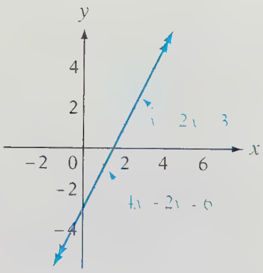


Example 3

Solve by graphing:

$$\begin{aligned} 4x - 2y &= 6 \\ y &= 2x - 3 \end{aligned}$$

Solution



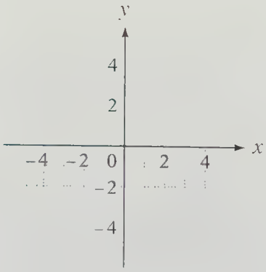
The solutions are the ordered pairs that satisfy the equation $y = 2x - 3$.

You Try It 3

Solve by graphing:

$$\begin{aligned} y &= 3x - 1 \\ 6x - 2y &= -6 \end{aligned}$$

Your solution

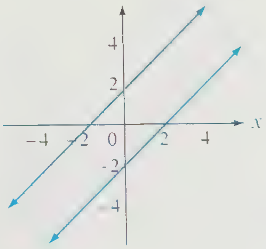


Solutions on p. S19

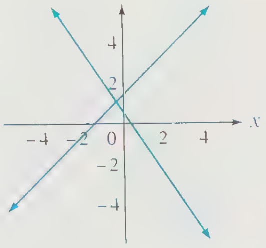
8.1 Exercises

For Exercises 1 and 2, match each system of equations (systems I, II, and III) with **a.** independent, **b.** dependent, or **c.** inconsistent.

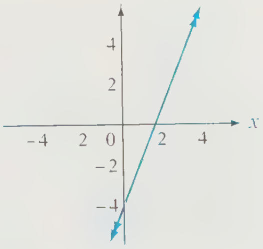
1. I



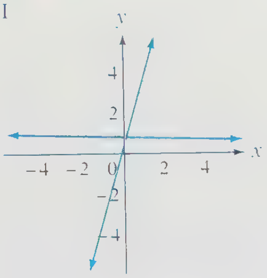
II



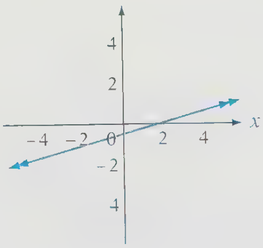
III



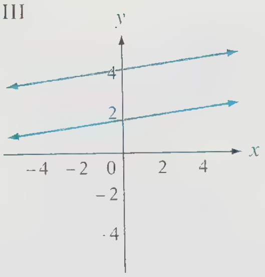
2. I



II

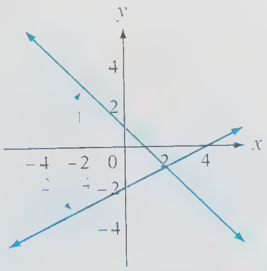


III

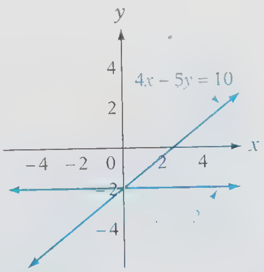


For Exercises 3 to 10, use the graphs of the equations of the system of equations to find the solution of the system of equations.

3.



4.



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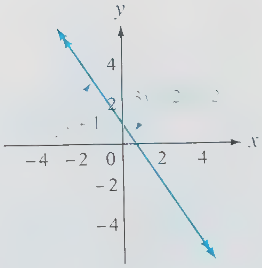

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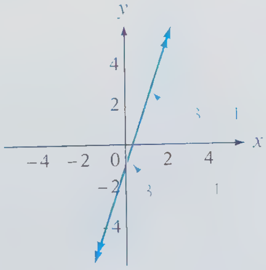

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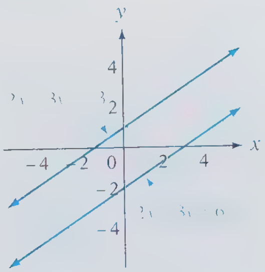

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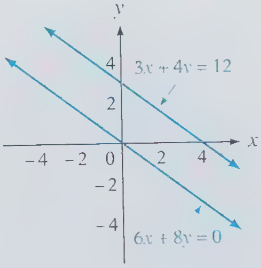
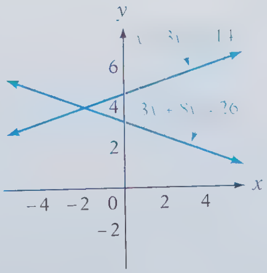
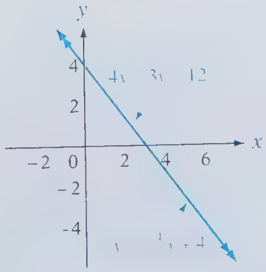


9.



10.



11. Is (2, 3) a solution of $\begin{aligned} 3x + 4y &= 18 \\ 2x - y &= 1 \end{aligned}$?

12. Is (2, −1) a solution of $\begin{aligned} x - 2y &= 4 \\ 2x + y &= 3 \end{aligned}$?

13. Is (4, 3) a solution of $\begin{aligned} 5x - 2y &= 14 \\ x + y &= 8 \end{aligned}$?

14. Is (2, 5) a solution of $\begin{aligned} 3x + 2y &= 16 \\ 2x - 3y &= 4 \end{aligned}$?

15. Is (2, −3) a solution of $\begin{aligned} y &= 2x - 7 \\ 3x - y &= 9 \end{aligned}$?

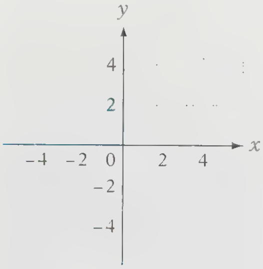
16. Is (−1, −2) a solution of $\begin{aligned} 3x - 4y &= 5 \\ y &= x - 1 \end{aligned}$?

17. Is (0, 0) a solution of $\begin{aligned} 3x + 4y &= 0 \\ y &= x \end{aligned}$?

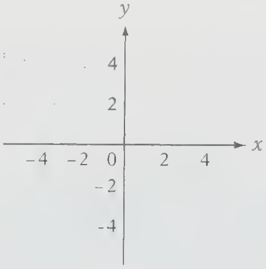
18. Is (3, −4) a solution of $\begin{aligned} 5x - 2y &= 23 \\ 2x - 5y &= 25 \end{aligned}$?

Solve by graphing.

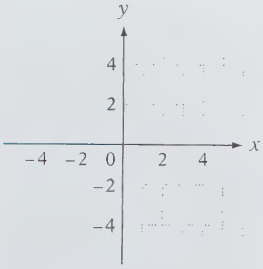
19. $\begin{aligned} x - y &= 3 \\ x + y &= 5 \end{aligned}$



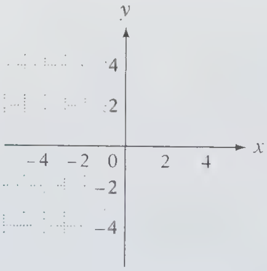
20. $\begin{aligned} 2x - y &= 4 \\ x + y &= 5 \end{aligned}$



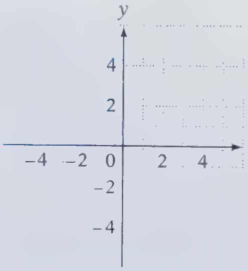
21. $\begin{aligned} x + 2y &= 6 \\ x - y &= 3 \end{aligned}$



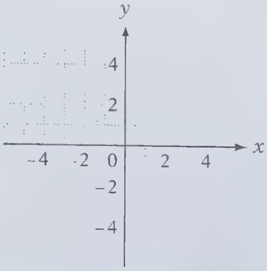
22. $\begin{aligned} 3x - y &= 3 \\ 2x + y &= 2 \end{aligned}$



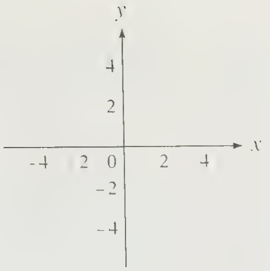
23. $\begin{aligned} 3x - 2y &= 6 \\ y &= 3 \end{aligned}$



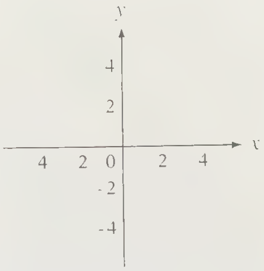
24. $\begin{aligned} x &= 2 \\ 3x + 2y &= 4 \end{aligned}$



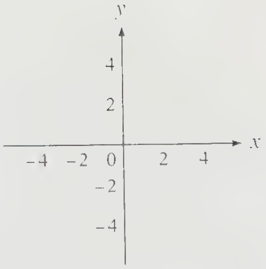
25. $x = 3$
$y = -2$



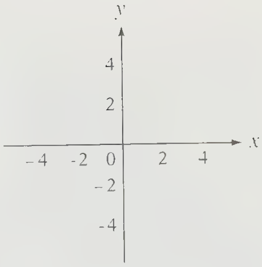
26. $x + 1 = 0$
$y - 3 = 0$



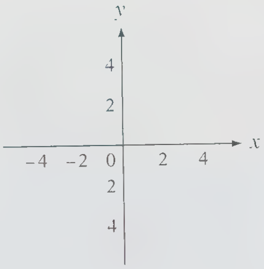
27. $y = 2x - 6$
$x + y = 0$



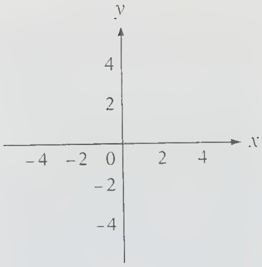
28. $5x - 2y = 11$
$y = 2x - 5$



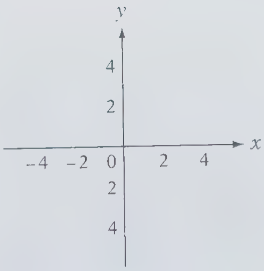
29. $2x + y = -2$
$6x + 3y = 6$



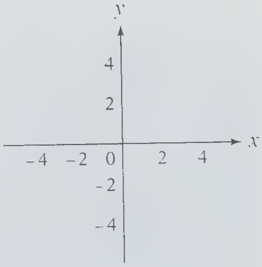
30. $x + y = 5$
$3x + 3y = 6$



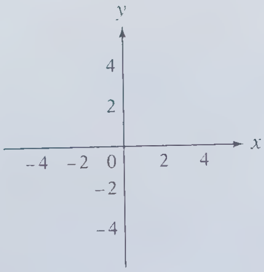
31. $y = 2x - 2$
$4x - 2y = 4$



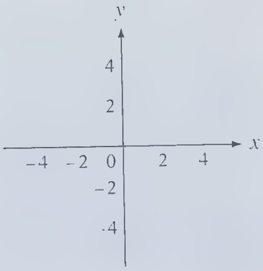
32. $y = -\frac{1}{3}x + 1$
$2x + 6y = 6$



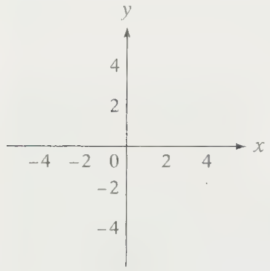
33. $x - y = 5$
$2x - y = 6$



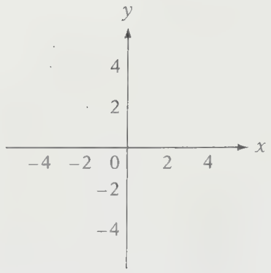
34. $5x - 2y = 10$
$3x + 2y = 6$



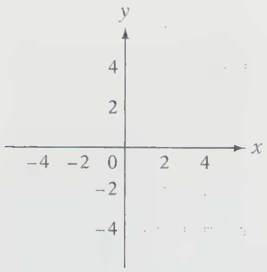
35. $3x + 4y = 0$
$2x - 5y = 0$



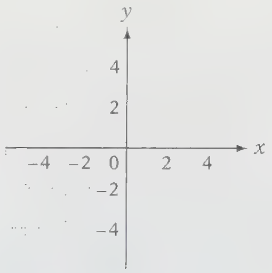
36. $2x - 3y = 0$
$y = -\frac{1}{3}x$



37. $x - 3y = 3$
$2x - 6y = 12$



38. $4x + 6y = 12$
$6x + 9y = 18$



APPLYING THE CONCEPTS

39. Determine whether the statement is always true, sometimes true, or never true.
- **a.** A solution of a system of two equations in two variables is a point in the plane.
- **b.** Two parallel lines have the same slope.
- **c.** Two different lines with the same y-intercept are parallel.
- **d.** Two different lines with the same slope are parallel.

40. Explain how you can determine from the graph of a system of two linear equations in two variables whether it is an independent system of equations.

41. Explain how you can determine from the graph of a system of two linear equations in two variables whether it is an inconsistent system of equations.

42. Write a system of equations that has $(-2, 4)$ as its only solution.

43. Write a system of equations for which there is no solution.

44. Write a system of equations that is a dependent system of equations.

8.2 Solving Systems of Linear Equations by the Substitution Method

[illegible]

To solve a system of linear equations by the substitution method 8

A graphical solution of a system of equations is based on approximating the coordinates of a point of intersection. An algebraic method called the **substitution method** can be used to find an exact solution of a system of equations.

Solve by the substitution method: (1) $2x + 5y = -11$
(2) $y = 3x - 9$

Equation (2) states that $y = 3x - 9$. Substitute $3x - 9$ for y in Equation (1). Then solve for x.

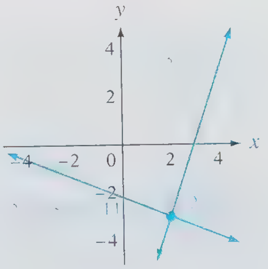
$$\begin{aligned} 2x + 5y &= -11 && \bullet\ \text{This is Equation (1).} \\ 2x + 5(3x - 9) &= -11 && \bullet\ \text{From Equation (2), substitute } 3x - 9 \text{ for } y. \\ 2x + 15x - 45 &= -11 && \bullet\ \text{Solve for } x. \\ 17x - 45 &= -11 \\ 17x &= 34 \\ x &= 2 \end{aligned}$$

Now substitute the value of x into Equation (2) and solve for y.

$$\begin{aligned} y &= 3x - 9 && \bullet\ \text{This is Equation (2).} \\ y &= 3(2) - 9 && \bullet\ \text{Substitute 2 for } x. \\ y &= 6 - 9 = -3 \end{aligned}$$

The solution is the ordered pair $(2, -3)$.

The graph of the equations in this system of equations is shown at the right. Note that the lines intersect at the point whose coordinates are $(2, -3)$, which is the algebraic solution we determined by the substitution method.



To solve a system of equations by the substitution method, we may need to solve one of the equations in the system of equations for one of its variables. For instance, the first step in solving the system of equations

$$\begin{aligned} (1) \quad x + 2y &= -3 \\ (2) \quad 2x - 3y &= 5 \end{aligned}$$

is to solve an equation of the system for one of its variables. Either equation can be used.

Solving Equation (1) for x:	Solving Equation (2) for x:
$x + 2y = -3$	$2x - 3y = 5$
$x = -2y - 3$	$2x = 3y + 5$
	$x = \dfrac{3y + 5}{2} = \dfrac{3}{2}y + \dfrac{5}{2}$

Because solving Equation (1) for x does not result in fractions, it is the easier of the two equations to use.

Here is the solution of the system of equations given on the preceding page.

Solve by the substitution method: (1) $x + 2y = -3$
(2) $2x - 3y = 5$

To use the substitution method, we must solve an equation for one of its variables. Equation (1) is used here because solving it for x does not result in fractions.

$$x + 2y = -3$$

(3) $x = -2y - 3$ • Solve for x. This is Equation (3).

Now substitute $-2y - 3$ for x in Equation (2) and solve for y.

$2x - 3y = 5$ • This is Equation (2).

$2(-2y - 3) - 3y = 5$ • From Equation (3), substitute $-2y - 3$ for x.

$-4y - 6 - 3y = 5$ • Solve for y.

$-7y - 6 = 5$

$-7y = 11$

$y = -\frac{11}{7}$

Substitute the value of y into Equation (3) and solve for x.

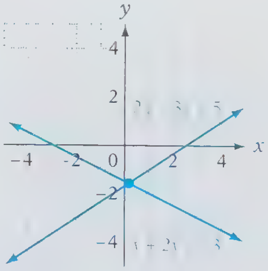
$x = -2y - 3$ • This is Equation (3).

$= -2\left(-\frac{11}{7}\right) - 3$ • Substitute $-\frac{11}{7}$ for y.

$= \frac{22}{7} - 3 = \frac{22}{7} - \frac{21}{7} = \frac{1}{7}$

The solution is $\left(\frac{1}{7}, -\frac{11}{7}\right)$.

The graph of the system of equations given above is shown at the right. It would be difficult to determine the exact solution of this system of equations from the graphs of the equations.



Solve by the substitution method: (1) $y = 3x - 1$
(2) $y = -2x - 6$

$y = -2x - 6$

$3x - 1 = -2x - 6$ • Substitute $3x - 1$ for y in Equation (2).

$5x = -5$ • Solve for x.

$x = -1$

Substitute this value of x into Equation (1) or Equation (2) and solve for y. Equation (1) is used here.

$y = 3x - 1$

$y = 3(-1) - 1 = -4$

The solution is $(-1, -4)$.

The substitution method can be used to analyze inconsistent and dependent systems of equations.

Solve by the substitution method: (1) $2x + 3y = 3$

(2) $y = -\frac{2}{3}x + 3$

$$2x + 3y = 3$$
• This is Equation (1).

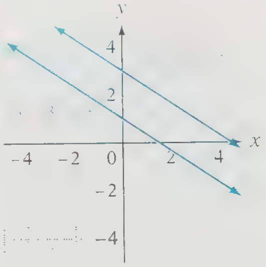
$$2x + 3\left(-\frac{2}{3}x + 3\right) = 3$$
• From Equation (2), replace y with $-\frac{2}{3}x + 3$.

$$2x - 2x + 9 = 3$$
• Solve for x.

$$9 = 3$$
• This is not a true equation.

Because $9 = 3$ is not a true equation, the system of equations has no solution.

Solving Equation (1) above for y, we have $y = -\frac{2}{3}x + 1$. Comparing this with Equation (2) reveals that the slopes are equal and the y-intercepts are different. The graphs of the equations that make up this system of equations are parallel and thus never intersect. Because the graphs do not intersect, there are no solutions of the system of equations. The system of equations is inconsistent.



Solve by the substitution method: (1) $x = 2y + 3$

(2) $4x - 8y = 12$

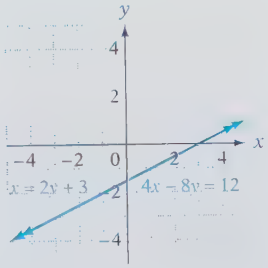
$$4x - 8y = 12$$
• This is Equation (2).

$$4(2y + 3) - 8y = 12$$
• From Equation (1), replace x by $2y + 3$.

$$8y + 12 - 8y = 12$$
• Solve for y.

$$12 = 12$$
• This is a true equation.

The true equation $12 = 12$ indicates that any ordered pair (x, y) that satisfies one equation of the system satisfies the other equation. Therefore, the system of equations has an infinite number of solutions. The solutions are the ordered pairs (x, y) that are solutions of $x = 2y + 3$.



If we write Equation (1) and Equation (2) in slope-intercept form, we have

$$\begin{aligned} x &= 2y + 3 \\ -2y &= -x + 3 \\ y &= \frac{1}{2}x - \frac{3}{2} \end{aligned} \qquad \begin{aligned} 4x - 8y &= 12 \\ -8y &= -4x + 12 \\ y &= \frac{1}{2}x - \frac{3}{2} \end{aligned}$$

The slope-intercept forms of the equations are the same, and therefore the graphs are the same. If we graph these two equations, we essentially graph one over the other. Accordingly, the graphs intersect at an infinite number of points.

TAKE NOTE

As we mentioned in the previous section, when the system of equations is dependent, either equation can be used to write the ordered-pair solutions. Thus we could have said, "The solutions are the ordered pairs (x, y) that are solutions of $4x - 8y = 12$." Also note that, as we show at the right, if we solve each equation for y, the equations have the same slope-intercept form. This means we could also say, "The solutions are the ordered pairs (x, y) that are solutions of $y = \frac{1}{2}x - \frac{3}{2}$." When a system of equations is dependent, there are many ways in which the solutions can be stated.

Example 1 Solve by substitution:

(1) $3x + 4y = -2$

(2) $-x + 2y = 4$

Solution Solve Equation (2) for x.

$$\begin{aligned} -x + 2y &= 4 \\ -x &= -2y + 4 \\ x &= 2y - 4 \end{aligned}$$

Substitute in Equation (1).

$$\begin{aligned} (1) \qquad 3x + 4y &= -2 \\ 3(2y - 4) + 4y &= -2 \\ 6y - 12 + 4y &= -2 \\ 10y - 12 &= -2 \\ 10y &= 10 \\ y &= 1 \end{aligned}$$

Substitute in Equation (2).

$$\begin{aligned} (2) \qquad -x + 2y &= 4 \\ -x + 2(1) &= 4 \\ -x + 2 &= 4 \\ -x &= 2 \\ x &= -2 \end{aligned}$$

The solution is $(-2, 1)$.

You Try It 1 Solve by substitution:

(1) $7x - y = 4$

(2) $3x + 2y = 9$

Your solution

Example 2 Solve by substitution:

$4x + 2y = 5$

$y = -2x + 1$

Solution

$$\begin{aligned} 4x + 2y &= 5 \\ 4x + 2(-2x + 1) &= 5 \\ 4x - 4x + 2 &= 5 \\ 2 &= 5 \end{aligned}$$

This is not a true equation. The system of equations is inconsistent and therefore does not have a solution.

You Try It 2 Solve by substitution:

$3x - y = 4$

$y = 3x + 2$

Your solution

Example 3 Solve by substitution:

$y = 3x - 2$

$6x - 2y = 4$

Solution

$$\begin{aligned} 6x - 2y &= 4 \\ 6x - 2(3x - 2) &= 4 \\ 6x - 6x + 4 &= 4 \\ 4 &= 4 \end{aligned}$$

This is a true equation. The system of equations is dependent. The solutions are the ordered pairs that satisfy the equation $y = 3x - 2$.

You Try It 3 Solve by substitution:

$y = -2x + 1$

$6x + 3y = 3$

Your solution

Solutions on pp. S19–S20

To solve investment problems

The annual simple interest that an investment earns is given by the equation $Pr = I$, where P is the principal, or the amount invested, r is the simple interest rate, and I is the simple interest.

For instance, if you invest \$750 at a simple interest rate of 6%, then the interest earned after 1 year is calculated as follows:

$$Pr = I$$

$$750(0.06) = I \qquad \text{• Replace } P \text{ by 750 and } r \text{ by 0.06 (6\%).}$$

$$45 = I \qquad \text{• Simplify.}$$

The amount of interest earned is \$45.

A medical lab technician decided to open an Individual Retirement Account (IRA) by placing \$2000 in two simple interest accounts. On one account, a corporate bond fund, the annual simple interest rate is 7.5%. On the second account, a real estate investment trust, the annual simple interest rate is 9%. If the technician wants to have annual earnings of \$168 from these two investments, how much must be invested in each account?

Strategy for Solving Simple-Interest Investment Problems

1. For each amount invested, use the equation $Pr = I$. Write a numerical or variable expression for the principal, the interest rate, and the interest earned.

Amount invested at 7.5%: x
Amount invested at 9%: y

	Principal, P	·	Interest rate, r	=	Interest earned, I
Amount at 7.5%	x	·	0.075	=	$0.075x$
Amount at 9%	y	·	0.09	=	$0.09y$

2. Write a system of equations. One equation will express the relationship among the amounts invested. The second equation will express the relationship among the amounts of interest earned by the investments.

The total amount invested is \$2000: $x + y = 2000$
The total annual interest earned is \$168: $0.075x + 0.09y = 168$

Solve the system of equations.

$$(1) \quad x + y = 2000$$

$$(2) \quad 0.075x + 0.09y = 168$$

Solve Equation (1) for y and substitute into Equation (2).

$$(3) \quad y = -x + 2000$$

$$\begin{aligned} 0.075x + 0.09(-x + 2000) &= 168 \\ 0.075x - 0.09x + 180 &= 168 \\ -0.015x &= -12 \\ x &= 800 \end{aligned}$$

Substitute the value of x into Equation (3) and solve for y.

$$y = -x + 2000$$

$$y = -800 + 2000 = 1200$$

The amount invested at 7.5% is \$800. The amount invested at 9% is \$1200.

Example 4

A hair stylist invested some money at an annual simple interest rate of 5.2%. A second investment, \$1000 more then the first, was invested at an annual simple interest rate of 7.2% so that the total interest earned was \$320. How much was invested in each account?

Strategy

- Amount invested at 5.2%: x
  Amount invested at 7.2%: y

	Principal	*Rate*	*Interest*
Amount at 5.2%	x	0.052	$0.052x$
Amount at 7.2%	y	0.072	$0.072y$

- The second investment is \$1000 more than the first investment:

  $y = x + 1000$

  The sum of the interest earned at 5.2% and the interest earned at 7.2% equals \$320.

  $0.052x + 0.072y = 320$

Solution

(1) $y = x + 1000$

(2) $0.052x + 0.072y = 320$

Replace y in Equation (2) by $x + 1000$ from Equation (1). Then solve for x.

$$\begin{aligned} 0.052x + 0.072y &= 320 \\ 0.052x + 0.072(x + 1000) &= 320 \\ 0.052x + 0.072x + 72 &= 320 \\ 0.124x + 72 &= 320 \\ 0.124x &= 248 \\ x &= 2000 \end{aligned}$$

$y = x + 1000 = 2000 + 1000 = 3000$

\$2000 was invested at an annual simple interest rate of 5.2%; \$3000 was invested at 7.2%.

You Try It 4

The manager of a city's investment income wished to place \$330,000 in two simple interest accounts. The first account earns 6.5% annual interest and the second account earns 4.5%. How much should be invested in each account so that both accounts earn the same annual interest?

Your strategy

Your solution

Solution on p. S20

8.2 Exercises

Objective A

1. Describe in your own words the process of solving a system of equations by the substitution method.

2. When you solve a system of equations by the substitution method, how do you determine whether the system of equations is dependent?

Solve by substitution.

3. $\begin{aligned} 2x + 3y &= 7 \\ x &= 2 \end{aligned}$

4. $\begin{aligned} y &= 3 \\ 3x - 2y &= 6 \end{aligned}$

5. $\begin{aligned} y &= x - 3 \\ x + y &= 5 \end{aligned}$

6. $\begin{aligned} y &= x + 2 \\ x + y &= 6 \end{aligned}$

7. $\begin{aligned} x &= y - 2 \\ x + 3y &= 2 \end{aligned}$

8. $\begin{aligned} x &= y + 1 \\ x + 2y &= 7 \end{aligned}$

9. $\begin{aligned} y &= 4 - 3x \\ 3x + y &= 5 \end{aligned}$

10. $\begin{aligned} y &= 2 - 3x \\ 6x + 2y &= 7 \end{aligned}$

11. $\begin{aligned} x &= 3y + 3 \\ 2x - 6y &= 12 \end{aligned}$

12. $\begin{aligned} x &= 2 - y \\ 3x + 3y &= 6 \end{aligned}$

13. $\begin{aligned} 3x + 5y &= -6 \\ x &= 5y + 3 \end{aligned}$

14. $\begin{aligned} y &= 2x + 3 \\ 4x - 3y &= 1 \end{aligned}$

15. $\begin{aligned} 3x + y &= 4 \\ 4x - 3y &= 1 \end{aligned}$

16. $\begin{aligned} x - 4y &= 9 \\ 2x - 3y &= 11 \end{aligned}$

17. $\begin{aligned} 3x - y &= 6 \\ x + 3y &= 2 \end{aligned}$

18. $\begin{aligned} 4x - y &= -5 \\ 2x + 5y &= 13 \end{aligned}$

19. $\begin{aligned} 3x - y &= 5 \\ 2x + 5y &= -8 \end{aligned}$

20. $\begin{aligned} 3x + 4y &= 18 \\ 2x - y &= 1 \end{aligned}$

21. $\begin{aligned} 4x + 3y &= 0 \\ 2x - y &= 0 \end{aligned}$

22. $\begin{aligned} 5x + 2y &= 0 \\ x - 3y &= 0 \end{aligned}$

23. $\begin{aligned} 2x - y &= 2 \\ 6x - 3y &= 6 \end{aligned}$

24. $3x + y = 4$
$9x + 3y = 12$

25. $x = 3y + 2$
$y = 2x + 6$

26. $x = 4 - 2y$
$y = 2x - 13$

27. $y = 2x + 11$
$y = 5x - 19$

28. $y = 2x - 8$
$y = 3x - 13$

29. $y = -4x + 2$
$y = -3x - 1$

30. $x = 3y + 7$
$x = 2y - 1$

31. $x = 4y - 2$
$x = 6y + 8$

32. $x = 3 - 2y$
$x = 5y - 10$

Application Problems

33. An investment of \$3500 is divided between two simple interest accounts. On one account, the annual simple interest rate is 5%, and on the second account, the annual simple interest rate is 7.5%. How much should be invested in each account so that the total interest from the two accounts is \$215?

34. A mortgage broker purchased two trust deeds for a total of \$250,000. One trust deed earns 7% simple annual interest, and the second one earns 8% simple annual interest. If the total annual interest from the two trust deeds is \$18,500, what was the purchase price of each trust deed?

35. When Sara Whitehorse changed jobs, she rolled over the \$6000 in her retirement account into two simple interest accounts. On one account, the annual simple interest rate is 9%; on the second account, the annual simple interest rate is 6%. How much must be invested in each account if the accounts earn the same amount of annual interest?

36. An animal trainer decided to take the \$15,000 won on a game show and deposit it in two simple interest accounts. Part of the winnings were placed in an account paying 7% annual simple interest, and the remainder was used to purchase a government bond that earns 6.5% annual simple interest. The amount of interest earned for 1 year was \$1020. How much was invested in each account?

37. A police officer has chosen a high-yield stock fund that earns 8% annual simple interest for part of a \$6000 investment. The remaining portion is used to purchase a preferred stock that earns 11% annual simple interest. How much should be invested in each account so that the amount earned on the 8% account is twice the amount earned on the 11% account?

38. To plan for the purchase of a new car, a deposit was made into an account that earns 7% annual simple interest. Another deposit, $1500 less than the first deposit, was placed in a certificate of deposit earning 9% annual simple interest. The total interest earned on both accounts for 1 year was $505. How much money was deposited in the certificate of deposit?

39. The Pacific Investment Group invested some money in a certificate of deposit (CD) that earns 6.5% annual simple interest. Twice the amount invested at 6.5% was invested in a second CD that earns 8.5% annual simple interest. If the total annual interest earned from the two investments was $4935, how much was invested at 6.5%?

40. A corporation gave a university $300,000 to support product safety research. The university deposited some of the money in a 10% simple interest account and the remainder in an 8.5% simple interest account. How much should be deposited in each account so that the annual interest earned is $28,500?

41. Ten co-workers formed an investment club, and each deposited $2000 in the club's account. They decided to take the total amount and invest some of it in preferred stock that pays 8% annual simple interest and the remainder in a municipal bond that pays 7% annual simple interest. The amount of interest earned each year from the investments was $1520. How much was invested in each?

42. A financial consultant advises a client to invest part of $30,000 in municipal bonds that earn 6.5% annual simple interest and the remainder of the money in 8.5% corporate bonds. How much should be invested in each so that the total interest earned each year is $2190?

43. Alisa Rhodes placed some money in a real estate investment trust that earns 7.5% annual simple interest. A second investment, which was one-half the amount placed in the real estate investment trust, was used to purchase a trust deed that earns 9% annual simple interest. If the total annual interest earned from the two investments was $900, how much was invested in the trust deed?

APPLYING THE CONCEPTS

44. Suppose a breadmaker costs $180 and that the ingredients and electricity needed to make one loaf of bread cost $.95. If a comparable loaf of bread at a grocery store costs $1.55, how many loaves of bread must you make before the breadmaker pays for itself?

45. Suppose a natural gas clothes dryer costs $240 and uses $.45 worth of gas to dry a load of clothes for one hour. If a laundromat charges $1.75 to use a dryer for one hour, how many loads of clothes must you dry before purchasing a gas dryer is more economical?

For what value of k does the system of equations have no solution?

46. $\begin{aligned} 2x - 3y &= 7 \\ kx - 3y &= 4 \end{aligned}$

47. $\begin{aligned} 8x - 4y &= 1 \\ 2x - ky &= 3 \end{aligned}$

48. $\begin{aligned} x &= 4y + 4 \\ kx - 8y &= 4 \end{aligned}$

49. The following was offered as a solution of the system of equations

(1) $y = \frac{1}{2}x + 2$

(2) $2x + 5y = 10$

$$\begin{aligned} 2x + 5y &= 10 && \bullet\ \text{Equation (2)} \\ 2x + 5\left(\frac{1}{2}x + 2\right) &= 10 && \bullet\ \text{Substitute } \frac{1}{2}x + 2 \text{ for } y. \\ 2x + \frac{5}{2}x + 10 &= 10 && \bullet\ \text{Solve for } x. \\ \frac{9}{2}x &= 0 \\ x &= 0 \end{aligned}$$

At this point the student stated that because $x = 0$, the system of equations has no solution. If this assertion is correct, is the system of equations independent, dependent, or inconsistent? If the assertion is not correct, what is the correct solution?

50. When you solve a system of equations by the substitution method, how do you determine whether the system of equations is inconsistent?

51. A sales representative invests in a stock paying a 9% dividend. A research consultant invests \$5000 more than the sales representative in bonds paying 8% annual simple interest. The research consultant's income from the investment is equal to the sales representative's. Find the amount of the research consultant's investment.

52. A plant manager invested \$3000 more in stocks than in bonds. The stocks paid 8% annual simple interest, and the bonds paid 9.5% annual simple interest. Both investments yielded the same income. Find the total annual interest received on both investments.

53. The exercises in this objective were based on annual *simple* interest, r, which means that the amount of interest earned after 1 year is given by $I = Pr$. For **compound interest,** the interest earned for a certain period of time (usually daily or monthly) is added to the principal before the interest for the next period is calculated. The compound interest earned in 1 year is given by the formula $I = P\left[\left(1 + \frac{r}{n}\right)^n - 1\right]$, where n is the number of times per year that interest is compounded. For instance, if interest is compounded daily, then $n = 365$; if interest is compounded monthly, then $n = 12$. Suppose an investment of \$5000 is made into three different accounts. The first account earns 8% annual simple interest, the second earns 8% compounded monthly ($n = 12$), and the third earns 8% compounded daily ($n = 365$). Find the amount of interest earned from each account.

8.3 Solving Systems of Equations by the Addition Method

Objective A

To solve a system of linear equations by the addition method

Another method of solving a system of equations is called the **addition method.** This method is based on the Addition Property of Equations.

Note, for the system of equations at the right, the effect of adding Equation (2) to Equation (1). Because $2y$ and $-2y$ are opposites, adding the equations results in an equation with only one variable.

$$\begin{array}{lr} (1) & 5x + 2y = 11 \\ (2) & \underline{3x - 2y = 13} \\ & 8x + 0y = 24 \\ & 8x = 24 \end{array}$$

Solving $8x = 24$ for x gives the first coordinate of the ordered-pair solution of the system of equations.

$$\begin{aligned} \frac{8x}{8} &= \frac{24}{8} \\ x &= 3 \end{aligned}$$

The second coordinate is found by substituting the value of x into Equation (1) or Equation (2) and then solving for y. Equation (1) is used here.

$$\begin{array}{lr} (1) & 5x + 2y = 11 \\ & 5(3) + 2y = 11 \\ & 15 + 2y = 11 \\ & 2y = -4 \\ & y = -2 \end{array}$$

The solution is $(3, -2)$.

Sometimes adding the two equations does not eliminate one of the variables. In this case, use the Multiplication Property of Equations to rewrite one or both of the equations such that the coefficients of one of the variables are opposites.

Solve by the addition method:
$$\begin{array}{lr} (1) & 4x + y = 5 \\ (2) & 2x - 5y = 19 \end{array}$$

Multiply Equation (2) by -2. The coefficients of x will then be opposites.

$$\begin{array}{lr} & -2(2x - 5y) = -2 \cdot 19 \\ (3) & -4x + 10y = -38 \end{array}$$

- Multiply Equation (2) by -2.
- Simplify. This is Equation (3).

Add Equation (1) to Equation (3). Then solve for y.

$$\begin{array}{lr} (1) & 4x + y = 5 \\ (3) & \underline{-4x + 10y = -38} \\ & 11y = -33 \\ & y = -3 \end{array}$$

- Note that the coefficients of x are opposites.
- Add the two equations.
- Solve for y.

Substitute the value of y into Equation (1) or Equation (2) and solve for x. Equation (1) will be used here.

$$\begin{array}{lr} (1) & 4x + y = 5 \\ & 4x + (-3) = 5 \\ & 4x - 3 = 5 \\ & 4x = 8 \\ & x = 2 \end{array}$$

- Substitute -3 for y.
- Solve for x.

The solution is $(2, -3)$.

Sometimes each equation of a system of equations must be multiplied by a constant so that the coefficients of one of the variable terms are opposites.

Solve by the addition method: (1) $3x + 7y = 2$
(2) $5x - 3y = -26$

To eliminate x, multiply Equation (1) by 5 and Equation (2) by -3. Note at the right how the constants are chosen.

$$5(3x + 7y) = 5 \cdot 2$$
$$-3(5x - 3y) = -3(-26)$$

• The negative is used so that the coefficients will be opposites.

$$\begin{aligned} 15x + 35y &= 10 && \bullet \text{ 5 times Equation (1).} \\ -15x + 9y &= 78 && \bullet\ -3 \text{ times Equation (2).} \\ \hline 44y &= 88 && \bullet \text{ Add the equations.} \\ y &= 2 && \bullet \text{ Solve for } y. \end{aligned}$$

Substitute the value of y into Equation (1) or Equation (2) and solve for x. Equation (1) will be used here.

$$\begin{aligned} (1) \quad 3x + 7y &= 2 \\ 3x + 7(2) &= 2 && \bullet \text{ Substitute 2 for } y. \\ 3x + 14 &= 2 && \bullet \text{ Solve for } x. \\ 3x &= -12 \\ x &= -4 \end{aligned}$$

The solution is $(-4, 2)$.

For the above system of equations, the value of x was determined by substitution. This value can also be determined by eliminating y from the system.

$$\begin{aligned} 9x + 21y &= 6 && \bullet \text{ 3 times Equation (1).} \\ 35x - 21y &= -182 && \bullet \text{ 7 times Equation (2).} \\ \hline 44x \qquad &= -176 && \bullet \text{ Add the equations.} \\ x &= -4 && \bullet \text{ Solve for } x. \end{aligned}$$

Note that this is the same value of x as was determined by using substitution.

Solve by the addition method: (1) $2x - 5y = 4$
(2) $4x - 10y = 8$

Eliminate x. Multiply Equation (1) by -2.

$$\begin{aligned} -2(2x - 5y) &= -2(4) && \bullet\ -2 \text{ times Equation (1).} \\ (3) \quad -4x + 10y &= -8 && \bullet \text{ This is Equation (3).} \end{aligned}$$

TAKE NOTE

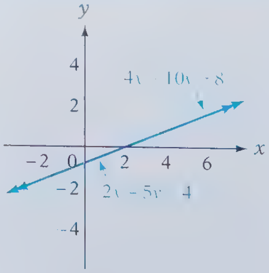
When you use the addition method to solve a system of equations and the result is an equation that is always true (like the one at the right), the system of equations is dependent. Compare this with the following example.

Add Equation (3) to Equation (2) and solve for y.

$$\begin{aligned} (2) \quad 4x - 10y &= 8 \\ (3) \quad -4x + 10y &= -8 \\ \hline 0x + 0y &= 0 \\ 0 &= 0 \end{aligned}$$

The equation $0 = 0$ means that the system of equations is dependent. Therefore, the solutions of the system of equations are the ordered pairs that satisfy $2x - 5y = 4$.

The graphs of the two equations in the system of equations above are shown at the right. Note that one line is on top of the other line and therefore they intersect infinitely often. The system of equations is dependent and the solutions are the ordered pairs that belong to either one of the equations.



Solve by the addition method:

$$\begin{array}{ll} (1) & 2x + y = 2 \\ (2) & 4x + 2y = -5 \end{array}$$

Eliminate y. Multiply Equation (1) by -2.

$(1)\qquad -2(2x + y) = -2 \cdot 2$ • −2 times Equation (1).

$(3)\qquad -4x - 2y = -4$ • This is Equation (3).

Add Equation (2) to Equation (3) and solve for x.

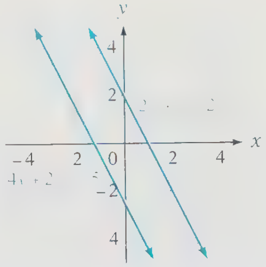
$$\begin{array}{lr} (3) & -4x - 2y = -4 \\ (2) & \underline{4x + 2y = -5} \end{array}$$

$0x + 0y = -9$ • Add Equation (2) to Equation (3).

$0 = -9$ • This is not a true equation.

The system of equations is inconsistent and therefore does not have a solution.

The graphs of the two equations in the system of equations above are shown at the right. Note that the graphs are parallel and therefore do not intersect. Thus the system of equations has no solution.



Example 1

Solve by the addition method:

$$\begin{array}{ll} (1) & 2x + 4y = 7 \\ (2) & 5x - 3y = -2 \end{array}$$

Solution

Eliminate x.

$5(2x + 4y) = 5 \cdot 7$ • 5 times Equation (1).

$-2(5x - 3y) = -2(-2)$ • −2 times Equation (2).

$$\begin{array}{r} 10x + 20y = 35 \\ \underline{-10x + 6y = 4} \end{array}$$

$26y = 39$ • Add the equations.

$y = \frac{39}{26} = \frac{3}{2}$ • Solve for y.

Substitute $\frac{3}{2}$ for y in Equation (1).

$(1)\qquad 2x + 4y = 7$

$2x + 4\left(\frac{3}{2}\right) = 7$ • Replace y by $\frac{3}{2}$.

$2x + 6 = 7$ • Solve for x.

$2x = 1$

$x = \frac{1}{2}$

The solution is $\left(\frac{1}{2}, \frac{3}{2}\right)$.

You Try It 1

Solve by the addition method:

$$\begin{array}{ll} (1) & x - 2y = 1 \\ (2) & 2x + 4y = 0 \end{array}$$

Your solution

Solution on p. S20

Example 2

Solve by the addition method:

$$\begin{aligned} &(1) \quad 6x + 9y = 15 \\ &(2) \quad 4x + 6y = 10 \end{aligned}$$

Solution

Eliminate x.

$$\begin{aligned} 4(6x + 9y) &= 4 \cdot 15 && \bullet \text{ 4 times Equation (1).} \\ -6(4x + 6y) &= -6 \cdot 10 && \bullet \text{ }-6\text{ times Equation (2).} \end{aligned}$$

$$\begin{aligned} 24x + 36y &= 60 \\ \underline{-24x - 36y} &\underline{= -60} \\ 0x + 0y &= 0 && \bullet \text{ Add the equations.} \\ 0 &= 0 \end{aligned}$$

The system of equations is dependent. The solutions are the ordered pairs that satisfy the equation $6x + 9y = 15$.

You Try It 2

Solve by the addition method:

$$\begin{aligned} 2x - 3y &= 4 \\ -4x + 6y &= -8 \end{aligned}$$

Your solution

Example 3

Solve by the addition method:

$$\begin{aligned} &(1) \quad 2x = y + 8 \\ &(2) \quad 3x + 2y = 5 \end{aligned}$$

Solution

Write Equation (1) in the form $Ax + By = C$.

$$\begin{aligned} 2x &= y + 8 \\ (3) \quad 2x - y &= 8 && \bullet \text{ This is Equation (3).} \end{aligned}$$

Eliminate y.

$$\begin{aligned} 2(2x - y) &= 2 \cdot 8 && \bullet \text{ 2 times Equation (3).} \\ 3x + 2y &= 5 && \bullet \text{ This is Equation (2).} \end{aligned}$$

$$\begin{aligned} 4x - 2y &= 16 \\ \underline{3x + 2y} &\underline{= 5} \\ 7x &= 21 && \bullet \text{ Add the equations.} \\ x &= 3 \end{aligned}$$

Replace x in Equation (1).

$$\begin{aligned} (1) \quad 2x &= y + 8 \\ 2 \cdot 3 &= y + 8 && \bullet \text{ Replace } x \text{ by 3.} \\ 6 &= y + 8 \\ -2 &= y \end{aligned}$$

The solution is $(3, -2)$.

You Try It 3

Solve by the addition method:

$$\begin{aligned} 4x + 5y &= 11 \\ 3y &= x + 10 \end{aligned}$$

Your solution

Solutions on p. S21

8.3 Exercises

[illegible]

Solve by the addition method.

1. $x + y = 4$
$x - y = 6$

2. $2x + y = 3$
$x - y = 3$

3. $x + y = 4$
$2x + y = 5$

4. $x - 3y = 2$
$x + 2y = -3$

5. $2x - y = 1$
$x + 3y = 4$

6. $x - 2y = 4$
$3x + 4y = 2$

7. $4x - 5y = 22$
$x + 2y = -1$

8. $3x - y = 11$
$2x + 5y = 13$

9. $2x - y = 1$
$4x - 2y = 2$

10. $x + 3y = 2$
$3x + 9y = 6$

11. $4x + 3y = 15$
$2x - 5y = 1$

12. $3x - 7y = 13$
$6x + 5y = 7$

13. $2x - 3y = 1$
$4x - 6y = 2$

14. $2x + 4y = 6$
$3x + 6y = 9$

15. $3x - 6y = -1$
$6x - 4y = 2$

16. $5x + 2y = 3$
$3x - 10y = -1$

17. $5x + 7y = 10$
$3x - 14y = 6$

18. $7x + 10y = 13$
$4x + 5y = 6$

19. $3x - 2y = 0$
$6x + 5y = 0$

20. $5x + 2y = 0$
$3x + 5y = 0$

21. $2x - 3y = 16$
$3x + 4y = 7$

22. $3x + 4y = 10$
$4x + 3y = 11$

23. $5x + 3y = 7$
$2x + 5y = 1$

24. $-2x + 7y = 9$
$3x + 2y = -1$

25. $3x + 4y = 4$
$5x + 12y = 5$

26. $2x + 5y = 2$
$3x + 3y = 1$

27. $8x - 3y = 11$
$6x - 5y = 11$

28. $4x - 8y = 36$
$3x - 6y = 15$

29. $5x + 15y = 20$
$2x + 6y = 12$

30. $y = 2x - 3$
$3x + 4y = -1$

31. $3x = 2y + 7$
$5x - 2y = 13$

32. $2y = 4 - 9x$
$9x - y = 25$

33. $2x + 9y = 16$
$5x = 1 - 3y$

34. $3x - 4 = y + 18$
$4x + 5y = -21$

35. $2x + 3y = 7 - 2x$
$7x + 2y = 9$

36. $5x - 3y = 3y + 4$
$4x + 3y = 11$

APPLYING THE CONCEPTS

37. Describe in your own words the process of solving a system of equations by the addition method.

38. The point of intersection of the graphs of the equations $Ax + 2y = 2$ and $2x + By = 10$ is $(2, -2)$. Find A and B.

39. The point of intersection of the graphs of the equations $Ax - 4y = 9$ and $4x + By = -1$ is $(-1, -3)$. Find A and B.

40. For what value of k is the system of equations dependent?

a. $2x + 3y = 7$
$4x + 6y = k$

b. $y = \frac{2}{3}x - 3$
$y = kx - 3$

c. $x = ky - 1$
$y = 2x + 2$

41. For what value of k is the system of equations inconsistent?

a. $x + y = 7$
$kx + y = 3$

b. $x + 2y = 4$
$kx + 3y = 2$

c. $2x + ky = 1$
$x + 2y = 2$

8.4 Application Problems in Two Variables

To solve rate-of-wind or rate-of-current problems

We normally need two variables to solve motion problems that involve an object moving with or against a wind or current.

Flying with the wind, a small plane can fly 600 mi in 3 h. Against the wind, the plane can fly the same distance in 4 h. Find the rate of the plane in calm air and the rate of the wind.

Strategy for Solving Rate-of-Wind or Rate-of-Current Problems

Choose one variable to represent the rate of the object in calm conditions and a second variable to represent the rate of the wind or current. Using these variables, express the rate of the object with and against the wind or current. Use the equation $rt = d$ to write expressions for the distance traveled by the object. The results can be recorded in a table.

Rate of plane in calm air: p
Rate of wind: w

	Rate	·	*Time*	=	*Distance*
With the wind	$p + w$	·	3	=	$3(p + w)$
Against the wind	$p - w$	·	4	=	$4(p - w)$

Determine how the expressions for distance are related.

The distance traveled with the wind is 600 mi. $3(p + w) = 600$
The distance traveled against the wind is 600 mi. $4(p - w) = 600$

Solve the system of equations.

$$\begin{aligned} 3(p + w) &= 600 \\ 4(p - w) &= 600 \end{aligned} \quad \longrightarrow \quad \begin{aligned} \frac{1}{3} \cdot 3(p + w) &= \frac{1}{3} \cdot 600 \\ \frac{1}{4} \cdot 4(p - w) &= \frac{1}{4} \cdot 600 \end{aligned} \quad \longrightarrow \quad \begin{aligned} p + w &= 200 \\ p - w &= 150 \\ \hline 2p &= 350 \\ p &= 175 \end{aligned}$$

$$\begin{aligned} p + w &= 200 \\ 175 + w &= 200 \\ w &= 25 \end{aligned}$$

The rate of the plane in calm air is 175 mph.
The rate of the wind is 25 mph.

Example 1

A 450-mile trip from one city to another takes 3 h when a plane is flying with the wind. The return trip, against the wind, takes 5 h. Find the rate of the plane in still air and the rate of the wind.

Strategy

- Rate of the plane in still air: p
  Rate of the wind: w

	Rate	*Time*	*Distance*
With wind	$p + w$	3	$3(p + w)$
Against wind	$p - w$	5	$5(p - w)$

- The distance traveled with the wind is 450 mi. The distance traveled against the wind is 450 mi.

Solution

$$3(p + w) = 450 \qquad \frac{1}{3} \cdot 3(p + w) = \frac{1}{3} \cdot 450$$

$$5(p - w) = 450 \qquad \frac{1}{5} \cdot 5(p - w) = \frac{1}{5} \cdot 450$$

$$\begin{aligned} p + w &= 150 \\ p - w &= 90 \\ 2p &= 240 \\ p &= 120 \end{aligned}$$

$$\begin{aligned} p + w &= 150 \\ 120 + w &= 150 \\ w &= 30 \end{aligned}$$

The rate of the plane in still air is 120 mph. The rate of the wind is 30 mph.

You Try It 1

A canoeist paddling with the current can travel 15 mi in 3 h. Against the current, it takes the canoeist 5 h to travel the same distance. Find the rate of the current and the rate of the canoeist in calm water.

Your strategy

Your solution

Solution on p. S21

Objective B To solve application problems using two variables

The application problems in this section are varieties of those problems solved earlier in the text. Each of the strategies for the problems in this section will result in a system of equations.

Point of Interest

The Babylonians had a method for solving a system of equations. Here is an adaptation of a problem from an ancient (around 1500 B.C.) Babylonian text. "There are two silver blocks. The sum of $\frac{1}{7}$ of the first block and $\frac{1}{11}$ of the second block is one sheqel (a weight). The first block diminished by $\frac{1}{7}$ of its weight equals the second diminished by $\frac{1}{11}$ of its weight. What are the weights of the two blocks?"

A jeweler purchased 5 oz of a gold alloy and 20 oz of a silver alloy for a total cost of \$540. The next day, at the same prices per ounce, the jeweler purchased 4 oz of the gold alloy and 25 oz of the silver alloy for a total cost of \$450. Find the cost per ounce of the gold and silver alloys.

Strategy for Solving an Application Problem in Two Variables

Choose one variable to represent one of the unknown quantities and a second variable to represent the other unknown quantity. Write numerical or variable expressions for all the remaining quantities. These results can be recorded in two tables, one for each of the conditions.

Cost per ounce of gold: g
Cost per ounce of silver: s

First day:

	Amount	·	*Unit Cost*	=	*Value*
Gold	5	·	g	=	$5g$
Silver	20	·	s	=	$20s$

Second day:

	Amount	·	*Unit Cost*	=	*Value*
Gold	4	·	g	=	$4g$
Silver	25	·	s	=	$25s$

Determine a system of equations. Each table will give one equation of the system.

The total value of the purchase on the first day was \$540. $5g + 20s = 540$

The total value of the purchase on the second day was \$450. $4g + 25s = 450$

Solve the system of equations.

$$\begin{aligned} 5g + 20s &= 540 \\ 4g + 25s &= 450 \end{aligned} \qquad \begin{aligned} 4(5g + 20s) &= 4 \cdot 540 \\ -5(4g + 25s) &= -5 \cdot 450 \end{aligned} \qquad \begin{aligned} 20g + 80s &= 2160 \\ -20g - 125s &= -2250 \\ \hline -45s &= -90 \\ s &= 2 \end{aligned}$$

$$\begin{aligned} 5g + 20s &= 540 \\ 5g + 20(2) &= 540 \qquad \bullet\ s = 2 \\ 5g + 40 &= 540 \\ 5g &= 500 \\ g &= 100 \end{aligned}$$

The cost per ounce of the gold alloy was \$100.
The cost per ounce of the silver alloy was \$2.

Example 2

A store owner purchased 20 incandescent light bulbs and 30 fluorescent bulbs for a total cost of $40. A second purchase, at the same prices, included 30 incandescent bulbs and 10 fluorescent bulbs for a total cost of $25. Find the cost of an incandescent bulb and of a fluorescent bulb.

Strategy

Cost of an incandescent bulb: b
Cost of a fluorescent bulb: f

First purchase:

	Amount	*Unit Cost*	*Value*
Incandescent	20	b	$20b$
Fluorescent	30	f	$30f$

Second purchase:

	Amount	*Unit Cost*	*Value*
Incandescent	30	b	$30b$
Fluorescent	10	f	$10f$

The total cost of the first purchase was $40.
The total cost of the second purchase was $25.

Solution

$$\begin{aligned} 20b + 30f &= 40 \qquad & 3(20b + 30f) &= 3 \cdot 40 \\ 30b + 10f &= 25 \qquad & -2(30b + 10f) &= -2 \cdot 25 \end{aligned}$$

$$\begin{aligned} 60b + 90f &= 120 \\ \underline{-60b - 20f} &\underline{= -50} \\ 70f &= 70 \\ f &= 1 \end{aligned}$$

$$\begin{aligned} 20b + 30f &= 40 \\ 20b + 30(1) &= 40 \\ 20b &= 10 \\ b &= \frac{1}{2} \end{aligned}$$

The cost of an incandescent bulb was $.50.
The cost of a fluorescent bulb was $1.00.

You Try It 2

A citrus grower purchased 25 orange trees and 20 grapefruit trees for $290. The next week, at the same prices, the grower bought 20 orange trees and 30 grapefruit trees for $330. Find the cost of an orange tree and the cost of a grapefruit tree.

Your strategy

Your solution

Solution on p. S21

8.4 Exercises

Application Problems

1. A whale swimming against an ocean current traveled 60 mi in 2 h. Swimming in the opposite direction, with the current, the whale was able to travel the same distance in 1.5 h. Find the speed of the whale in calm water and the rate of the ocean current.
2. A plane flying with the jet stream flew from Los Angeles to Chicago, a distance of 2250 mi, in 5 h. Flying against the jet stream, the plane could fly only 1750 mi in the same amount of time. Find the rate of the plane in calm air and the rate of the wind.
3. A rowing team rowing with the current traveled 40 km in 2 h. Rowing against the current, the team could travel only 16 km in 2 h. Find the rowing rate in calm water and the rate of the current.
4. The bird capable of the fastest flying speed is the swift. A swift flying with the wind to a favorite feeding spot traveled 26 mi in 0.2 h. On returning, now against the wind, the swift was able to travel only 16 mi in the same amount of time. What is the rate of the swift in calm air and what was the rate of the wind?
5. A private Learjet 31A was flying with a tailwind and traveled 1120 mi in 2 h. Flying against the wind on the return trip, the jet was able to travel only 980 mi in 2 h. Find the speed of the jet in calm air and the rate of the wind.
6. A plane flying with a tailwind flew 300 mi in 2 h. Against the wind, it took 3 h to travel the same distance. Find the rate of the plane in calm air and the rate of the wind.
7. A Boeing Apache Longbow military helicopter traveling directly into a strong headwind was able to travel 450 mi in 2.5 h. The return trip, now with a tailwind, took 1 h 40 min. Find the speed of the helicopter in calm air and the rate of the wind.
8. A seaplane pilot flying with the wind flew from an ocean port to a lake, a distance of 240 mi, in 2 h. Flying against the wind, the pilot flew from the lake to the ocean port in 2 h 40 min. Find the rate of the plane in calm air and the rate of the wind.
9. Rowing with the current, a canoeist paddled 14 mi in 2 h. Against the current, the canoeist could paddle only 10 mi in the same amount of time. Find the rate of the canoeist in calm water and the rate of the current.

Application Problems

10. A computer online service charges one hourly price for regular use but a higher hourly rate for designated "premium" areas. One customer was charged \$28 after spending 2 h in premium areas and 9 regular hours; another spent 3 h in the premium areas and 6 regular hours and was charged \$27. What does the online service charge per hour for regular and premium services?
11. A baker purchased 12 lb of wheat flour and 15 lb of rye flour for a total cost of \$18.30. A second purchase, at the same prices, included 15 lb of wheat flour and 10 lb of rye flour. The cost of the second purchase was \$16.75. Find the cost per pound of the wheat flour and of the rye flour.

12. An investor owned 300 shares of an oil company and 200 shares of a movie company. The quarterly dividend from the two stocks was \$165. After the investor sold 100 shares of the oil company and bought an additional 100 shares of the movie company, the quarterly dividend became \$185. Find the dividend per share for each stock.

13. A lab technician mixed 40 ml of reagent I with 60 ml of reagent II to produce a solution that was 31% hydrochloric acid. A second solution, which was 31.5% hydrochloric acid, was created by mixing 35 ml of reagent I with 65 ml of reagent II. What is the percent hydrochloric acid in the two reagents?

14. A pastry chef created a 50-ounce sugar solution that was 34% sugar from a 20% sugar solution and a 40% sugar solution. How much of the 20% sugar solution and how much of the 40% sugar solution were used?

15. The octane number of 87 on gasoline means that it will fight engine "knock" as effectively as a reference fuel that is 87% isooctane, a type of gas. Suppose you want to fill an 18-gallon empty tank with some 87-octane gasoline and some 93-octane fuel to produce a mixture that is 89-octane. How much of each type of gasoline must you use?

APPLYING THE CONCEPTS

16. Two angles are supplementary. The larger angle is 15° more than twice the measure of the smaller angle. Find the measures of the two angles. (Supplementary angles are two angles whose sum is 180°.)

17. The value of the nickels and dimes in a coin bank is \$.25. If the number of nickels and the number of dimes were doubled, the value of the coins would be \$.50. How many nickels and how many dimes are in the bank?

18. An investor has \$5000 to invest in two accounts. The first account earns 8% annual simple interest, and the second account earns 10% annual simple interest. How much money should be invested in each account so that the annual simple interest earned is \$600?

19. Solve the following problem, which dates from a Chinese manuscript called the Jinzhang that is approximately 2100 years old. "The price of 1 acre of good land is 300 pieces of gold; the price of 7 acres of bad land is 500 pieces of gold. One has purchased altogether 100 acres. The price was 10,000 pieces of gold. How much good land and how much bad land was bought?" Adapted from Victor J. Katz, *A History of Mathematics, An Introduction* (New York: HarperCollins, 1993), p. 15.

20. A coin bank contains only nickels or dimes, but there are no more than 27 coins. The value of the coins is \$2.10. How many different combinations of nickels and dimes could be in the coin bank?

Focus on Problem Solving

Using a Table and Searching for a Pattern

Consider the numbers 10, 12, and 28 and the sum of the proper factors (the natural number factors less than the number) of those numbers.

10: $1 + 2 + 5 = 8$ 12: $1 + 2 + 3 + 4 + 6 = 16$ 28: $1 + 2 + 4 + 7 + 14 = 28$

10 is called a **deficient number** because the sum of its proper factors is less than the number ($8 < 10$). 12 is called an **abundant number** because the sum of its proper factors is greater than the number ($16 > 12$), and 28 is called a **perfect number** because the sum of its proper divisors equals the number ($28 = 28$).

Our goal for this Focus on Problem Solving is to try to find a method that will determine whether a number is deficient, abundant, or perfect without having to first find all the factors and then add them up. We will use a table and search for a pattern.

Before we begin, recall that a prime number is a number greater than 1 whose only factors are itself and 1, and that each natural number greater than 1 has a unique prime factorization. For instance, the prime factorization of 36 is given by $36 = 2^2 \cdot 3^2$. Note that the proper factors of 36 (1, 2, 3, 4, 6, 9, 12, 18) can be represented in terms of the same prime numbers.

$$1 = 2^0, \quad 2 = 2^1, \quad 3 = 3^1, \quad 4 = 2^2, \quad 6 = 2 \cdot 3, \quad 9 = 3^2, \quad 12 = 2^2 \cdot 3, \quad 18 = 2 \cdot 3^2$$

Now let us consider a trial problem of determining whether 432 is deficient, abundant, or perfect.

TAKE NOTE
This table contains the factor $432 = 2^4 \cdot 3^3$, which is not a proper factor.

We write the prime factorization of 432 as $2^4 \cdot 3^3$ and place the factors of 432 in a table as shown at the right. This table contains *all* the factors of 432 represented in terms of the prime number factors. The sum of each column is shown at the bottom.

	1	$1 \cdot 3$	$1 \cdot 3^2$	$1 \cdot 3^3$
	2	$2 \cdot 3$	$2 \cdot 3^2$	$2 \cdot 3^3$
	2^2	$2^2 \cdot 3$	$2^2 \cdot 3^2$	$2^2 \cdot 3^3$
	2^3	$2^3 \cdot 3$	$2^3 \cdot 3^2$	$2^3 \cdot 3^3$
	2^4	$2^4 \cdot 3$	$2^4 \cdot 3^2$	$2^4 \cdot 3^3$
Sum	31	$31 \cdot 3$	$31 \cdot 3^2$	$31 \cdot 3^3$

Here is the calculation of the sum for the column headed by $1 \cdot 3$.

$$1 \cdot 3 + 2 \cdot 3 + 2^2 \cdot 3 + 2^3 \cdot 3 + 2^4 \cdot 3 = (1 + 2 + 2^2 + 2^3 + 2^4)3 = 31(3)$$

For the column headed by $1 \cdot 3^2$ there is a similar situation.

$$1 \cdot 3^2 + 2 \cdot 3^2 + 2^2 \cdot 3^2 + 2^3 \cdot 3^2 + 2^4 \cdot 3^2 = (1 + 2 + 2^2 + 2^3 + 2^4)3^2 = 31(3^2)$$

The sum of *all* the factors (including 432) is the sum of the last row.

$$\text{Sum of all factors} = 31 + 31 \cdot 3 + 31 \cdot 3^2 + 31 \cdot 3^3 = 31(1 + 3 + 3^2 + 3^3) = 31(40) = 1240$$

To find the sum of the *proper* factors, we must subtract 432 from 1240; we get 808 (see Take Note). Thus 432 is abundant.

We now look for some pattern for the sum of all the factors. Note that

$$\text{Sum of left column} \quad 1 + 2 + 2^2 + 2^3 + 2^4 = \downarrow \quad \downarrow = 1 + 3 + 3^2 + 3^3 \quad \text{Sum of top row}$$

$$31(40) = 1240$$

This suggests that the sum of the proper factors can be found by finding the sum of all the prime power factors for each prime, multiplying those numbers, and then subtracting the original number. Although we have not proved this for all cases, it is a true statement.

For instance, to find the sum of the proper factors of 3240, first find the prime factorization.

$$3240 = 2^3 \cdot 3^4 \cdot 5$$

Now find the following sums:

$$1 + 2 + 2^2 + 2^3 = 15 \qquad 1 + 3 + 3^2 + 3^3 + 3^4 = 121 \qquad 1 + 5 = 6$$

The sum of the proper factors $= (15)(121)6 - 3240 = 7650$. Thus 3240 is abundant.

Determine whether the number is deficient, abundant, or perfect.

1. 200 **2.** 3125 **3.** 8128 **4.** 10,000

5. Is a prime number deficient, abundant, or perfect?

Projects and Group Activities

Finding a Pattern

The Focus on Problem Solving involved finding the sum of $1 + 3 + 3^2$ and $1 + 2 + 2^2 + 2^3$. For sums that contain a larger number of terms, it may be difficult or time-consuming to try to evaluate the sum. Perhaps there is a pattern for these sums that we can use to calculate them without having to evaluate each exponential expression and then add the results.

Look at the following from the calculation of the sum of the factors of 3240.

$$1 + 2 + 2^2 + 2^3 = 1 + 2 + 4 + 8 = 15$$

$$\frac{2^{3+1} - 1}{2 - 1} = \frac{2^4 - 1}{1} = 16 - 1 = 15$$

$$1 + 3 + 3^2 + 3^3 + 3^4 = 1 + 3 + 9 + 27 + 81 = 121$$

$$\frac{3^{4+1} - 1}{3 - 1} = \frac{3^5 - 1}{2} = \frac{243 - 1}{2} = \frac{242}{2} = 121$$

Consider another sum of this type.

$$1 + 7 + 7^2 + 7^3 + 7^4 + 7^5 = 1 + 7 + 49 + 343 + 2401 + 16{,}807 = 19{,}608$$

$$\frac{7^{5+1} - 1}{7 - 1} = \frac{7^6 - 1}{6} = \frac{117{,}649 - 1}{6} = \frac{117{,}648}{6} = 19{,}608$$

On the basis of the examples shown above, make a conjecture about the value of $1 + n + n^2 + n^3 + n^4 + \cdots + n^k$, where n and k are natural numbers greater than 1.

Solving a System of Equations with a Scientific Calculator

By using the addition method, it is possible to solve the system of equations

$$\begin{aligned} ax + by &= c \\ dx + ey &= f \end{aligned}$$

The solution is

$$x = \frac{ce - bf}{ae - bd} \quad \text{and} \quad y = \frac{af - cd}{ae - bd} \quad \text{where } ae - bd \neq 0$$

Using this solution, we can solve a system of two equations in two variables with a calculator. It is helpful to observe that the denominators of both expressions are identical. The calculation of the denominator is done first and then stored in the calculator's memory. If the value of the denominator is zero, the system of equations is dependent or inconsistent, and this calculator method cannot be used.

Solve: $\begin{aligned} 2x - 5y &= 9 \\ 4x + 3y &= 2 \end{aligned}$

Make a list of the values of a, b, c, d, e, and f.

$$a = 2 \qquad b = -5 \qquad c = 9$$
$$d = 4 \qquad e = 3 \qquad f = 2$$

Calculate the denominator $D = ae - bd$.

2 $\boxed{\times}$ 3 $\boxed{-}$ 5 $\boxed{+/-}$ $\boxed{\times}$ 4 $\boxed{=}$ $\boxed{M+}$[1]

Find x. Replace the variables in $x = \dfrac{ce - bf}{ae - bd}$ by their values.

$\boxed{(}$ 9 $\boxed{\times}$ 3 $\boxed{-}$ 5 $\boxed{+/-}$ $\boxed{\times}$ 2 $\boxed{)}$ $\boxed{\div}$ $\boxed{MR}$ $\boxed{=}$

The result should be approximately 1.423077.

Find y. Replace the variables in $y = \dfrac{af - cd}{ae - bd}$ by their values.

$\boxed{(}$ 2 $\boxed{\times}$ 2 $\boxed{-}$ 9 $\boxed{\times}$ 4 $\boxed{)}$ $\boxed{\div}$ $\boxed{MR}$ $\boxed{=}$

The result should be approximately −1.230769.

The approximate solution is the ordered pair (1.423077, −1.230769). The result is an approximation because the calculator display is an approximation to the exact result.

Solve by using a scientific calculator.

1. $\begin{aligned} 3.29x + 4.17y &= -2.34 \\ 0.34x - 0.17y &= 0.1 \end{aligned}$

2. $\begin{aligned} 27x + 17y &= 50 \\ 4.5x - 9.2y &= 18.3 \end{aligned}$

3. $\begin{aligned} y &= \frac{2}{3}x - 4 \\ 2x + y &= 5 \end{aligned}$

4. $\begin{aligned} y &= \frac{7}{4}x + 2 \\ y &= \frac{2}{3}x - 3 \end{aligned}$

5. Use the addition method to show that the solution of the system of equations $\begin{aligned} ax + by &= c \\ dx + ey &= f \end{aligned}$ is given by $x = \dfrac{ce - bf}{ae - bd}$ and $y = \dfrac{af - cd}{ae - bd}$, where $ae - bd \neq 0$.

Solving a System of Equations with a Graphing Calculator

A graphing calculator can also be used to approximate the solution of a system of equations in two variables. Graph each equation of the system of equations and then approximate the coordinates of the point of intersection. The process by which you approximate the solution depends on what model of calculator you have. In all cases, however, you must first solve each equation in the system of equations for y.

Solve: $\begin{aligned} 2x - 5y &= 9 \\ 4x + 3y &= 2 \end{aligned}$

Solve each equation for y.

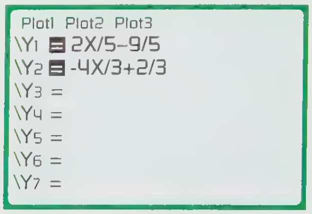
$$\begin{aligned} 2x - 5y &= 9 \\ -5y &= -2x + 9 \\ y &= \frac{2}{5}x - \frac{9}{5} \end{aligned} \qquad\qquad \begin{aligned} 4x + 3y &= 2 \\ 3y &= -4x + 2 \\ y &= -\frac{4}{3}x + \frac{2}{3} \end{aligned}$$

---

[1] Some calculators use STO to store a result and use RCL to recall the result.

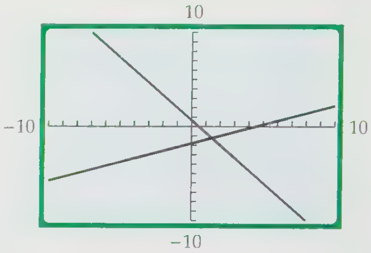
The graphing calculator screens shown below are taken from a TI-83 Plus. Similar screens would appear if we used a different graphing calculator.

For the TI-83 and TI-83 Plus, press Y=. Enter one equation as Y1 and the other as Y2. The result should be similar to the screen at the right.

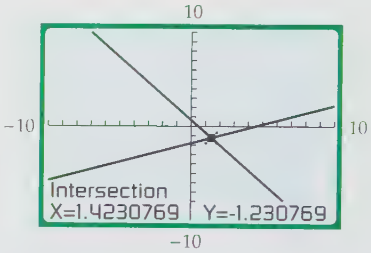


```
Plot1 Plot2 Plot3
\Y1 ■2X/5-9/5
\Y2 ■-4X/3+2/3
\Y3 =
\Y4 =
\Y5 =
\Y6 =
\Y7 =
```

Press GRAPH. The graphs of the two equations should appear on the screen. If the point of intersection is not on the screen, adjust the viewing window by pressing the WINDOW key.



Press 2nd CALC 5 ENTER ENTER ENTER. After a few seconds, the point of intersection will show on the bottom of the screen as X = 1.4230769, Y = −1.230769.



```
Intersection
X=1.4230769  Y=-1.230769
```

Solve by using a graphing calculator.

1. $4x - 5y = 8$
$5x + 7y = 7$

2. $3x + 2y = 11$
$7x - 6y = 13$

3. $x = 3y + 2$
$y = 4x - 2$

4. $x = 2y - 5$
$x = 3y + 2$

Chapter Summary

Key Words

Equations considered together are called a *system of equations*. [p. 369]

A *solution of a system of equations in two variables* is an ordered pair that is a solution of each equation of the system. [p. 369]

An *independent system of equations* has one solution. [p. 370]

A *dependent system of equations* has an infinite number of solutions. [p. 370]

An *inconsistent system of equations* has no solution. [p. 370]

Essential Rules

A system of equations can be solved by the *graphing method*, the *substitution method*, or the *addition method*.

Annual Simple Interest Equation

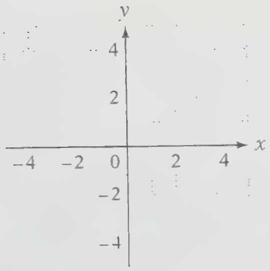
$I = Pr$, where I is the annual simple interest, P is the principal, and r is the annual interest rate as a decimal. [p. 381]

Chapter Review

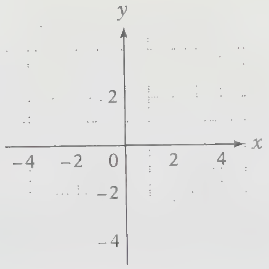
1. Is $(-1, -3)$ a solution of this system of equations?
$$\begin{aligned} 5x + 4y &= -17 \\ 2x - y &= 1 \end{aligned}$$

2. Is $(-2, 0)$ a solution of this system of equations?
$$\begin{aligned} -x + 9y &= 2 \\ 6x - 4y &= 12 \end{aligned}$$

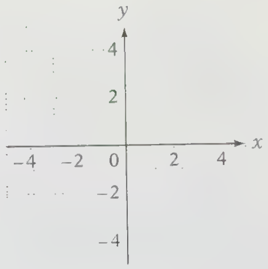
3. Solve by graphing:
$$\begin{aligned} 3x - y &= 6 \\ y &= -3 \end{aligned}$$



4. Solve by graphing:
$$\begin{aligned} 4x - 2y &= 8 \\ y &= 2x - 4 \end{aligned}$$



5. Solve by graphing:
$$\begin{aligned} x + 2y &= 3 \\ y &= -\frac{1}{2}x + 1 \end{aligned}$$



6. Solve by substitution:
$$\begin{aligned} 4x + 7y &= 3 \\ x &= y - 2 \end{aligned}$$

7. Solve by substitution:
$$\begin{aligned} 6x - y &= 0 \\ 7x - y &= 1 \end{aligned}$$

8. Solve by the addition method:
$$\begin{aligned} 3x + 8y &= -1 \\ x - 2y &= -5 \end{aligned}$$

9. Solve by the addition method:
$$\begin{aligned} 6x + 4y &= -3 \\ 12x - 10y &= -15 \end{aligned}$$

10. Solve by substitution:
$$\begin{aligned} 12x - 9y &= 18 \\ y &= \frac{4}{3}x - 3 \end{aligned}$$

11. Solve by substitution:
$$\begin{aligned} 8x - y &= 2 \\ y &= 5x + 1 \end{aligned}$$

12. Solve by the addition method:
$$\begin{aligned} 4x - y &= 9 \\ 2x + 3y &= -13 \end{aligned}$$

13. Solve by the addition method:
$$\begin{aligned} 5x + 7y &= 21 \\ 20x + 28y &= 63 \end{aligned}$$

14. Solve by substitution:
$$\begin{aligned} 4x + 3y &= 12 \\ y &= -\frac{4}{3}x + 4 \end{aligned}$$

15. Solve by substitution:
$7x + 3y = -16$
$x - 2y = 5$

16. Solve by the addition method:
$3x + y = -2$
$-9x - 3y = 6$

17. Solve by the addition method:
$6x - 18y = 7$
$9x + 24y = 2$

18. A sculling team rowing with the current went 24 mi in 2 h. Rowing against the current, the sculling team went 18 mi in 3 h. Find the rate of the sculling team in calm water and the rate of the current.

19. An investor bought 1500 shares of stock, some at \$6 per share and the rest at \$25 per share. If \$12,800 worth of stock was purchased, how many shares of each kind did the investor buy?

20. A flight crew flew 420 km in 3 h with a tailwind. Flying against the wind, the flight crew flew 440 km in 4 h. Find the rate of the flight crew in calm air and the rate of the wind.

21. A small plane flying with the wind flew 360 mi in 3 h. Against a headwind, the plane took 4 h to fly the same distance. Find the rate of the plane in calm air and the rate of the wind.

22. A small wood-carving company mailed 190 advertisements, some requiring \$.25 postage and others requiring \$.45. The total cost for mailing was \$59.50. Find the number of advertisements mailed at each rate.

23. A Terra Cotta Art Center receives an annual income of \$915 from two simple interest investments. One investment, in a corporate bond fund, earns 8.5% annual simple interest. The second investment, in a real estate investment trust, earns 7% annual simple interest. If the total amount invested in the two accounts is \$12,000, how much is invested in each account?

24. A silo contains a mixture of lentils and corn. If 50 bushels of lentils were added, there would be twice as many bushels of lentils as of corn; if 150 bushels of corn were added instead, there would be the same amount of corn as of lentils. How many bushels of each were originally in the silo?

25. Mosher Children's Hospital received a \$300,000 donation that it invested in two simple interest accounts, one earning 5.4% and the other earning 6.6%. If each account earned the same amount of annual interest, how much was invested in each account?

Chapter Test

1. Is $(-2, 3)$ a solution of this system?

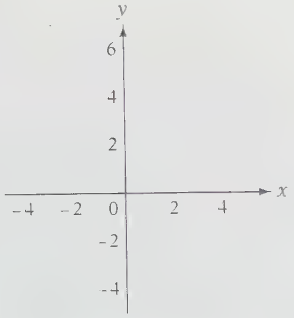
$$\begin{aligned} 2x + 5y &= 11 \\ x + 3y &= 7 \end{aligned}$$

2. Is $(1, -3)$ a solution of this system?

$$\begin{aligned} 3x - 2y &= 9 \\ 4x + y &= 1 \end{aligned}$$

3. Solve by graphing:

$$\begin{aligned} 3x + 2y &= 6 \\ 5x + 2y &= 2 \end{aligned}$$



4. Solve by substitution:

$$\begin{aligned} 4x - y &= 11 \\ y &= 2x - 5 \end{aligned}$$

5. Solve by substitution:

$$\begin{aligned} x &= 2y + 3 \\ 3x - 2y &= 5 \end{aligned}$$

6. Solve by substitution:

$$\begin{aligned} 3x + 5y &= 1 \\ 2x - y &= 5 \end{aligned}$$

7. Solve by substitution:

$$\begin{aligned} 3x - 5y &= 13 \\ x + 3y &= 1 \end{aligned}$$

8. Solve by substitution:

$$\begin{aligned} 2x - 4y &= 1 \\ y &= \frac{1}{2}x + 3 \end{aligned}$$

9. Solve by the addition method:

$$\begin{aligned} 4x + 3y &= 11 \\ 5x - 3y &= 7 \end{aligned}$$

10. Solve by the addition method:

$$\begin{aligned} 2x - 5y &= 6 \\ 4x + 3y &= -1 \end{aligned}$$

11. Solve by the addition method:

$x + 2y = 8$

$3x + 6y = 24$

12. Solve by the addition method:

$7x + 3y = 11$

$2x - 5y = 9$

13. Solve by the addition method:

$5x + 6y = -7$

$3x + 4y = -5$

14. With the wind, a plane flies 240 mi in 2 h. Against the wind, the plane requires 3 h to fly the same distance. Find the rate of the plane in calm air and the rate of the wind.

15. For the first performance of a play in a community theater, 50 reserved-seat tickets and 80 general-admission tickets were sold. The total receipts were $980. For the second performance, 60 reserved-seat tickets and 90 general-admission tickets were sold. The total receipts were $1140. Find the price of a reserved-seat ticket and the price of a general-admission ticket.

16. Bernardo Community Library received a $28,000 donation that it invested in two accounts, one earning 7.6% simple interest and the other earning 6.4% simple interest. If both accounts earned the same amount of annual interest, how much was invested in each account?

Cumulative Review

1. Evaluate $\frac{a^2 - b^2}{2a}$ when $a = 4$ and $b = -2$.

2. Solve: $-\frac{3}{4}x = \frac{9}{8}$

3. Given $f(x) = x^2 + 2x - 1$, find $f(2)$.

4. Simplify: $(2a^2 - 3a + 1)(2 - 3a)$

5. Simplify: $\frac{(-2x^2y)^4}{-8x^3y^2}$

6. Simplify: $(4b^2 - 8b + 4) \div (2b - 3)$

7. Simplify: $\frac{8x^{-2}y^5}{-2xy^4}$

8. Factor: $4x^2y^4 - 64y^2$

9. Solve: $(x - 5)(x + 2) = -6$

10. Simplify: $\frac{x^2 - 6x + 8}{2x^3 + 6x^2} \div \frac{2x - 8}{4x^3 + 12x^2}$

11. Simplify: $\frac{x - 1}{x + 2} + \frac{2x + 1}{x^2 + x - 2}$

12. Simplify: $\dfrac{x + 4 - \frac{7}{x - 2}}{x + 8 + \frac{21}{x - 2}}$

13. Solve: $\frac{x}{2x - 3} + 2 = \frac{-7}{2x - 3}$

14. Solve $A = P + Prt$ for r.

15. Find the x- and y-intercepts for $2x - 3y = 12$.

16. Find the slope of the line that passes through the points $(2, -3)$ and $(-3, 4)$.

17. Find the equation of the line that passes through the point $(-2, 3)$ and has slope $-\frac{3}{2}$.

18. Is $(2, 0)$ a solution of this system?

$$\begin{aligned} 5x - 3y &= 10 \\ 4x + 7y &= 8 \end{aligned}$$

19. Solve by substitution:
$3x - 5y = -23$
$x + 2y = -4$

20. Solve by the addition method:
$5x - 3y = 29$
$4x + 7y = -5$

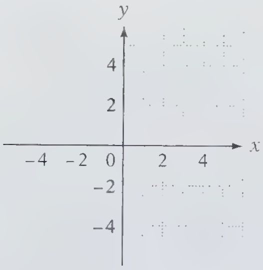
21. A total of \$8750 is invested in two accounts. On one account, the annual simple interest rate is 9.6%; on the second account, the annual simple interest rate is 7.2%. How much should be invested in each account so that both accounts earn the same interest?

22. A passenger train leaves a train depot $\frac{1}{2}$ h after a freight train leaves the same depot. The freight train is traveling 8 mph slower than the passenger train. Find the rate of each train if the passenger train overtakes the freight train in 3 h.

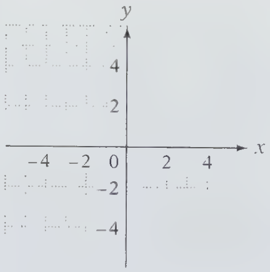
23. The length of each side of a square is extended 4 in. The area of the resulting square is 144 in^2. Find the length of a side of the original square.

24. A plane can travel 160 mph in calm air. Flying with the wind, the plane can fly 570 mi in the same amount of time as it takes to fly 390 mi against the wind. Find the rate of the wind.

25. Graph $2x - 3y = 6$.



26. Solve by graphing: $3x + 2y = 6$
$3x - 2y = 6$



27. With the current, a motorboat can travel 48 mi in 3 h. Against the current, the boat requires 4 h to travel the same distance. Find the rate of the boat in calm water.

28. A child adds 8 g of sugar to a 50-gram serving of a breakfast cereal that is 25% sugar. What is now the percent concentration of sugar in the mixture? Round to the nearest tenth of a percent.

Chapter 9 Inequalities

[illegible]

Section 9.1

- **A** To write a set using the roster method
- **B** To write a set using set-builder notation
- **C** To graph an inequality on the number line

Section 9.2

- **A** To solve an inequality using the Addition Property of Inequalities
- **B** To solve an inequality using the Multiplication Property of Inequalities
- **C** To solve application problems

Section 9.3

- **A** To solve general inequalities
- **B** To solve application problems

Section 9.4

- **A** To graph an inequality in two variables

Making decisions is a part of life, and having a good understanding of the alternatives makes deciding between options much easier. For example, choosing which of two rental car companies will provide the more economical lease may be easier if you know how many miles you will drive the car. With that information, you can determine the "better deal" by writing and solving an inequality, such as those in the applications **on pages 427 and 428.**

Need help? For online student resources, such as section quizzes, visit this textbook's website at **math.college.hmco.com/students.**

Prep Test

1. Place the correct symbol, < or >, between the two numbers.

$-45 \quad -27$

2. Simplify: $3x - 5(2x - 3)$

3. State the Addition Property of Equations.

4. State the Multiplication Property of Equations.

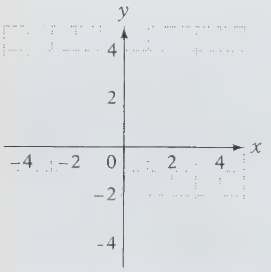
5. A certain grade of hamburger contains 15% fat. How many pounds of fat are in 3 lb of this hamburger?

6. Solve: $4x - 5 = -7$

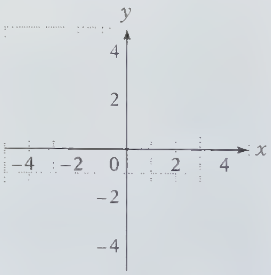
7. Solve: $4 = 2 - \frac{3}{4}x$

8. Solve: $7 - 2(2x - 3) = 3x - 1$

9. Graph: $y = \frac{2}{3}x - 3$



10. Graph: $3x + 4y = 12$



Go Figure

Without using a calculator, which is the largest number: 2^{150}, 3^{100}, or 5^{50}?

9.1 Sets

[illegible]

To write a set using the roster method

Recall that a *set* is a collection of objects, which are called the *elements* of the set.

The **roster method** of writing a set encloses a list of the elements in braces.

The set of the last three letters of the alphabet is written {x, y, z}.

The set of the positive integers less than 5 is written {1, 2, 3, 4}.

Use the roster method to write the set of integers between 0 and 10.

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

- **A set can be designated by a capital letter. Note that 0 and 10 are not elements of the set.**

Use the roster method to write the set of natural numbers.

$A = \{1, 2, 3, 4, \ldots\}$

- **The three dots mean that the pattern of numbers continues without end.**

The **empty set,** or **null set,** is the set that contains no elements. The symbol $\emptyset$ or { } is used to represent the empty set.

The set of people who have run a 2-minute mile is the empty set.

The **union** of two sets, written $A \cup B$, is the set that contains the elements of A and the elements of B.

Find $A \cup B$, given $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$.

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

- **The union of *A* and *B* contains all the elements of *A* and all the elements of *B*. Any elements that are in both *A* and *B* are listed only once.**

The **intersection** of two sets, written $A \cap B$, is the set that contains the elements that are common to both A and B.

Find $A \cap B$, given $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$.

$A \cap B = \{3, 4\}$

- **The intersection of *A* and *B* contains the elements common to *A* and *B*.**

Example 1
Use the roster method to write the set of the odd positive integers less than 12.

Solution
$A = \{1, 3, 5, 7, 9, 11\}$

You Try It 1
Use the roster method to write the set of the odd negative integers greater than −10.

Your solution

Solution on p. S22

Example 2
Use the roster method to write the set of the even positive integers.

Solution
$A = \{2, 4, 6, \ldots\}$

You Try It 2
Use the roster method to write the set of the odd positive integers.

Your solution

Example 3
Find $D \cup E$, given
$D = \{6, 8, 10, 12\}$ and
$E = \{-8, -6, 10, 12\}$.

Solution
$D \cup E = \{-8, -6, 6, 8, 10, 12\}$

You Try It 3
Find $A \cup B$, given
$A = \{-2, -1, 0, 1, 2\}$ and
$B = \{0, 1, 2, 3, 4\}$.

Your solution

Example 4
Find $A \cap B$, given
$A = \{5, 6, 9, 11\}$ and
$B = \{5, 9, 13, 15\}$.

Solution
$A \cap B = \{5, 9\}$

You Try It 4
Find $C \cap D$, given
$C = \{10, 12, 14, 16\}$ and
$D = \{10, 16, 20, 26\}$.

Your solution

Example 5
Find $A \cap B$, given
$A = \{1, 2, 3, 4\}$ and
$B = \{8, 9, 10, 11\}$.

Solution
$A \cap B = \emptyset$

You Try It 5
Find $A \cap B$, given
$A = \{-5, -4, -3, -2\}$ and
$B = \{2, 3, 4, 5\}$.

Your solution

Solutions on p. S22

Objective B To write a set using set-builder notation

Point of Interest

The symbol ∈ was first used in the book *Arithmeticae Principia*, published in 1889. It is the first letter of the Greek word εστι which means "is." The symbols for union and intersection were also introduced at that time.

Another method of representing sets is called **set-builder notation.** Using set-builder notation, we represent the set of all positive integers less than 10 as

$\{x \mid x < 10, x \in \text{positive integers}\}$, which is read "the set of all x such that x is less than 10 and x is an element of the positive integers."

Use set-builder notation to write the set of real numbers greater than 4.

$\{x \mid x > 4, x \in \text{real numbers}\}$

- "$x \in$ real numbers" is read "x is an element of the real numbers."

Example 6
Use set-builder notation to write the set of negative integers greater than -100.

Solution
$\{x \mid x > -100, x \in \text{negative integers}\}$

You Try It 6
Use set-builder notation to write the set of positive even integers less than 59.

Your solution

Example 7
Use set-builder notation to write the set of real numbers less than 60.

Solution
$\{x \mid x < 60, x \in \text{real numbers}\}$

You Try It 7
Use set-builder notation to write the set of real numbers greater than -3.

Your solution

Solutions on p. S22

[illegible]

To graph an inequality on the number line

9

An expression that contains the symbol $>$, $<$, $\geq$ (is greater than or equal to), or $\leq$ (is less than or equal to) is called an **inequality.** An inequality expresses the relative order of two mathematical expressions. The expressions can be either numerical or variable.

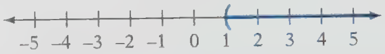
$$\left.\begin{array}{l} 4 > 2 \\ 3x \leq 7 \\ x^2 - 2x > y + 4 \end{array}\right\} \text{Inequalities}$$

An **inequality** can be graphed on the number line.

TAKE NOTE
For the remainder of this section, all variables will represent real numbers. Given this convention, the expression $\{x \mid x > 1, x \in \text{real numbers}\}$ will be written as $\{x \mid x > 1\}$, as shown in the example at the right.

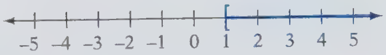
Graph: $\{x \mid x > 1\}$

The graph is the real numbers greater than 1. The parenthesis at 1 indicates that 1 is not included in the graph.



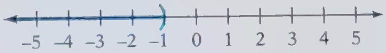
Graph: $\{x \mid x \geq 1\}$

The bracket at 1 indicates that 1 is included in the solution set.



Graph: $\{x \mid -1 > x\}$

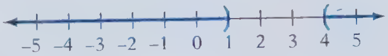
$-1 > x$ is equivalent to $x < -1$. The numbers less than -1 are to the left of -1 on the number line.



The union of two sets is the set that contains all the elements of each set.

Graph: $\{x \mid x > 4\} \cup \{x \mid x < 1\}$

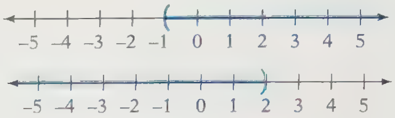
The graph is the numbers greater than 4 and the numbers less than 1.



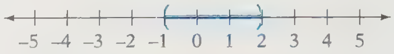
The intersection of two sets is the set that contains the elements common to both sets.

Graph: $\{x \mid x > -1\} \cap \{x \mid x < 2\}$

The graphs of $\{x \mid x > -1\}$ and $\{x \mid x < 2\}$ are shown at the right.

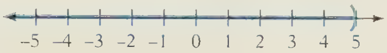


The graph of $\{x \mid x > -1\} \cap \{x \mid x < 2\}$ is the numbers between -1 and 2.



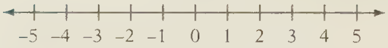
Example 8
Graph: $\{x \mid x < 5\}$

Solution
The solution set is the numbers less than 5.



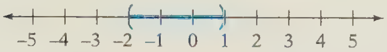
You Try It 8
Graph: $\{x \mid -2 < x\}$

Your solution



Example 9
Graph: $\{x \mid x > -2\} \cap \{x \mid x < 1\}$

Solution
The solution set is the numbers between -2 and 1.

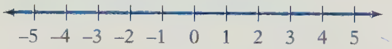


You Try It 9
Graph: $\{x \mid x > -1\} \cup \{x \mid x < -3\}$

Your solution

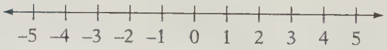
Example 10
Graph: $\{x \mid x \le 5\} \cup \{x \mid x \ge -3\}$

Solution
The solution set is the real numbers.



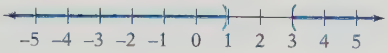
You Try It 10
Graph: $\{x \mid x < 2\} \cup \{x \mid x \ge -2\}$

Your solution



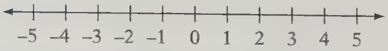
Example 11
Graph: $\{x \mid x > 3\} \cup \{x \mid x < 1\}$

Solution
The solution set is the numbers greater than 3 and the numbers less than 1.



You Try It 11
Graph: $\{x \mid x \le 4\} \cap \{x \mid x \ge -4\}$

Your solution



Solutions on p. S22

9.1 Exercises

[illegible]

1. Explain how to find the union of two sets.

2. Explain how to find the intersection of two sets.

Use the roster method to write the set.

3. The integers between 15 and 22

4. The integers between −10 and −4

5. The odd integers between 8 and 18

6. The even integers between −11 and −1

7. The letters of the alphabet between a and d

8. The letters of the alphabet between p and v

Find $A \cup B$.

9. $A = \{3, 4, 5\}$ $B = \{4, 5, 6\}$

10. $A = \{-3, -2, -1\}$ $B = \{-2, -1, 0\}$

11. $A = \{-10, -9, -8\}$ $B = \{8, 9, 10\}$

12. $A = \{\text{a, b, c}\}$ $B = \{\text{x, y, z}\}$

13. $A = \{\text{a, b, d, e}\}$ $B = \{\text{c, d, e, f}\}$

14. $A = \{\text{m, n, p, q}\}$ $B = \{\text{m, n, o}\}$

15. $A = \{1, 3, 7, 9\}$ $B = \{7, 9, 11, 13\}$

16. $A = \{-3, -2, -1\}$ $B = \{-1, 1, 2\}$

Find $A \cap B$.

17. $A = \{3, 4, 5\}$ $B = \{4, 5, 6\}$

18. $A = \{-4, -3, -2\}$ $B = \{-6, -5, -4\}$

19. $A = \{-4, -3, -2\}$ $B = \{2, 3, 4\}$

20. $A = \{1, 2, 3, 4\}$ $B = \{1, 2, 3, 4\}$

21. $A = \{\text{a, b, c, d, e}\}$ $B = \{\text{c, d, e, f, g}\}$

22. $A = \{\text{m, n, o, p}\}$ $B = \{\text{k, l, m, n}\}$

[illegible]

Use set-builder notation to write the set.

23. The negative integers greater than −5

24. The positive integers less than 5

25. The integers greater than 30

26. The integers less than −70

27. The even integers greater than 5

28. The odd integers less than −2

29. The real numbers greater than 8

30. The real numbers less than 57

Graph.

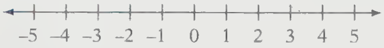
31. $\{x \mid x > 2\}$

32. $\{x \mid x \geq -1\}$

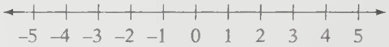
33. $\{x \mid x \leq 0\}$

34. $\{x \mid x < 4\}$

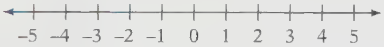
35. $\{x \mid x > -2\} \cup \{x \mid x < -4\}$



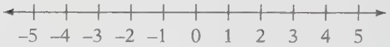
36. $\{x \mid x > 4\} \cup \{x \mid x < -2\}$



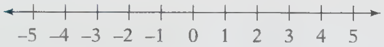
37. $\{x \mid x > -2\} \cap \{x \mid x < 4\}$



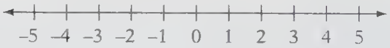
38. $\{x \mid x > -3\} \cap \{x \mid x < 3\}$



39. $\{x \mid x \geq -2\} \cup \{x \mid x < 4\}$



40. $\{x \mid x > 0\} \cup \{x \mid x \leq 4\}$



APPLYING THE CONCEPTS

41. Determine whether the statement is always true, sometimes true, or never true.
   a. Given that $a > 0$ and $b < 0$, then $ab > 0$.
   b. Given that $a < 0$, then $a^2 > 0$.
   c. Given that $a > 0$ and $b < 0$, then $a^2 > b$.

42. By trying various sets, make a conjecture about whether the union of two sets is
   a. a commutative operation
   b. an associative operation

43. By trying various sets, make a conjecture about whether the intersection of two sets is
   a. a commutative operation
   b. an associative operation

9.2

The Addition and Multiplication Properties of Inequalities

[illegible]

To solve an inequality using the Addition Property of Inequalities

The **solution set of an inequality** is a set of numbers each element of which, when substituted for the variable, results in a true inequality.

The inequality at the right is true if the variable is replaced by 7, 9.3, or $\frac{15}{2}$.

$$x + 5 > 8$$

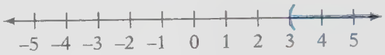
$$\left.\begin{aligned} 7 + 5 &> 8 \\ 9.3 + 5 &> 8 \\ \tfrac{15}{2} + 5 &> 8 \end{aligned}\right\} \text{True inequalities}$$

The inequality $x + 5 > 8$ is false if the variable is replaced by 2, 1.5, or $-\frac{1}{2}$.

$$\left.\begin{aligned} 2 + 5 &> 8 \\ 1.5 + 5 &> 8 \\ -\tfrac{1}{2} + 5 &> 8 \end{aligned}\right\} \text{False inequalities}$$

There are many values of the variable x that will make the inequality $x + 5 > 8$ true. The solution set of $x + 5 > 8$ is any number greater than 3.

At the right is the graph of the solution set of $x + 5 > 8$.



In solving an inequality, the goal is to rewrite the given inequality in the form *variable* > *constant* or *variable* < *constant*. The Addition Property of Inequalities is used to rewrite an inequality in this form.

Addition Property of Inequalities

The same term can be added to each side of an inequality without changing the solution set of the inequality.

If $a > b$, then $a + c > b + c$.
If $a < b$, then $a + c < b + c$.

The Addition Property of Inequalities also holds true for an inequality containing the symbol ≥ or ≤.

The Addition Property of Inequalities is used when, in order to rewrite an inequality in the form *variable* > *constant* or *variable* < *constant*, we must remove a term from one side of the inequality. Add the opposite of that term to each side of the inequality.

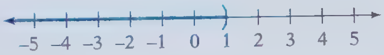
Solve: $x - 4 < -3$

$$x - 4 < -3$$

$$x - 4 + 4 < -3 + 4$$
- Add 4 to each side of the inequality.

$$x < 1$$
- Simplify.

At the right is the graph of the solution set of $x - 4 < -3$.



Because subtraction is defined in terms of addition, the Addition Property of Inequalities allows the same term to be subtracted from each side of an inequality.

Solve: $5x - 6 \le 4x - 4$

$5x - 6 \le 4x - 4$

$5x - 4x - 6 \le 4x - 4x - 4$ • Subtract $4x$ from each side of the inequality.

$x - 6 \le -4$ • Simplify.

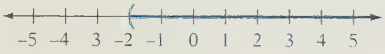
$x - 6 + 6 \le -4 + 6$ • Add 6 to each side of the inequality.

$x \le 2$ • Simplify.

Example 1
Solve $3 < x + 5$ and graph the solution set.

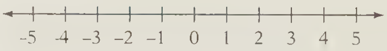
Solution

$$3 < x + 5$$
$$3 - 5 < x + 5 - 5$$
$$-2 < x$$



You Try It 1
Solve $x + 2 < -2$ and graph the solution set.

Your solution



Example 2
Solve: $7x - 14 \le 6x - 16$

Solution

$$7x - 14 \le 6x - 16$$
$$7x - 6x - 14 \le 6x - 6x - 16$$
$$x - 14 \le -16$$
$$x - 14 + 14 \le -16 + 14$$
$$x \le -2$$

You Try It 2
Solve: $5x + 3 > 4x + 5$

Your solution

Solutions on p. S22

[illegible]

To solve an inequality using the Multiplication Property of Inequalities

9

In solving an inequality, the goal is to rewrite the given inequality in the form *variable* > *constant* or *variable* < *constant*. The Multiplication Property of Inequalities is used when, in order to rewrite an inequality in this form, we must remove a coefficient from one side of the inequality.

Multiplication Property of Inequalities

Each side of an inequality can be multiplied by the same positive number without changing the solution set of the inequality.

If $a > b$ and $c > 0$, then $ac > bc$.
If $a < b$ and $c > 0$, then $ac < bc$.

If each side of an inequality is multiplied by the same negative number and the inequality symbol is reversed, then the solution set of the inequality is not changed.

If $a > b$ and $c < 0$, then $ac < bc$.
If $a < b$ and $c < 0$, then $ac > bc$.

TAKE NOTE

Any time an inequality is multiplied or divided by a negative number, the inequality symbol must be reversed. Compare the next two examples.

$2x < -4$ $\frac{2x}{2} < \frac{-4}{2}$ $x < -2$	Divide each side by *positive* 2. Inequality *is not* reversed.
$-2x < 4$ $\frac{-2x}{-2} > \frac{4}{-2}$ $x > -2$	Divide each side by *negative* 2. Inequality *is* reversed.

$5 > 4$	• A true inequality
$5(2) > 4(2)$	• Multiply by *positive* 2.
$10 > 8$	• Still a true inequality

$6 < 9$	• A true inequality
$6(-3) > 9(-3)$	• Multiply by *negative* 3 and *reverse* the inequality.
$-18 > -27$	• Still a true inequality

The Multiplication Property of Inequalities also holds true for an inequality containing the symbol ≥ or ≤.

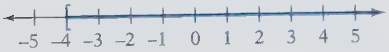
➡ Solve $-\frac{3}{2}x \le 6$ and graph the solution set.

$$-\frac{3}{2}x \le 6$$

$$-\frac{2}{3}\left(-\frac{3}{2}x\right) \ge -\frac{2}{3}(6)$$

$$x \ge -4$$

• Multiply each side of the inequality by $-\frac{2}{3}$. Because $-\frac{2}{3}$ is a negative number, the inequality symbol must be reversed.



• Graph $x \ge -4$.

Because division is defined in terms of multiplication, the Multiplication Property of Inequalities allows each side of an inequality to be divided by a nonzero constant.

TAKE NOTE

As shown in the example at the right, the goal in solving an inequality can be *constant* < *variable* or *constant* > *variable*. We could have written the answer to this example as $x > -\frac{2}{3}$.

➡ Solve: $-4 < 6x$

$-4 < 6x$	
$\frac{-4}{6} < \frac{6x}{6}$	• Divide each side of the inequality by 6.
$-\frac{2}{3} < x$	• Simplify: $\frac{-4}{6} = -\frac{2}{3}$

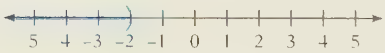
Example 3 Solve $-7x > 14$ and graph the solution set.

Solution

$$-7x > 14$$

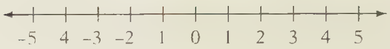
$$\frac{-7x}{-7} < \frac{14}{-7}$$

$$x < -2$$



You Try It 3 Solve $-3x > -9$ and graph the solution set.

Your solution



Example 4

Solve: $-\frac{5}{8}x \leq \frac{5}{12}$

Solution

$$-\frac{5}{8}x \leq \frac{5}{12}$$

$$-\frac{8}{5}\left(-\frac{5}{8}x\right) \geq -\frac{8}{5}\left(\frac{5}{12}\right)$$

$$x \geq -\frac{2}{3}$$

You Try It 4

Solve: $-\frac{3}{4}x \geq 18$

Your solution

Solutions on p. S22

Objective C To solve application problems 9

Example 5

A student must have at least 450 points out of 500 points on five tests to receive an A in a course. One student's results on the first four tests were 94, 87, 77, and 95. What scores on the last test will enable this student to receive an A in the course?

Strategy

To find the scores, write and solve an inequality using N to represent the possible scores on the last test.

Solution

Total number of points on the five tests	is greater than or equal to	450

$$94 + 87 + 77 + 95 + N \geq 450$$

$$353 + N \geq 450$$

$$353 - 353 + N \geq 450 - 353$$

$$N \geq 97$$

The student's score on the last test must be greater than or equal to 97.

You Try It 5

An appliance dealer will make a profit on the sale of a television set if the cost of the new set is less than 70% of the selling price. What selling prices will enable the dealer to make a profit on a television set that costs the dealer $314?

Your strategy

Your solution

Solution on p. S22

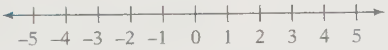
9.2 Exercises

Solve and graph the solution set.

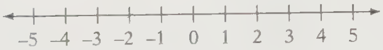
1. $x + 1 < 3$

2. $y + 2 < 2$

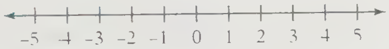
3. $x - 5 > -2$



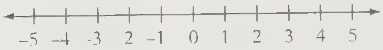
4. $x - 3 > -2$



5. $7 \leq n + 4$

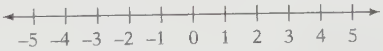


6. $3 \leq 5 + x$



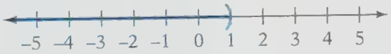
7. $x - 6 \leq -10$

8. $y - 8 \leq -11$



Write an inequality that represents the set of numbers shown in the graph.

9.



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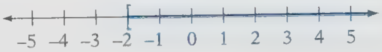

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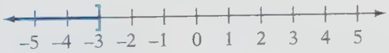

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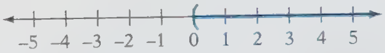


Solve.

13. $y - 3 \geq -12$

14. $x + 8 \geq -14$

15. $3x - 5 < 2x + 7$

16. $5x + 4 < 4x - 10$

17. $8x - 7 \geq 7x - 2$

18. $3n - 9 \geq 2n - 8$

19. $2x + 4 < x - 7$

20. $9x + 7 < 8x - 7$

21. $4x - 8 \leq 2 + 3x$

22. $5b - 9 < 3 + 4b$

23. $6x + 4 \geq 5x - 2$

24. $7x - 3 \geq 6x - 2$

25. $2x - 12 > x - 10$

26. $3x + 9 > 2x + 7$

27. $d + \frac{1}{2} < \frac{1}{3}$

28. $x - \frac{3}{8} < \frac{5}{6}$

29. $x + \frac{5}{8} \geq -\frac{2}{3}$

30. $y + \frac{5}{12} \geq -\frac{3}{4}$

31. $x - \frac{3}{8} < \frac{1}{4}$

32. $y + \frac{5}{9} \leq \frac{5}{6}$

33. $2x - \frac{1}{2} < x + \frac{3}{4}$

34. $6x - \frac{1}{3} \leq 5x - \frac{1}{2}$

35. $3x + \frac{5}{8} > 2x + \frac{5}{6}$

36. $4b - \frac{7}{12} \geq 3b - \frac{9}{16}$

37. $3.8x < 2.8x - 3.8$

38. $1.2x < 0.2x - 7.3$

39. $x + 5.8 \leq 4.6$

40. $n - 3.82 \leq 3.95$

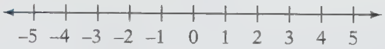
41. $x - 3.5 < 2.1$

42. $x - 0.23 \leq 0.47$

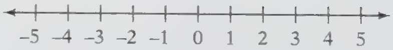
[illegible]

Solve and graph the solution set.

43. $3x < 12$



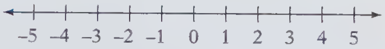
44. $8x \leq -24$



45. $15 \leq 5y$

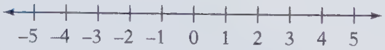
46. $-48 < 24x$

47. $16x \leq 16$

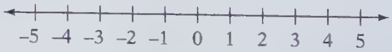


48. $3x > 0$

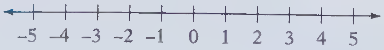
49. $-8x > 8$



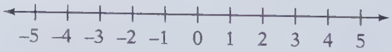
50. $-2n \leq -8$



51. $-6b > 24$



52. $-4x < 8$



Solve.

53. $-5y \geq 0$

54. $-3z < 0$

55. $7x > 2$

56. $6x \leq -1$

57. $2x \leq -5$

58. $\frac{5}{6}n < 15$

59. $\frac{3}{4}x < 12$

60. $\frac{2}{3}y \geq 4$

61. $10 \leq \frac{5}{8}x$

62. $4 \geq \frac{2}{3}x$

63. $-\frac{3}{7}x \leq 6$

64. $-\frac{2}{11}b \geq -6$

65. $-\frac{4}{7}x \geq -12$

66. $\frac{2}{3}n < \frac{1}{2}$

67. $-\frac{3}{5}x < 0$

68. $-\frac{2}{3}x \geq 0$

69. $-\frac{3}{8}x \geq \frac{9}{14}$

70. $-\frac{3}{5}x < -\frac{6}{7}$

71. $-\frac{4}{5}x < -\frac{8}{15}$

72. $-\frac{3}{4}y \geq -\frac{5}{8}$

73. $-\frac{8}{9}x \geq -\frac{16}{27}$

74. $1.5x \leq 6.30$

75. $2.3x \leq 5.29$

76. $-3.5d > 7.35$

77. $-0.24x > 0.768$

78. $4.25m > -34$

79. $-3.9x \geq -19.5$

Application Problems

80. Three-fifths of a number is greater than two-thirds. Find the smallest integer that satisfies this inequality.

81. To be eligible for a basketball tournament, a basketball team must win at least 60% of its remaining games. If the team has 17 games remaining, how many games must the team win to qualify for the tournament?

82. To avoid a tax penalty, at least 90% of a self-employed person's total annual income tax liability must be paid in estimated tax payments during the year. What amount of income tax must a person with an annual income tax liability of $3500 pay in estimated tax payments?

83. A service organization will receive a bonus of $200 for collecting more than 1850 lb of aluminum cans during its four collection drives. On the first three drives, the organization collected 505 lb, 493 lb, and 412 lb. How many pounds of cans must the organization collect on the fourth drive to receive the bonus?

84. Computer software engineers are fond of saying that software takes at least twice as long to develop as they think it will. According to that saying, how many hours will it take to develop a software product that an engineer thinks can be finished in 50 h?

85. A government agency recommends a minimum daily allowance of vitamin C of 60 mg. How many additional milligrams of vitamin C does a person who has already drunk a glass of orange juice with 10 mg of vitamin C need in order to satisfy the recommended daily allowance?

86. To pass a course with a B grade, a student must have an average of 80 points on five tests. The student's grades on the first four tests were 75, 83, 86, and 78. What scores can the student receive on the fifth test to earn a B grade?

87. A professor scores all tests with a maximum of 100 points. To earn an A grade in this course, a student must have an average of 92 on four tests. One student's grades on the first three tests were 89, 86, and 90. Can this student earn an A grade?

88. A health official recommends a maximum cholesterol level of 200 units. How many units must a patient with a cholesterol level of 275 units reduce her cholesterol level to satisfy the recommended maximum level?

APPLYING THE CONCEPTS

Given that $a > b$ and that a and b are real numbers, determine for which real numbers c the statement is true. Use set-builder notation to write the answer.

89. $ac > bc$

90. $ac < bc$

91. $a + c > b + c$

92. $a + c < b + c$

93. $\dfrac{a}{c} > \dfrac{b}{c}$

94. $\dfrac{a}{c} < \dfrac{b}{c}$

95. In your own words, state the Addition Property of Inequalities.

96. In your own words, state the Multiplication Property of Inequalities.

9.3 General Inequalities

To solve general inequalities

Solving an inequality frequently requires application of both the Addition and the Multiplication Properties of Inequalities.

Solve: $4y - 3 \geq 6y + 5$

$4y - 3 \geq 6y + 5$	
$4y - 6y - 3 \geq 6y - 6y + 5$	• Subtract $6y$ from each side of the inequality.
$-2y - 3 \geq 5$	• Simplify.
$-2y - 3 + 3 \geq 5 + 3$	• Add 3 to each side of the inequality.
$-2y \geq 8$	• Simplify.
$\dfrac{-2y}{-2} \leq \dfrac{8}{-2}$	• Divide each side of the inequality by -2. Because -2 is a negative number, the inequality symbol must be reversed.
$y \leq -4$	

When an inequality contains parentheses, one of the steps in solving the inequality requires the use of the Distributive Property.

Solve: $-2(x - 7) > 3 - 4(2x - 3)$

$-2(x - 7) > 3 - 4(2x - 3)$	
$-2x + 14 > 3 - 8x + 12$	• Use the Distributive Property to remove parentheses.
$-2x + 14 > -8x + 15$	• Simplify.
$-2x + 8x + 14 > -8x + 8x + 15$	• Add $8x$ to each side of the inequality.
$6x + 14 > 15$	• Simplify.
$6x + 14 - 14 > 15 - 14$	• Subtract 14 from each side of the inequality.
$6x > 1$	• Simplify.
$\dfrac{6x}{6} > \dfrac{1}{6}$	• Divide each side of the inequality by 6.
$x > \dfrac{1}{6}$	

Example 1 Solve: $7x - 3 \leq 3x + 17$

Solution

$$7x - 3 \leq 3x + 17$$
$$7x - 3x - 3 \leq 3x - 3x + 17$$
$$4x - 3 \leq 17$$
$$4x - 3 + 3 \leq 17 + 3$$
$$4x \leq 20$$
$$\frac{4x}{4} \leq \frac{20}{4}$$
$$x \leq 5$$

You Try It 1 Solve: $5 - 4x > 9 - 8x$

Your solution

Solution on p. S22

Example 2
Solve:
$3(3 - 2x) \geq -5x - 2(3 - x)$

Solution

$$\begin{aligned} 3(3 - 2x) &\geq -5x - 2(3 - x) \\ 9 - 6x &\geq -5x - 6 + 2x \\ 9 - 6x &\geq -3x - 6 \\ 9 - 6x + 3x &\geq -3x + 3x - 6 \\ 9 - 3x &\geq -6 \\ 9 - 9 - 3x &\geq -6 - 9 \\ -3x &\geq -15 \\ \frac{-3x}{-3} &\leq \frac{-15}{-3} \\ x &\leq 5 \end{aligned}$$

You Try It 2
Solve:
$8 - 4(3x + 5) \leq 6(x - 8)$

Your solution

Solution on p. S22

To solve application problems

Example 3
A rectangle is 10 ft wide and $(2x + 4)$ ft long. Express as an integer the maximum length of the rectangle when the area is less than 200 ft^2. (The area of a rectangle is equal to its length times its width.)

Strategy
To find the maximum length:

- Replace the variables in the area formula by the given values and solve for x.
- Replace the variable in the expression $2x + 4$ with the value found for x.

Solution

Length times width	is less than	200 ft^2

$$\begin{aligned} 10(2x + 4) &< 200 \\ 20x + 40 &< 200 \\ 20x + 40 - 40 &< 200 - 40 \\ 20x &< 160 \\ \frac{20x}{20} &< \frac{160}{20} \\ x &< 8 \end{aligned}$$

The length is $(2x + 4)$ ft. Because $x < 8$, $2x + 4 < 2(8) + 4 = 20$. Therefore, the length is less than 20 ft. The maximum length is 19 ft.

You Try It 3
Company A rents cars for \$8 a day and \$.10 for every mile driven. Company B rents cars for \$10 a day and \$.08 per mile driven. You want to rent a car for 1 week. What is the maximum number of miles you can drive a Company A car if it is to cost you less than a Company B car?

Your strategy

Your solution

Solution on pp. S22–S23

9.3 Exercises

[illegible]

Solve.

1. $4x - 8 < 2x$

2. $7x - 4 < 3x$

3. $2x - 8 > 4x$

4. $3y + 2 > 7y$

5. $8 - 3x \le 5x$

6. $10 - 3x \le 7x$

7. $3x + 2 > 5x - 8$

8. $2n - 9 \ge 5n + 4$

9. $5x - 2 < 3x - 2$

10. $8x - 9 > 3x - 9$

11. $0.1(180 + x) > x$

12. $x > 0.2(50 + x)$

13. $2(2y - 5) \le 3(5 - 2y)$

14. $2(5x - 8) \le 7(x - 3)$

15. $5(2 - x) > 3(2x - 5)$

16. $4(3d - 1) > 3(2 - 5d)$

17. $4 - 3(3 - n) \le 3(2 - 5n)$

18. $15 - 5(3 - 2x) \le 4(x - 3)$

19. $2x - 3(x - 4) \ge 4 - 2(x - 7)$

20. $4 + 2(3 - 2y) \le 4(3y - 5) - 6y$

[illegible] *Application Problems*

21. The sales agent for a jewelry company is offered a flat monthly salary of \$3200 or a salary of \$1000 plus an 11% commission on the selling price of each item sold by the agent. If the agent chooses the \$3200, what dollar amount does the agent expect to sell in 1 month?

22. A baseball player is offered an annual salary of \$200,000 or a base salary of \$100,000 plus a bonus of \$1000 for each hit over 100 hits. How many hits must the baseball player make to earn more than \$200,000?

23. A computer bulletin board service charges a flat fee of \$10 per month or a fee of \$4 per month plus \$.10 for each minute the service is used. How many minutes must a person use this service to exceed \$10?

24. A site licensing fee for a computer program is \$1500. Paying this fee allows the company to use the program at any computer terminal within the company. Alternatively, the company can choose to pay \$200 for each individual computer it has. How many individual computers must a company have for the site license to be more economical for the company?

25. For a product to be labeled orange juice, a state agency requires that at least 80% of the drink be real orange juice. How many ounces of artificial flavors can be added to 32 oz of real orange juice and have it still be legal to label the drink orange juice?

26. Grade A hamburger cannot contain more than 20% fat. How much fat can a butcher mix with 300 lb of lean meat to meet the 20% requirement?

27. A shuttle service taking skiers to a ski area charges \$8 per person each way. Four skiers are debating whether to take the shuttle bus or rent a car for \$45 plus \$.25 per mile. Assuming that the skiers will share the cost of the car and that they want the least expensive method of transportation, find how far away the ski area is if they choose the shuttle service.

APPLYING THE CONCEPTS

28. Determine whether the statement is always true, sometimes true, or never true, given that a, b, and c are real numbers.

a. If $a > b$, then $-a > -b$.
b. If $a < b$, then $ac < bc$.
c. If $a > b$, then $a + c > b + c$.
d. If $a \neq 0$, $b \neq 0$, and $a > b$, then $\frac{1}{a} > \frac{1}{b}$.

Use the roster method to list the set of positive integers that are solutions of the inequality.

29. $7 - 2b \leq 15 - 5b$

30. $-6(2 - d) \geq 4d - 9$

Use the roster method to list the set of integers that are common to the solution sets of the two inequalities.

31. $5x - 12 \leq x + 8$
$3x - 4 \geq 2 + x$

32. $3(x + 2) > 9x - 2$
$4(x + 5) > 3(x + 6)$

33. Determine the solution set of $2 - 3(x + 4) < 5 - 3x$.

34. Determine the solution set of $3x + 2(x - 1) > 5(x + 1)$.

9.4 Graphing Linear Inequalities

Objective A

To graph an inequality in two variables

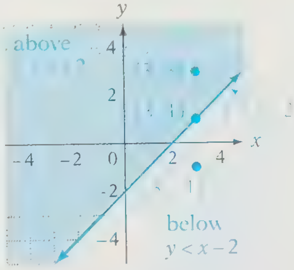
9

Point of Interest

Linear inequalities play an important role in applied mathematics. They are used in a branch of mathematics called *linear programming*, which was developed during World War II to solve problems in supplying the Air Force with the machine parts necessary to keep planes flying. Today, its applications extend to many other disciplines.

The graph of the linear equation $y = x - 2$ separates a plane into three sets:

The set of points on the line
The set of points above the line
The set of points below the line



The point $(3, 1)$ is a solution of $y = x - 2$.

$$\begin{array}{c|c} y & = x - 2 \\ \hline 1 & 3 - 2 \\ 1 & = 1 \end{array}$$

The point $(3, 3)$ is a solution of $y > x - 2$.

$$\begin{array}{c|c} y & > x - 2 \\ \hline 3 & 3 - 2 \\ 3 & > 1 \end{array}$$

Any point above the line is a solution of $y > x - 2$.

The point $(3, -1)$ is a solution of $y < x - 2$.

$$\begin{array}{c|c} y & < x - 2 \\ \hline -1 & 3 - 2 \\ -1 & < 1 \end{array}$$

Any point below the line is a solution of $y < x - 2$.

The solution set of $y = x - 2$ is all points on the line. The solution set of $y > x - 2$ is all points above the line. The solution set of $y < x - 2$ is all points below the line. The solution set of an inequality in two variables is a **half-plane.**

The following illustrates the procedure for graphing a linear inequality.

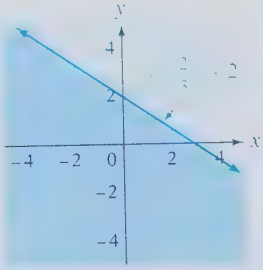
⇨ Graph the solution set of $2x + 3y \leq 6$.

Solve the inequality for y.

$$\begin{aligned} 2x + 3y &\leq 6 \\ 2x - 2x + 3y &\leq -2x + 6 && \bullet \text{ Subtract } 2x \text{ from each side.} \\ 3y &\leq -2x + 6 && \bullet \text{ Simplify.} \\ \frac{3y}{3} &\leq \frac{-2x + 6}{3} && \bullet \text{ Divide each side by 3.} \\ y &\leq -\frac{2}{3}x + 2 && \bullet \text{ Simplify.} \end{aligned}$$

Change the inequality to an equality and graph $y = -\frac{2}{3}x + 2$. If the inequality is $\geq$ **or** $\leq$, the line is in the solution set and is shown by a **solid line.** If the inequality is $>$ **or** $<$, the line is not a part of the solution set and is shown by a **dotted line.**

If the inequality is $>$ **or** $\geq$, shade the **upper half-plane.** If the inequality is $<$ **or** $\leq$, shade the **lower half-plane.**



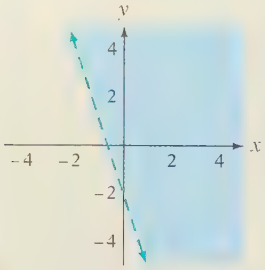
Example 1

Graph the solution set of $3x + y > -2$.

Solution

$$\begin{aligned} 3x + y &> -2 \\ 3x - 3x + y &> -3x - 2 \\ y &> -3x - 2 \end{aligned}$$

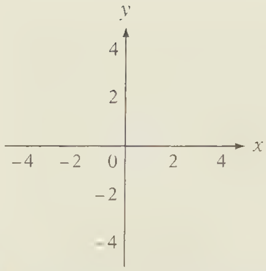
Graph $y = -3x - 2$ as a dotted line.
Shade the upper half-plane.



You Try It 1

Graph the solution set of $x - 3y < 2$.

Your solution



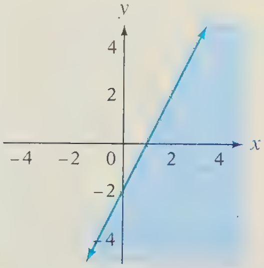
Example 2

Graph the solution set of $2x - y \geq 2$.

Solution

$$\begin{aligned} 2x - y &\geq 2 \\ 2x - 2x - y &\geq -2x + 2 \\ -y &\geq -2x + 2 \\ -1(-y) &\leq -1(-2x + 2) \\ y &\leq 2x - 2 \end{aligned}$$

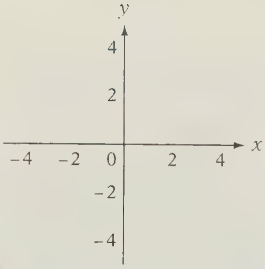
Graph $y = 2x - 2$ as a solid line. Shade the lower half-plane.



You Try It 2

Graph the solution set of $2x - 4y \leq 8$.

Your solution

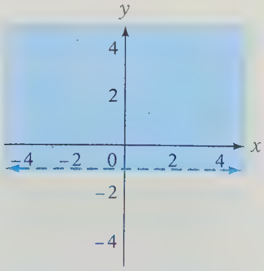


Example 3

Graph the solution set of $y > -1$.

Solution

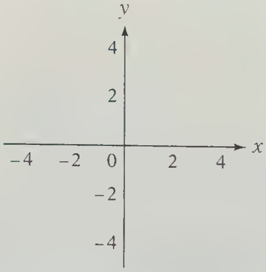
Graph $y = -1$ as a dotted line.
Shade the upper half-plane.



You Try It 3

Graph the solution set of $x < 3$.

Your solution



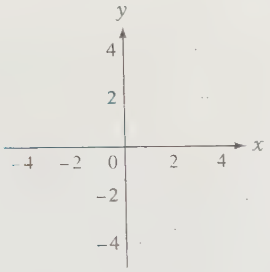
Solutions on p. S23

9.4 Exercises

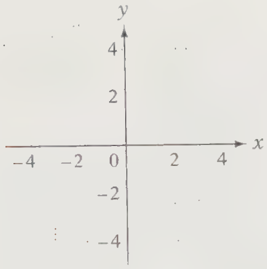
[illegible]

Graph the solution set.

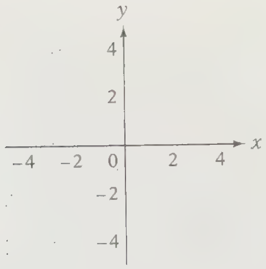
1. $x + y > 4$



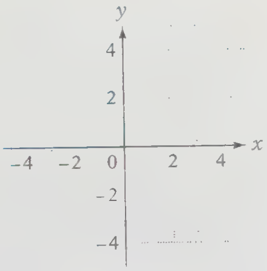
2. $x - y > -3$



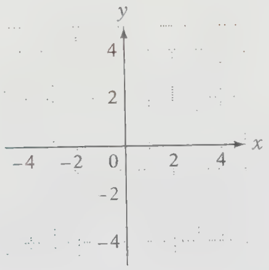
3. $2x - y < -3$



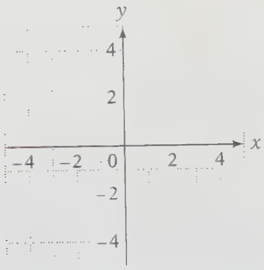
4. $3x - y < 9$



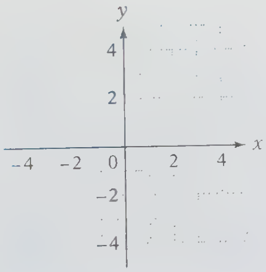
5. $2x + y \geq 4$



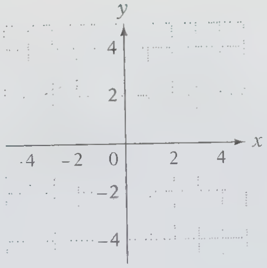
6. $3x + y \geq 6$



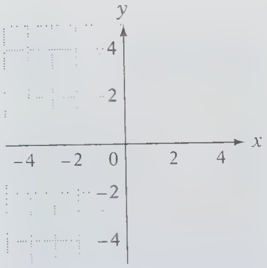
7. $y \leq -2$



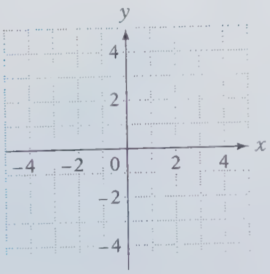
8. $y > 3$



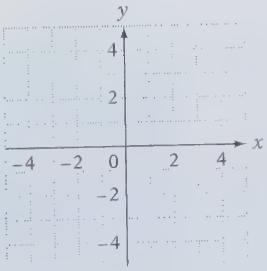
9. $3x - 2y < 8$



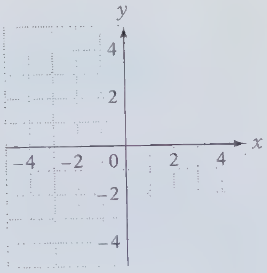
10. $5x + 4y > 4$



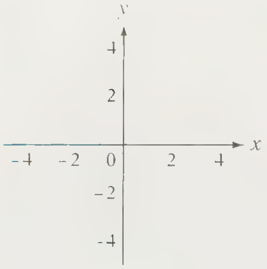
11. $-3x - 4y \geq 4$



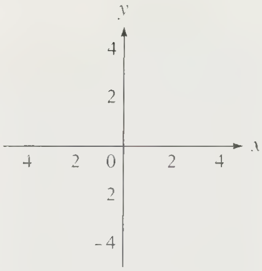
12. $-5x - 2y \geq 8$



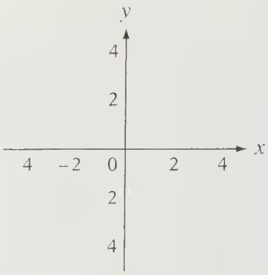
13. $6x + 5y \leq -10$



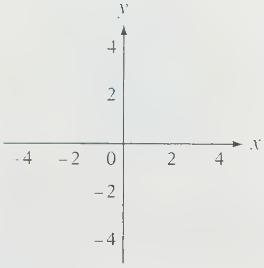
14. $2x + 2y \leq -4$



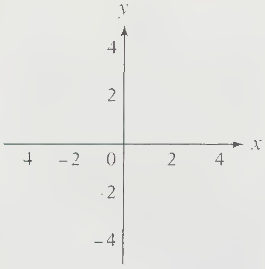
15. $-4x + 3y < -12$



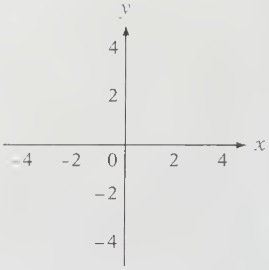
16. $-4x + 5y < 15$



17. $-2x + 3y \leq 6$



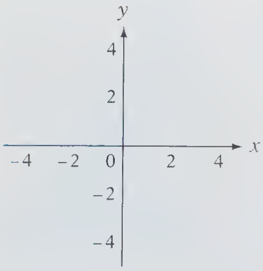
18. $3x - 4y > 12$



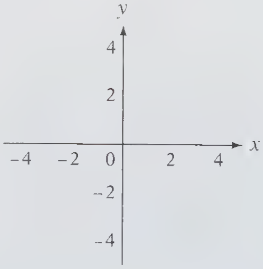
APPLYING THE CONCEPTS

Graph the solution set.

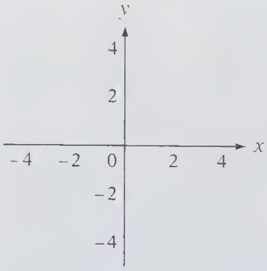
19. $\frac{x}{4} + \frac{y}{2} > 1$



20. $2x - 3(y + 1) > y - (4 - x)$

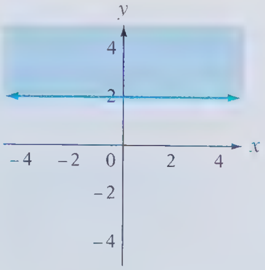


21. $4y - 2(x + 1) \geq 3(y - 1) + 3$

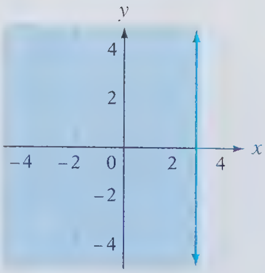


Write the inequality given its graph.

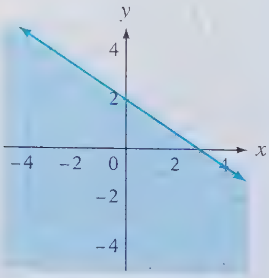
22.



23.



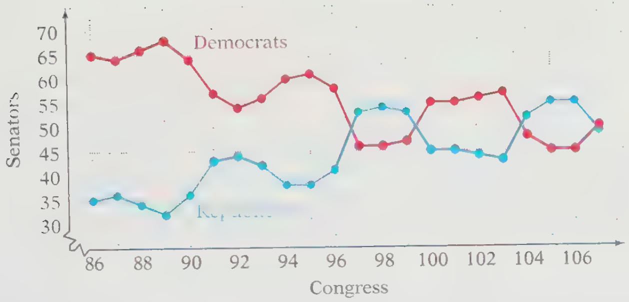
24.



Focus on Problem Solving

Graphing Data

Graphs are very useful in displaying data. By studying a graph, we can reach various conclusions about the data.



The double-line graph at the left shows the number of Democrats and the number of Republicans in the U.S. Senate for the 86th Congress (1959–1961) through the 107th Congress (2001–2003).

1. How many Democratic and how many Republican senators were in the 90th Congress?
2. In which Congress was the difference between the numbers of Democrats and Republicans the greatest?
3. In which Congress did the majority first change from Democratic to Republican?
4. Between which two Congresses did the number of Republican senators increase but the number of Democratic senators remain the same?
5. In what percent of the Congresses did the number of Democrats exceed the number of Republicans?
6. In which Congresses were there a greater number of Republican senators than Democratic senators?

The table at the right, based on data from Dataquest, shows the number (in millions) of Internet subscribers who used cable modems and the number (in millions) who used digital subscriber line (DSL) modems for the years 1998 through 2002.

Year	*Modem*	*DSL*
1998	0.8	0.4
1999	1.9	1.0
2000	3.0	2.9
2001	3.8	4.2
2002(est.)	4.6	6.5

Source: Dataquest as reported at **internet.about.com** on March 15, 2001

7. Make a double-line graph of the data.
8. In which year did the number of DSL subscribers first exceed the number of cable modem subscribers?
9. For the year 2003, would you expect the number of cable modem subscribers to increase more or less than the number of DSL subscribers?

Projects and Group Activities

Mean and Standard Deviation

An automotive engineer tests the miles-per-gallon ratings of 15 cars and records the results as follows:

25 22 21 27 25 35 29 31 25 26 21 39 34 32 28

The **mean** of the data is the sum of the measurements divided by the number of measurements. The symbol for the mean is $\bar{x}$.

$$\text{Mean} = \bar{x} = \frac{\text{sum of all data values}}{\text{number of data values}}$$

To find the mean for the data above, add the numbers and then divide by 15.

$$\bar{x} = \frac{25 + 22 + 21 + 27 + 25 + 35 + 29 + 31 + 25 + 26 + 21 + 39 + 34 + 32 + 28}{15}$$

$$= \frac{420}{15} = 28$$

The mean number of miles per gallon for the 15 cars tested was 28 mi/gal.

The mean is one of the most frequently computed averages. It is the one that is commonly used to calculate a student's performance in a class.

The scores for a history student on five tests were 78, 82, 91, 87, and 93. What was the mean score for this student?

To find the mean, add the numbers. Then divide by 5.

$$\bar{x} = \frac{78 + 82 + 91 + 87 + 93}{5}$$

$$= \frac{431}{5} = 86.2$$

The mean score for the history student was 86.2.

Consider two students, each of whom has taken five exams.

Scores for student A

84	86	83	85	87

$$\bar{x} = \frac{84 + 86 + 83 + 85 + 87}{5} = \frac{425}{5} = 85$$

The mean for student A is 85.

Scores for student B

90	75	94	68	98

$$\bar{x} = \frac{90 + 75 + 94 + 68 + 98}{5} = \frac{425}{5} = 85$$

The mean for student B is 85.

For each of these students, the mean (average) for the five exams is 85. However, student A has a more consistent record of scores than student B. One way to measure the consistency, or "clustering" near the mean, of data is to use the **standard deviation.**

To calculate the standard deviation:

Step 1. Sum the squares of the differences between each value of the data and the mean.

Step 2. Divide the result in Step 1 by the number of items in the set of data.

Step 3. Take the square root of the result in Step 2.

The calculation for student A is shown at the right.

Step 1:

x	$x - \overline{x}$	$(x - \overline{x})^2$
84	$84 - 85$	$(-1)^2 = 1$
86	$86 - 85$	$1^2 = 1$
83	$83 - 85$	$(-2)^2 = 4$
85	$85 - 85$	$0^2 = 0$
87	$87 - 85$	$2^2 = 4$
		Total = 10

The symbol for standard deviation is the lower-case Greek letter *sigma*, σ.

Step 2: $\frac{10}{5} = 2$

Step 3: $\sigma = \sqrt{2} \approx 1.414$

The standard deviation for student A's scores is approximately 1.414.

Following a similar procedure for student B shows that the standard deviation for student B's scores is approximately 11.524. Because the standard deviation of student B's scores is greater than that of student A's ($11.524 > 1.414$), student B's scores are not as consistent as those of student A.

1. The weights in ounces of 6 newborn infants were recorded by a hospital. The weights were 96, 105, 84, 90, 102, and 99. Find the standard deviation of the weights.
2. The numbers of rooms occupied in a hotel on 6 consecutive days were 234, 321, 222, 246, 312, and 396. Find the standard deviation for the number of rooms occupied.
3. Seven coins were tossed 100 times. The numbers of heads recorded for each coin were 56, 63, 49, 50, 48, 53, and 52. Find the standard deviation of the number of heads.
4. The temperatures, in degrees Fahrenheit, for 11 consecutive days at a desert resort were 95°, 98°, 98°, 104°, 97°, 100°, 96°, 97°, 108°, 93°, and 104°. For the same days, temperatures in Antarctica were 27°, 28°, 28°, 30°, 28°, 27°, 30°, 25°, 24°, 26°, and 21°. Which location has the greater standard deviation of temperatures?
5. The scores for 5 college basketball games were 56, 68, 60, 72, and 64. The scores for 5 professional basketball games were 106, 118, 110, 122, and 114. Which scores have the greater standard deviation?
6. The weights in pounds of the 5-man front line of a college football team are 210, 245, 220, 230, and 225. Find the standard deviation of the weights.
7. One student received test scores of 85, 92, 86, and 89. A second student received scores of 90, 97, 91, and 94 (exactly 5 points more on each test). Are the means of the two students the same? If not, what is the relationship between the means of the two students? Are the standard deviations of the scores of the two students the same? If not, what is the relationship between the standard deviations of the scores of the two students?
8. Grade-point average (GPA) is a *weighted* mean. It is called a weighted mean because a grade in a 5-unit course has more influence on your GPA than a grade in a 2-unit course. GPA is calculated by multiplying the numerical equivalent of each grade by the number of units, adding those products, and then dividing by the total number of units. Calculate your GPA for the last quarter or semester.

9. If you average 40 mph for 1 h and then 50 mph for 1 h, is your average speed $\frac{40 + 50}{2} = 45$ mph? Why or why not?

10. A company is negotiating with its employees the terms of a raise in salary. One proposal would add \$500 a year to each employee's salary. The second proposal would give each employee a 4% raise. Explain how each of these proposals would affect the current mean and standard deviation of salaries for the company.

Chapter Summary

Key Words

A *set* is a collection of objects. The objects of a set are called the *elements* of the set. [p. 411]

The *roster method* of writing a set encloses a list of the elements in braces. [p. 411]

The *empty set,* or *null set,* written $\emptyset$ or $\{\ \}$, is the set that contains no elements. [p. 411]

The *union* of two sets, written $A \cup B$, is the set that contains all the elements of A and all the elements of B (any elements that are in both sets A and B are listed only once). [p. 411]

The *intersection* of two sets, written $A \cap B$, is the set that contains the elements that are common to both A and B. [p. 411]

An *inequality* is an expression that contains the symbol $<$, $>$, $\le$, or $\ge$. [p. 413]

The *solution set of an inequality* is a set of numbers each element of which, when substituted for the variable, results in a true inequality. The solution set of an inequality can be graphed on the number line. [p. 417]

The solution set of an inequality in two variables is a *half-plane*. [p. 429]

Essential Rules

Addition Property of Inequalities

The same term can be added to each side of an inequality without changing the solution set of the inequality.

$$\text{If } a > b, \text{ then } a + c > b + c.$$
$$\text{If } a < b, \text{ then } a + c < b + c.$$

The Addition Property of Inequalities also holds true for an inequality containing the symbol $\ge$ or $\le$. [p. 417]

Multiplication Property of Inequalities

Each side of an inequality can be multiplied by the same **positive number** without changing the solution set of the inequality.

$$\text{If } a > b \text{ and } c > 0, \text{ then } ac > bc.$$
$$\text{If } a < b \text{ and } c > 0, \text{ then } ac < bc.$$

If each side of an inequality is multiplied by the same **negative number** and the inequality symbol is reversed, then the solution set of the inequality is not changed.

$$\text{If } a > b \text{ and } c < 0, \text{ then } ac < bc.$$
$$\text{If } a < b \text{ and } c < 0, \text{ then } ac > bc.$$

The Multiplication Property of Inequalities also holds true for an inequality containing the symbol $\ge$ or $\le$. [p. 419]

Chapter Review

1. Solve: $2x - 3 > x + 15$

2. Find $A \cap B$, given $A = \{0, 2, 4, 6, 8\}$ and $B = \{-2, -4\}$.

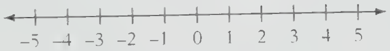
3. Use set-builder notation to write the set of odd integers greater than -8.

4. Find $A \cup B$, given $A = \{6, 8, 10\}$ and $B = \{2, 4, 6\}$.

5. Use the roster method to write the set of odd positive integers less than 8.

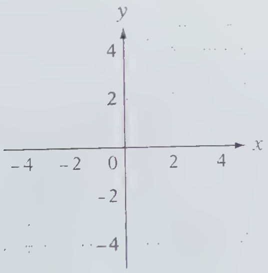
6. Solve: $12 - 4(x - 1) \leq 5(x - 4)$

7. Graph: $\{x \mid x > 3\}$

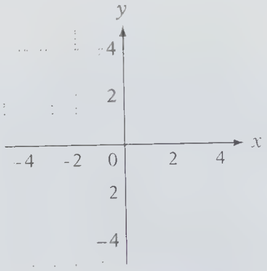


8. Solve: $3x + 4 \geq -8$

9. Graph: $3x + 2y \leq 12$

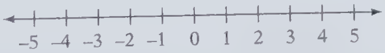


10. Graph: $5x + 2y < 6$



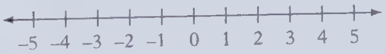
11. Use set-builder notation to write the set of real numbers greater than 3.

12. Solve and graph the solution set of $x - 3 > -1$.

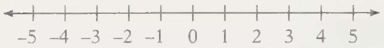


13. Find $A \cap B$, given $A = \{1, 5, 9, 13\}$ and $B = \{1, 3, 5, 7, 9\}$.

14. Graph: $\{x \mid x < 2\} \cup \{x \mid x > 5\}$



15. Graph: $\{x|x > -1\} \cap \{x|x \leq 2\}$



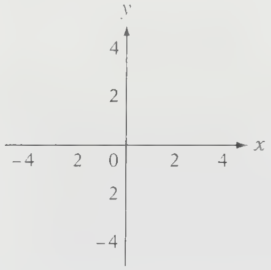
16. Solve: $-15x \leq 45$

17. Solve: $6x - 9 < 4x + 3(x + 3)$

18. Solve: $5 - 4(x + 9) > 11(12x - 9)$

19. Solve: $-\frac{3}{4}x > \frac{2}{3}$

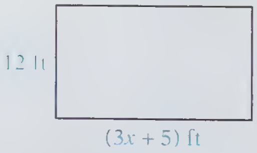
20. Graph: $2x - 3y < 9$



21. Solve: $7x - 2(x + 3) \geq x + 10$

22. Florist A charges a \$3 delivery fee plus \$21 per bouquet delivered. Florist B charges a \$15 delivery fee plus \$18 per bouquet delivered. A church wants to supply each resident of a small nursing home with a bouquet for Grandparents Day. Find the number of residents of the nursing home if using florist B is more economical than using florist A.

23. The width of a rectangular garden is 12 ft. The length of the garden is $(3x + 5)$ ft. Express as an integer the minimum length of the garden when the area is greater than 276 ft^2. (The area of a rectangle is equal to its length times its width.)

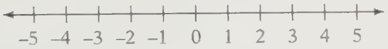


24. Six less than a number is greater than twenty-five. Find the smallest integer that will satisfy the inequality.

25. A student's grades on five sociology tests were 68, 82, 90, 73, and 95. What is the lowest score the student can receive on the next test and still be able to attain a minimum of 480 points?

Chapter Test

1. Graph: $\{x|x < 5\} \cap \{x|x > 0\}$



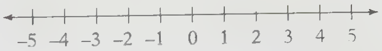
2. Use set-builder notation to write the set of the positive integers less than 50.

3. Use the roster method to write the set of the even positive integers between 3 and 9.

4. Solve: $3(2x - 5) \geq 8x - 9$

5. Solve: $x + \frac{1}{2} > \frac{5}{8}$

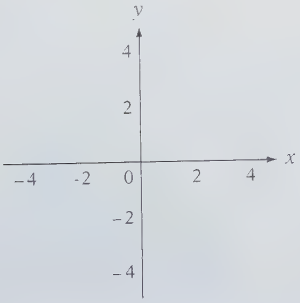
6. Graph: $\{x|x > -2\}$



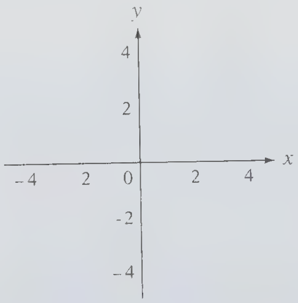
7. Solve: $5 - 3x > 8$

8. Use set-builder notation to write the set of the real numbers greater than -23.

9. Graph the solution set of $3x + y > 4$.

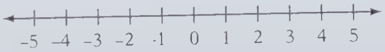


10. Graph the solution set of $4x - 5y \geq 15$.



11. Find $A \cap B$, given $A = \{6, 8, 10, 12\}$ and $B = \{12, 14, 16\}$.

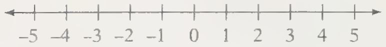
12. Solve and graph the solution set of $4 + x < 1$.



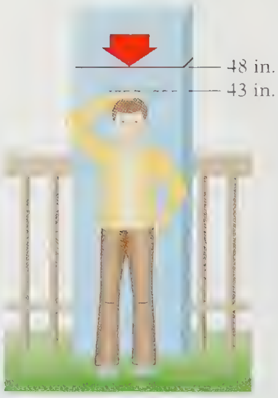
13. Solve: $-\frac{3}{8}x \leq 5$

14. Solve: $6x - 3(2 - 3x) < 4(2x - 7)$

15. Solve and graph the solution set of $\frac{2}{3}x \geq 2$.

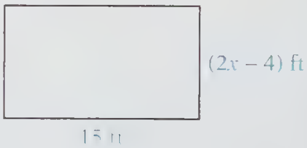


16. Solve: $2x - 7 \leq 6x + 9$

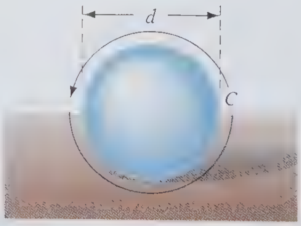


17. To ride a certain roller coaster at an amusement park, a person must be at least 48 in. tall. How many inches must a child who is 43 in. tall grow to be eligible to ride the roller coaster?

18. A rectangle is 15 ft long and $(2x - 4)$ ft wide. Express as an integer the maximum width of the rectangle if the area is less than 180 ft^2. (The area of a rectangle is equal to its length times its width.)



19. A ball bearing for a rotary engine must have a circumference between 0.1220 in. and 0.1240 in. What are the allowable diameters for the bearing to the nearest ten-thousandth of an inch? Recall that $C = \pi d$.



20. A stockbroker receives a monthly salary that is the greater of \$2500 or \$1000 plus 2% of the total value of all stock transactions the broker processes during the month. What dollar amounts of transactions did the broker process in a month for which the broker's salary was \$2500?

Cumulative Review

1. Simplify: $2[5a - 3(2 - 5a) - 8]$

2. Solve: $\frac{5}{8} \quad 4x = \frac{1}{8}$

3. Solve: $2x - 3[x - 2(x - 3)] = 2$

4. Simplify: $(-3a)(-2a^3b^2)^2$

5. Simplify: $\frac{27a^3b^2}{(-3ab^2)^3}$

6. Simplify: $(16x^2 - 12x - 2) \div (4x - 1)$

7. Given $f(x) = x^2 - 4x - 5$, find $f(-1)$.

8. Factor: $27a^2x^2 - 3a^2$

9. Simplify: $\frac{x^2 - 2x}{x^2 - 2x - 8} \div \frac{x^3 - 5x^2 + 6x}{x^2 - 7x + 12}$

10. Simplify: $\frac{4a}{2a - 3} - \frac{2a}{a + 3}$

11. Solve: $\frac{5y}{6} - \frac{5}{9} = \frac{y}{3} - \frac{5}{6}$

12. Solve $R = \frac{C - S}{t}$ for C.

13. Find the slope of the line that passes through the points $(2, -3)$ and $(-1, 4)$.

14. Find the equation of the line that passes through the point $(1, -3)$ and has slope $-\frac{3}{2}$.

15. Solve by substitution.

$$x = 3y + 1$$
$$2x + 5y = 13$$

16. Solve by the addition method.

$$9x - 2y = 17$$
$$5x + 3y = -7$$

17. Find $A \cup B$, given $A = \{0, 1, 2\}$ and $B = \{-10, -2\}$.

18. Use set-builder notation to write the set of the real numbers less than 48.

19. Graph: $\{x \mid x > 1\} \cup \{x \mid x < -1\}$

20. Graph the solution set of $\frac{3}{8}x > -\frac{3}{4}$.

21. Solve: $-\frac{4}{5}x > 12$

22. Solve: $15 - 3(5x - 7) < 2(7 - 2x)$

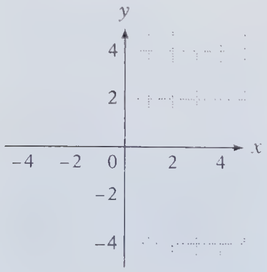
23. Three-fifths of a number is less than negative fifteen. What integers satisfy this inequality? Write the answer in set-builder notation.

24. Company A rents cars for \$6 a day and \$.25 for every mile driven. Company B rents cars for \$15 a day and \$.10 per mile. You want to rent a car for 6 days. What is the maximum number of miles you can drive a Company A car if it is to cost you less than a Company B car?

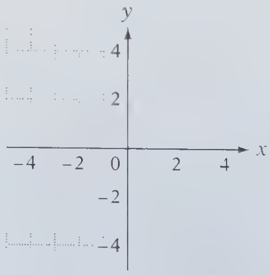
25. In a lake, 100 fish are caught, tagged, and then released. Later, 150 fish are caught. Three of these 150 fish are found to have tags. Estimate the number of fish in the lake.

26. The first angle of a triangle is 30° more than the second angle. The third angle is 10° more than twice the second angle. Find the measure of each angle.

27. Graph: $y = 2x - 1$



28. Graph the solution set of $6x - 3y \geq 6$.



Chapter 10 Radical Expressions

These floating icebergs stand out dramatically against the horizon. A submarine periscope would magnify and bring an object like this one clearly into focus. The periscope could also be used to determine the distance from the submarine to the iceberg. A simple formula involving a radical expression is used to calculate this distance, given the height of the periscope above the surface of the water. The **Project on page 469** shows how to use a radical equation to calculate the distance.

Objectives

Section 10.1

- **A** To simplify numerical radical expressions
- **B** To simplify variable radical expressions

Section 10.2

- **A** To add and subtract radical expressions

Section 10.3

- **A** To multiply radical expressions
- **B** To divide radical expressions

Section 10.4

- **A** To solve an equation containing a radical expression
- **B** To solve application problems

Need help? For online student resources, such as section quizzes, visit this textbook's website at **math.college.hmco.com/students.**

Prep Test

1. Evaluate: $-|-14|$

2. Simplify: $3x^2y - 4xy^2 - 5x^2y$

3. Solve: $1.5h = 21$

4. Solve: $3x - 2 = 5 - 2x$

5. Simplify: $x^3 \cdot x^3$

6. Simplify: $(x + y)^2$

7. Simplify: $(2x - 3)^2$

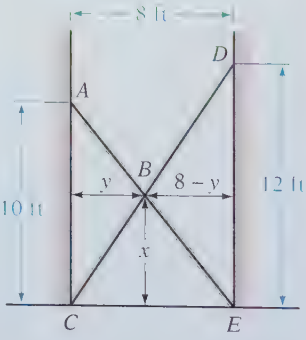
8. Simplify: $(2 - 3v)(2 + 3v)$

9. Simplify: $(a - 5)(a + 5)$

10. Simplify: $\dfrac{2x^4y^3}{18x^2y}$

Go Figure

Two pieces of lumber are used to support two walls as shown in the figure. How high above the ground do the pieces of lumber cross? (Hint: Triangles *ABC* and *DBE* are similar triangles. Therefore, the ratio of the corresponding heights equals the ratio of corresponding sides. That is, $\frac{y}{8-y} = \frac{10}{12}$. Solve for y and use this value to write another equation based on different similar triangles.)



10.1 Introduction to Radical Expressions

Objective A

To simplify numerical radical expressions

10

Point of Interest

The radical symbol was first used in 1525 but was written as √. Some historians suggest that the radical symbol also developed into the symbols for "less than" and "greater than." Because typesetters of that time did not want to make additional symbols, the radical was rotated to the position > and used as a "greater than" symbol and rotated to < and used for the "less than" symbol. Other evidence, however, suggests that the "less than" and "greater than" symbols were developed independently of the radical symbol.

A **square root** of a positive number x is a number whose square is x.

A square root of 16 is 4 because $4^2 = 16$.
A square root of 16 is -4 because $(-4)^2 = 16$.

Every positive number has two square roots, one a positive and one a negative number. The symbol $\sqrt{\ }$, called a **radical sign,** is used to indicate the positive or **principal square root** of a number. For example, $\sqrt{16} = 4$ and $\sqrt{25} = 5$. The number under the radical sign is called the **radicand.**

When the negative square root of a number is to be found, a negative sign is placed in front of the radical. For example, $-\sqrt{16} = -4$ and $-\sqrt{25} = -5$.

The square of an integer is a **perfect square.** For instance, 49, 81, and 144 are perfect squares.

$$7^2 = 49$$
$$9^2 = 81$$
$$12^2 = 144$$

The principal square root of a perfect-square integer is a positive integer.

$$\sqrt{49} = 7$$
$$\sqrt{81} = 9$$
$$\sqrt{144} = 12$$

If a number is not a perfect square, its square root can only be approximated. For example, 2 and 7 are not perfect squares. The square roots of these numbers are **irrational numbers.** Their decimal approximations never terminate or repeat.

$$\sqrt{2} \approx 1.4142135\ldots \qquad \sqrt{7} \approx 2.6457513\ldots$$

Radical expressions that contain radicands that are not perfect squares are frequently written in simplest form. A radical expression is in **simplest form** when the radicand contains no factor greater than 1 that is a perfect square. For instance, $\sqrt{50}$ is not in simplest form because 25 is a perfect-square factor of 50. The radical expression $\sqrt{15}$ is in simplest form because there are no perfect-square factors of 15 that are greater than 1.

TAKE NOTE

Recall that a factor of a number divides the number evenly. For instance, 6 is a factor of 18. The perfect square 9 is also a factor of 18. It is a *perfect-square factor* of 18, whereas 6 is not a perfect-square factor of 18.

The Product Property of Square Roots and a knowledge of perfect squares is used to simplify radicands that are not perfect squares.

The Product Property of Square Roots

If a and b are positive real numbers, then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

The chart below shows the square roots of some perfect squares.

Square Roots of Perfect Squares

$\sqrt{1} = 1$
$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\sqrt{25} = 5$
$\sqrt{36} = 6$
$\sqrt{49} = 7$
$\sqrt{64} = 8$
$\sqrt{81} = 9$
$\sqrt{100} = 10$
$\sqrt{121} = 11$
$\sqrt{144} = 12$

⇨ Simplify: $\sqrt{72}$

$\sqrt{72} = \sqrt{36 \cdot 2}$
- Write the radicand as the product of a perfect square and a factor that does not contain a perfect square.

$= \sqrt{36}\sqrt{2}$
- Use the Product Property of Square Roots to write the expression as a product.

$= 6\sqrt{2}$
- Simplify.

Note that 72 must be written as the product of a perfect square and a *factor that does not contain a perfect square.* Therefore, it would not be correct to simplify $\sqrt{72}$ as $\sqrt{9 \cdot 8}$. Although 9 is a perfect-square factor of 72, 8 contains a perfect-square factor $(8 = 4 \cdot 2)$. Therefore, $\sqrt{8}$ is not in simplest form. Remember to find the largest perfect-square factor of the radicand.

$$\begin{aligned}\sqrt{72} &= \sqrt{9 \cdot 8} \\ &= \sqrt{9} \cdot \sqrt{8} \\ &= 3\sqrt{8}\end{aligned}$$

Not in simplest form

Simplify: $\sqrt{147}$

$\sqrt{147} = \sqrt{49 \cdot 3}$
- Write the radicand as the product of a perfect square and a factor that does not contain a perfect square.

$= \sqrt{49}\sqrt{3}$
- Use the Product Property of Square Roots to write the expression as a product.

$= 7\sqrt{3}$
- Simplify.

Simplify: $\sqrt{360}$

$\sqrt{360} = \sqrt{36 \cdot 10}$
- Write the radicand as the product of a perfect square and a factor that does not contain a perfect square.

$= \sqrt{36}\sqrt{10}$
- Use the Product Property of Sqaure Roots to write the expression as a product.

$= 6\sqrt{10}$
- Simplify.

From the last example, note that $\sqrt{360} = 6\sqrt{10}$. The two expressions are different representations of the same number. Using a calculator, we find that $\sqrt{360} \approx 18.973666$ and $6\sqrt{10} \approx 6(3.1622777) = 18.9736662$.

Simplify: $\sqrt{-16}$

Because the square of any real number is positive, there is no real number whose square is -16. $\sqrt{-16}$ is not a real number.

Example 1 Simplify: $3\sqrt{90}$

Solution

$$\begin{aligned}3\sqrt{90} &= 3\sqrt{9 \cdot 10} \\ &= 3\sqrt{9}\sqrt{10} \\ &= 3 \cdot 3\sqrt{10} \\ &= 9\sqrt{10}\end{aligned}$$

You Try It 1 Simplify: $-5\sqrt{32}$

Your solution

Example 2 Simplify: $\sqrt{252}$

Solution

$$\begin{aligned}\sqrt{252} &= \sqrt{36 \cdot 7} \\ &= \sqrt{36}\sqrt{7} \\ &= 6\sqrt{7}\end{aligned}$$

You Try It 2 Simplify: $\sqrt{216}$

Your solution

Solutions on p. S23

To simplify variable radical expressions

Variable expressions that contain radicals do not always represent real numbers. For example, if $a = -4$, then

$$\sqrt{a^3} = \sqrt{(-4)^3} = \sqrt{-64}$$

and $\sqrt{-64}$ is not a real number.

Now consider the expression $\sqrt{x^2}$. Evaluate this expression for $x = -2$ and $x = 2$.

$$\sqrt{x^2} \qquad\qquad \sqrt{x^2}$$
$$\sqrt{(-2)^2} = \sqrt{4} = 2 = |-2| \qquad\qquad \sqrt{2^2} = \sqrt{4} = 2 = |2|$$

This suggests the following:

For any real number a, $\sqrt{a^2} = |a|$. If $a \geq 0$, then $\sqrt{a^2} = a$.

In order to avoid variable expressions that do not represent real numbers, and so that absolute-value signs are not needed for certain expressions, the variables in this chapter will represent *positive* numbers unless otherwise stated.

A variable or a product of variables written in exponential form is a perfect square when each exponent is an even number.

To find the square root of a perfect square, remove the radical sign and multiply each exponent by $\frac{1}{2}$.

Simplify: $\sqrt{a^6}$

$\sqrt{a^6} = a^3$
- Remove the radical sign and multiply the exponent by $\frac{1}{2}$.

A variable radical expression is in simplest form when the radicand contains no factor greater than 1 that is a perfect square.

Simplify: $\sqrt{x^7}$

$\sqrt{x^7} = \sqrt{x^6 \cdot x}$
- Write x^7 as the product of a perfect square and x.

$= \sqrt{x^6}\sqrt{x}$
- Use the Product Property of Square Roots.

$= x^3\sqrt{x}$
- Simplify the perfect square.

Simplify: $3x\sqrt{8x^3y^{13}}$

$3x\sqrt{8x^3y^{13}} = 3x\sqrt{4x^2y^{12}(2xy)}$
- Write the radicand as the product of perfect squares and factors that do not contain a perfect square.

$= 3x\sqrt{4x^2y^{12}}\sqrt{2xy}$
- Use the Product Property of Square Roots.

$= 3x \cdot 2xy^6\sqrt{2xy}$
- Simplify.

$= 6x^2y^6\sqrt{2xy}$

Simplify: $\sqrt{25(x+2)^2}$

$$\begin{aligned}\sqrt{25(x+2)^2} &= 5(x+2)\\ &= 5x + 10\end{aligned}$$

Example 3

Simplify: $\sqrt{b^{15}}$

Solution

$\sqrt{b^{15}} = \sqrt{b^{14} \cdot b} = \sqrt{b^{14}} \cdot \sqrt{b} = b^7\sqrt{b}$

You Try It 3

Simplify: $\sqrt{y^{19}}$

Your solution

Example 4

Simplify: $\sqrt{24x^5}$

Solution

$$\begin{aligned}\sqrt{24x^5} &= \sqrt{4x^4(6x)}\\ &= \sqrt{4x^4}\sqrt{6x}\\ &= 2x^2\sqrt{6x}\end{aligned}$$

You Try It 4

Simplify: $\sqrt{45b^7}$

Your solution

Example 5

Simplify: $2a\sqrt{18a^3b^{10}}$

Solution

$$\begin{aligned}2a\sqrt{18a^3b^{10}} &= 2a\sqrt{9a^2b^{10}(2a)}\\ &= 2a\sqrt{9a^2b^{10}}\sqrt{2a}\\ &= 2a \cdot 3ab^5\sqrt{2a}\\ &= 6a^2b^5\sqrt{2a}\end{aligned}$$

You Try It 5

Simplify: $3a\sqrt{28a^9b^{18}}$

Your solution

Example 6

Simplify: $\sqrt{16(x+5)^2}$

Solution

$\sqrt{16(x+5)^2} = 4(x+5) = 4x + 20$

You Try It 6

Simplify: $\sqrt{25(a+3)^2}$

Your solution

Example 7

Simplify: $\sqrt{x^2 + 10x + 25}$

Solution

$\sqrt{x^2 + 10x + 25} = \sqrt{(x+5)^2} = x + 5$

You Try It 7

Simplify: $\sqrt{x^2 + 14x + 49}$

Your solution

Solutions on p. S23

10.1 Exercises

Objective A

1. Describe in your own words how to simplify a radical expression.

2. Explain why $2\sqrt{2}$ is in simplest form and $\sqrt{8}$ is not in simplest form.

Simplify.

3. $\sqrt{16}$ **4.** $\sqrt{64}$ **5.** $\sqrt{49}$ **6.** $\sqrt{144}$ **7.** $\sqrt{32}$ **8.** $\sqrt{50}$

9. $\sqrt{8}$ **10.** $\sqrt{12}$ **11.** $6\sqrt{18}$ **12.** $-3\sqrt{48}$ **13.** $5\sqrt{40}$ **14.** $2\sqrt{28}$

15. $\sqrt{15}$ **16.** $\sqrt{21}$ **17.** $\sqrt{29}$ **18.** $\sqrt{13}$ **19.** $-9\sqrt{72}$ **20.** $11\sqrt{80}$

21. $\sqrt{45}$ **22.** $\sqrt{225}$ **23.** $\sqrt{0}$ **24.** $\sqrt{210}$ **25.** $6\sqrt{128}$ **26.** $9\sqrt{288}$

Find the decimal approximation rounded to the nearest thousandth.

27. $\sqrt{240}$ **28.** $\sqrt{300}$ **29.** $\sqrt{288}$ **30.** $\sqrt{600}$ **31.** $\sqrt{256}$ **32.** $\sqrt{324}$

Objective B

Simplify.

33. $\sqrt{x^6}$ **34.** $\sqrt{x^{12}}$ **35.** $\sqrt{y^{15}}$ **36.** $\sqrt{y^{11}}$

37. $\sqrt{a^{20}}$ **38.** $\sqrt{a^{16}}$ **39.** $\sqrt{x^4y^4}$ **40.** $\sqrt{x^{12}y^8}$

41. $\sqrt{4x^4}$ **42.** $\sqrt{25y^8}$ **43.** $\sqrt{24x^2}$ **44.** $\sqrt{x^3y^{15}}$

45. $\sqrt{60x^5}$ **46.** $\sqrt{72y^7}$ **47.** $\sqrt{49a^4b^8}$ **48.** $\sqrt{144x^2y^8}$

49. $\sqrt{18x^5y^7}$ **50.** $\sqrt{32a^5b^{15}}$ **51.** $\sqrt{40x^{11}y^7}$ **52.** $\sqrt{72x^9y^3}$

53. $\sqrt{80a^9b^{10}}$ **54.** $\sqrt{96a^5b^7}$ **55.** $2\sqrt{16a^2b^3}$ **56.** $5\sqrt{25a^4b^7}$

57. $x\sqrt{x^4y^2}$	**58.** $y\sqrt{x^3y^6}$	**59.** $4\sqrt{20a^4b^7}$	**60.** $5\sqrt{12a^3b^4}$
61. $3x\sqrt{12x^2y^7}$	**62.** $4y\sqrt{18x^5y^4}$	**63.** $2x^2\sqrt{8x^2y^3}$	**64.** $3y^2\sqrt{27x^4y^3}$
65. $\sqrt{25(a+4)^2}$	**66.** $\sqrt{81(x+y)^4}$	**67.** $\sqrt{4(x+2)^4}$	**68.** $\sqrt{9(x+2)^8}$
69. $\sqrt{x^2+4x+4}$	**70.** $\sqrt{b^2+8b+16}$	**71.** $\sqrt{y^2+2y+1}$	**72.** $\sqrt{a^2+6a+9}$

APPLYING THE CONCEPTS

73. Traffic accident investigators can estimate the speed S, in miles per hour, of a car from the length of its skid mark by using the formula $S = \sqrt{30fl}$, where f is the coefficient of friction (which depends of the type of road surface) and l is the length, in feet, of the skid mark. Say the coefficient of friction is 1.2 and the length of a skid mark is 60 ft. **a.** Determine the speed of the car as a radical expression in simplest form. **b.** Write the answer to part a as a decimal rounded to the nearest integer.

74. The distance a passenger in an airplane can see to the horizon can be approximated by $d = 1.2\sqrt{h}$, where d is the distance to the horizon in miles and h is the height of the plane in feet. To the nearest tenth of a mile, what is the distance to the horizon of a passenger who is flying at an altitude of 5000 ft?

75. If a and b are positive real numbers, does $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$? If not, give an example in which the expressions are not equal.

76. **a.** Find the two-digit perfect square that has exactly nine factors.
b. Find two whole numbers such that their difference is 10, the smaller number is a perfect square, and the larger number is two less than a perfect square.

77. You are to grade this solution to the problem "Write $\sqrt{72}$ in simplest form." Is the solution correct? If not, what error was made? What is the correct solution?

$$\begin{aligned}\sqrt{72} &= \sqrt{4}\sqrt{18}\\ &= 2\sqrt{18}\end{aligned}$$

78. Simplify: **a.** $\sqrt{\sqrt{16}}$ **b.** $\sqrt{\sqrt{81}}$

79. Given $f(x) = \sqrt{2x-1}$, find each of the following. Write your answer in simplest form.
a. $f(1)$ **b.** $f(5)$ **c.** $f(14)$

10.2 Addition and Subtraction of Radical Expressions

Objective A

To add and subtract radical expressions 10

The Distributive Property is used to simplify the sum or difference of radical expressions with like radicands.

$$5\sqrt{2} + 3\sqrt{2} = (5 + 3)\sqrt{2} = 8\sqrt{2}$$

$$6\sqrt{2x} - 4\sqrt{2x} = (6 - 4)\sqrt{2x} = 2\sqrt{2x}$$

Radical expressions that are in simplest form and have unlike radicands cannot be simplified by the Distributive Property.

$2\sqrt{3} + 4\sqrt{2}$ cannot be simplified by the Distributive Property.

⇒ Simplify: $4\sqrt{8} - 10\sqrt{2}$

$$\begin{aligned} 4\sqrt{8} - 10\sqrt{2} &= 4\sqrt{4 \cdot 2} - 10\sqrt{2} \\ &= 4\sqrt{4}\sqrt{2} - 10\sqrt{2} \\ &= 4 \cdot 2\sqrt{2} - 10\sqrt{2} \\ &= 8\sqrt{2} - 10\sqrt{2} \\ &= (8 - 10)\sqrt{2} \\ &= -2\sqrt{2} \end{aligned}$$

- Use the Product Property of Square Roots.
- Simplify the expression by using the Distributive Property.

⇒ Simplify: $8\sqrt{18x} - 2\sqrt{32x}$

$$\begin{aligned} 8\sqrt{18x} - 2\sqrt{32x} &= 8\sqrt{9 \cdot 2x} - 2\sqrt{16 \cdot 2x} \\ &= 8\sqrt{9}\sqrt{2x} - 2\sqrt{16}\sqrt{2x} \\ &= 8 \cdot 3\sqrt{2x} - 2 \cdot 4\sqrt{2x} \\ &= 24\sqrt{2x} - 8\sqrt{2x} \\ &= (24 - 8)\sqrt{2x} \\ &= 16\sqrt{2x} \end{aligned}$$

- Use the Product Property of Square Roots.
- Simplify the expression by using the Distributive Property.

Example 1
Simplify: $5\sqrt{2} - 3\sqrt{2} + 12\sqrt{2}$

Solution

$$\begin{aligned} 5\sqrt{2} - 3\sqrt{2} + 12\sqrt{2} &= (5 - 3 + 12)\sqrt{2} \\ &= 14\sqrt{2} \end{aligned}$$

You Try It 1
Simplify: $9\sqrt{3} + 3\sqrt{3} - 18\sqrt{3}$

Your solution

Example 2
Simplify: $3\sqrt{12} - 5\sqrt{27}$

Solution

$$\begin{aligned} 3\sqrt{12} - 5\sqrt{27} &= 3\sqrt{4 \cdot 3} - 5\sqrt{9 \cdot 3} \\ &= 3\sqrt{4}\sqrt{3} - 5\sqrt{9}\sqrt{3} \\ &= 3 \cdot 2\sqrt{3} - 5 \cdot 3\sqrt{3} \\ &= 6\sqrt{3} - 15\sqrt{3} \\ &= -9\sqrt{3} \end{aligned}$$

You Try It 2
Simplify: $2\sqrt{50} - 5\sqrt{32}$

Your solution

Example 3
Simplify: $3\sqrt{12x^3} - 2x\sqrt{3x}$

Solution

$$\begin{aligned} &3\sqrt{12x^3} - 2x\sqrt{3x} \\ &\quad = 3\sqrt{4x^2 \cdot 3x} - 2x\sqrt{3x} \\ &\quad = 3\sqrt{4x^2}\sqrt{3x} - 2x\sqrt{3x} \\ &\quad = 3 \cdot 2x\sqrt{3x} - 2x\sqrt{3x} \\ &\quad = 6x\sqrt{3x} - 2x\sqrt{3x} \\ &\quad = 4x\sqrt{3x} \end{aligned}$$

You Try It 3
Simplify: $y\sqrt{28y} + 7\sqrt{63y^3}$

Your solution

Example 4
Simplify: $2x\sqrt{8y} - 3\sqrt{2x^2y} + 2\sqrt{32x^2y}$

Solution

$$\begin{aligned} &2x\sqrt{8y} - 3\sqrt{2x^2y} + 2\sqrt{32x^2y} \\ &\quad = 2x\sqrt{4 \cdot 2y} - 3\sqrt{x^2 \cdot 2y} + 2\sqrt{16x^2 \cdot 2y} \\ &\quad = 2x\sqrt{4}\sqrt{2y} - 3\sqrt{x^2}\sqrt{2y} + 2\sqrt{16x^2}\sqrt{2y} \\ &\quad = 2x \cdot 2\sqrt{2y} - 3 \cdot x\sqrt{2y} + 2 \cdot 4x\sqrt{2y} \\ &\quad = 4x\sqrt{2y} - 3x\sqrt{2y} + 8x\sqrt{2y} \\ &\quad = 9x\sqrt{2y} \end{aligned}$$

You Try It 4
Simplify: $2\sqrt{27a^5} - 4a\sqrt{12a^3} + a^2\sqrt{75a}$

Your solution

Solutions on p. S24

10.2 Exercises

[illegible]

1. Which of the numbers 2, 9, 20, 25, 50, 81, and 100 are *not* perfect squares?

2. Write down a number that has a perfect-square factor that is greater than 1.

3. Write a sentence or two that you could email to a friend to explain the concept of a perfect-square factor.

4. Name the perfect-square factors of 540. What number is the largest perfect-square factor of 540?

Simplify.

5. $2\sqrt{2} + \sqrt{2}$

6. $3\sqrt{5} + 8\sqrt{5}$

7. $-3\sqrt{7} + 2\sqrt{7}$

8. $4\sqrt{5} - 10\sqrt{5}$

9. $-3\sqrt{11} - 8\sqrt{11}$

10. $-3\sqrt{3} - 5\sqrt{3}$

11. $2\sqrt{x} + 8\sqrt{x}$

12. $3\sqrt{y} + 2\sqrt{y}$

13. $8\sqrt{y} - 10\sqrt{y}$

14. $-5\sqrt{2a} + 2\sqrt{2a}$

15. $-2\sqrt{3b} - 9\sqrt{3b}$

16. $-7\sqrt{5a} - 5\sqrt{5a}$

17. $3x\sqrt{2} - x\sqrt{2}$

18. $2y\sqrt{3} - 9y\sqrt{3}$

19. $2a\sqrt{3a} - 5a\sqrt{3a}$

20. $-5b\sqrt{3x} - 2b\sqrt{3x}$

21. $3\sqrt{xy} - 8\sqrt{xy}$

22. $-4\sqrt{xy} + 6\sqrt{xy}$

23. $\sqrt{45} + \sqrt{125}$

24. $\sqrt{32} - \sqrt{98}$

25. $2\sqrt{2} + 3\sqrt{8}$

26. $4\sqrt{128} - 3\sqrt{32}$

27. $5\sqrt{18} - 2\sqrt{75}$

28. $5\sqrt{75} - 2\sqrt{18}$

29. $5\sqrt{4x} - 3\sqrt{9x}$

30. $-3\sqrt{25y} + 8\sqrt{49y}$

31. $3\sqrt{3x^2} - 5\sqrt{27x^2}$

32. $-2\sqrt{8y^2} + 5\sqrt{32y^2}$

33. $2x\sqrt{xy^2} - 3y\sqrt{x^2y}$

34. $4a\sqrt{b^2a} - 3b\sqrt{a^2b}$

35. $3x\sqrt{12x} - 5\sqrt{27x^3}$

36. $2a\sqrt{50a} + 7\sqrt{32a^3}$

37. $4y\sqrt{8y^3} - 7\sqrt{18y^5}$

38. $2a\sqrt{8ab^2} - 2b\sqrt{2a^3}$

39. $b^2\sqrt{a^5b} + 3a^2\sqrt{ab^5}$

40. $y^2\sqrt{x^5y} + x\sqrt{x^3y^5}$

41. $4\sqrt{2} - 5\sqrt{2} + 8\sqrt{2}$

42. $3\sqrt{3} + 8\sqrt{3} - 16\sqrt{3}$

43. $5\sqrt{x} - 8\sqrt{x} + 9\sqrt{x}$

44. $\sqrt{x} - 7\sqrt{x} + 6\sqrt{x}$

45. $8\sqrt{2} - 3\sqrt{y} - 8\sqrt{2}$

46. $8\sqrt{3} - 5\sqrt{2} - 5\sqrt{3}$

47. $8\sqrt{8} - 4\sqrt{32} - 9\sqrt{50}$

48. $2\sqrt{12} - 4\sqrt{27} + \sqrt{75}$

49. $-2\sqrt{3} + 5\sqrt{27} - 4\sqrt{45}$

50. $-2\sqrt{8} - 3\sqrt{27} + 3\sqrt{50}$

51. $4\sqrt{75} + 3\sqrt{48} - \sqrt{99}$

52. $2\sqrt{75} - 5\sqrt{20} + 2\sqrt{45}$

53. $\sqrt{25x} - \sqrt{9x} + \sqrt{16x}$

54. $\sqrt{4x} - \sqrt{100x} - \sqrt{49x}$

55. $3\sqrt{3x} + \sqrt{27x} - 8\sqrt{75x}$

56. $5\sqrt{5x} + 2\sqrt{45x} - 3\sqrt{80x}$

57. $2a\sqrt{75b} - a\sqrt{20b} + 4a\sqrt{45b}$

58. $2b\sqrt{75a} - 5b\sqrt{27a} + 2b\sqrt{20a}$

59. $x\sqrt{3y^2} - 2y\sqrt{12x^2} + xy\sqrt{3}$

60. $a\sqrt{27b^2} + 3b\sqrt{147a^2} - ab\sqrt{3}$

APPLYING THE CONCEPTS

61. Given $G(x) = \sqrt{x + 5} + \sqrt{5x + 3}$, write $G(3)$ in simplest form.

62. Is the equation $\sqrt{a^2 + b^2} = \sqrt{a} + \sqrt{b}$ true for all real numbers a and b?

63. Explain the steps in simplifying $4\sqrt{2a^3b} + 5\sqrt{8a^3b}$.

64. For each equation, write "ok" if the equation is correct. If the equation is incorrect, correct the right-hand side.

a. $3\sqrt{ab} + 5\sqrt{ab} = 8\sqrt{2ab}$
b. $7\sqrt{x^3} - 3x\sqrt{x} - x\sqrt{16x} = 0$
c. $5 - 2\sqrt{y} = 3\sqrt{y}$

10.3 Multiplication and Division of Radical Expressions

Objective A

To multiply radical expressions

10

The Product Property of Square Roots is used to multiply variable radical expressions.

$$\sqrt{2x}\sqrt{3y} = \sqrt{2x \cdot 3y} = \sqrt{6xy}$$

Simplify: $\sqrt{2x^2}\sqrt{32x^5}$

$$\begin{aligned}\sqrt{2x^2}\sqrt{32x^5} &= \sqrt{2x^2 \cdot 32x^5}\\ &= \sqrt{64x^7}\\ &= \sqrt{64x^6 \cdot x}\\ &= \sqrt{64x^6}\sqrt{x}\\ &= 8x^3\sqrt{x}\end{aligned}$$

- Use the Product Property of Square Roots.
- Multiply the radicands.
- Simplify.

Simplify: $\sqrt{2x}(x + \sqrt{2x})$

$$\begin{aligned}\sqrt{2x}(x + \sqrt{2x}) &= \sqrt{2x}(x) + \sqrt{2x}\sqrt{2x}\\ &= x\sqrt{2x} + \sqrt{4x^2}\\ &= x\sqrt{2x} + 2x\end{aligned}$$

- Use the Distributive Property to remove parentheses.
- Simplify.

Simplify: $(\sqrt{2} - 3x)(\sqrt{2} + x)$

$$\begin{aligned}(\sqrt{2} - 3x)(\sqrt{2} + x) &= \sqrt{2 \cdot 2} + x\sqrt{2} - 3x\sqrt{2} - 3x^2\\ &= \sqrt{4} + (x - 3x)\sqrt{2} - 3x^2\\ &= 2 - 2x\sqrt{2} - 3x^2\end{aligned}$$

- Use the FOIL method to remove parentheses.

The expressions $a + b$ and $a - b$, which are the sum and difference of two terms, are called **conjugates** of each other. You will recall that $(a + b)(a - b) = a^2 - b^2$.

TAKE NOTE

For $x > 0$, $(\sqrt{x})^2 = x$ because $(\sqrt{x})^2 = \sqrt{x} \cdot \sqrt{x} = \sqrt{x^2} = x$.

Simplify: $(2 + \sqrt{7})(2 - \sqrt{7})$

$$\begin{aligned}(2 + \sqrt{7})(2 - \sqrt{7}) &= 2^2 - (\sqrt{7})^2\\ &= 4 - 7\\ &= -3\end{aligned}$$

- $(2 + \sqrt{7})(2 - \sqrt{7})$ is the product of conjugates.

Simplify: $(3 + \sqrt{y})(3 - \sqrt{y})$

$$\begin{aligned}(3 + \sqrt{y})(3 - \sqrt{y}) &= 3^2 - (\sqrt{y})^2\\ &= 9 - y\end{aligned}$$

- $(3 + \sqrt{y})(3 - \sqrt{y})$ is the product of conjugates.

Example 1
Simplify: $\sqrt{3x^4}\sqrt{2x^2y}\sqrt{6xy^2}$

Solution

$$\begin{aligned}\sqrt{3x^4}\sqrt{2x^2y}\sqrt{6xy^2} &= \sqrt{36x^7y^3}\\ &= \sqrt{36x^6y^2 \cdot xy}\\ &= \sqrt{36x^6y^2}\sqrt{xy}\\ &= 6x^3y\sqrt{xy}\end{aligned}$$

You Try It 1
Simplify: $\sqrt{5a}\sqrt{15a^3b^4}\sqrt{20b^5}$

Your solution

Example 2
Simplify: $\sqrt{3ab}(\sqrt{3a} + \sqrt{9b})$

Solution

$$\begin{aligned}&\sqrt{3ab}(\sqrt{3a} + \sqrt{9b})\\ &= \sqrt{9a^2b} + \sqrt{27ab^2}\\ &= \sqrt{9a^2 \cdot b} + \sqrt{9b^2 \cdot 3a}\\ &= \sqrt{9a^2}\sqrt{b} + \sqrt{9b^2}\sqrt{3a}\\ &= 3a\sqrt{b} + 3b\sqrt{3a}\end{aligned}$$

You Try It 2
Simplify: $\sqrt{5x}(\sqrt{5x} - \sqrt{25y})$

Your solution

Example 3
Simplify: $(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$

Solution

$$\begin{aligned}(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) &= (\sqrt{a})^2 - (\sqrt{b})^2\\ &= a - b\end{aligned}$$

You Try It 3
Simplify: $(2\sqrt{x} + 7)(2\sqrt{x} - 7)$

Your solution

Example 4
Simplify: $(2\sqrt{x} - \sqrt{y})(5\sqrt{x} - 2\sqrt{y})$

Solution

$$\begin{aligned}&(2\sqrt{x} - \sqrt{y})(5\sqrt{x} - 2\sqrt{y})\\ &\quad= 10(\sqrt{x})^2 - 4\sqrt{xy} - 5\sqrt{xy} + 2(\sqrt{y})^2\\ &\quad= 10x - 9\sqrt{xy} + 2y\end{aligned}$$

You Try It 4
Simplify: $(3\sqrt{x} - \sqrt{y})(5\sqrt{x} - 2\sqrt{y})$

Your solution

Solutions on p. S24

Objective B To divide radical expressions 10

The Quotient Property of Square Roots

If *a* and *b* are positive real numbers, then

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad \text{and} \quad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

The square root of a quotient is equal to the quotient of the square roots.

Point of Interest

A radical expression that occurs in Einstein's Theory of Relativity is

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where *v* is the velocity of an object and *c* is the speed of light.

Simplify: $\sqrt{\frac{4x^2}{z^6}}$

$$\sqrt{\frac{4x^2}{z^6}} = \frac{\sqrt{4x^2}}{\sqrt{z^6}}$$

- Rewrite the radical expression as the quotient of the square roots.

$$= \frac{2x}{z^3}$$

- Simplify.

Simplify: $\sqrt{\frac{24x^3y^7}{3x^7y^2}}$

$$\sqrt{\frac{24x^3y^7}{3x^7y^2}} = \sqrt{\frac{8y^5}{x^4}}$$

- Simplify the radicand.

$$= \frac{\sqrt{8y^5}}{\sqrt{x^4}}$$

- Rewrite the radical expression as the quotient of the square roots.

$$= \frac{\sqrt{4y^4 \cdot 2y}}{\sqrt{x^4}}$$

- Simplify.

$$= \frac{\sqrt{4y^4}\sqrt{2y}}{\sqrt{x^4}}$$

$$= \frac{2y^2\sqrt{2y}}{x^2}$$

Simplify: $\frac{\sqrt{4x^2y}}{\sqrt{xy}}$

$$\frac{\sqrt{4x^2y}}{\sqrt{xy}} = \sqrt{\frac{4x^2y}{xy}}$$

- Use the Quotient Property of Square Roots.

$$= \sqrt{4x}$$

- Simplify the radicand.

$$= \sqrt{4}\sqrt{x}$$

- Simplify the radical expression.

$$= 2\sqrt{x}$$

A radical expression is not considered to be in simplest form if a radical remains in the denominator. The procedure used to remove a radical from the denominator is called **rationalizing the denominator.**

Simplify: $\frac{2}{\sqrt{3}}$

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \boxed{\frac{\sqrt{3}}{\sqrt{3}}}$$

- Multiply the expression by $\frac{\sqrt{3}}{\sqrt{3}}$, which equals 1.

$$= \frac{2\sqrt{3}}{(\sqrt{3})^2}$$

- The radicand in the denominator is a perfect square.

$$= \frac{2\sqrt{3}}{3}$$

- Simplify.

The radical expression is in simplest form because no radical remains in the denominator and the radical in the numerator contains no perfect-square factors other than 1.

When the denominator contains a radical expression with two terms, simplify the radical expression by multiplying the numerator and denominator by the conjugate of the denominator.

Simplify: $\dfrac{\sqrt{2y}}{\sqrt{y}+3}$

$$\frac{\sqrt{2y}}{\sqrt{y}+3} = \frac{\sqrt{2y}}{\sqrt{y}+3} \cdot \frac{\sqrt{y}-3}{\sqrt{y}-3}$$

- Multiply the numerator and denominator by $\sqrt{y}-3$, the conjugate of $\sqrt{y}+3$.

$$= \frac{\sqrt{2y^2}-3\sqrt{2y}}{(\sqrt{y})^2-3^2}$$

- Simplify.

$$= \frac{y\sqrt{2}-3\sqrt{2y}}{y-9}$$

Example 5

Simplify: $\dfrac{\sqrt{4x^2y^5}}{\sqrt{3x^4y}}$

Solution

$$\frac{\sqrt{4x^2y^5}}{\sqrt{3x^4y}} = \sqrt{\frac{4x^2y^5}{3x^4y}} = \sqrt{\frac{4y^4}{3x^2}} = \frac{\sqrt{4y^4}}{\sqrt{3x^2}} = \frac{2y^2}{x\sqrt{3}}$$

$$= \frac{2y^2}{x\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2y^2\sqrt{3}}{3x}$$

You Try It 5

Simplify: $\dfrac{\sqrt{15x^6y^7}}{\sqrt{3x^7y^9}}$

Your solution

Example 6

Simplify: $\dfrac{\sqrt{2}}{\sqrt{2}+\sqrt{6}}$

Solution

The denominator contains a radical expression with two terms. Multiply the numerator and denominator by the conjugate of the denominator.

$$\frac{\sqrt{2}}{\sqrt{2}+\sqrt{6}} = \frac{\sqrt{2}}{\sqrt{2}+\sqrt{6}} \cdot \frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}-\sqrt{6}}$$

$$= \frac{(\sqrt{2})^2-\sqrt{12}}{2-6} = \frac{2-2\sqrt{3}}{-4} = \frac{2(1-\sqrt{3})}{-4}$$

$$= \frac{1-\sqrt{3}}{-2} = -\frac{1-\sqrt{3}}{2}$$

You Try It 6

Simplify: $\dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{6}}$

Your solution

Example 7

Simplify: $\dfrac{3-\sqrt{5}}{2+3\sqrt{5}}$

Solution

$$\frac{3-\sqrt{5}}{2+3\sqrt{5}} = \frac{3-\sqrt{5}}{2+3\sqrt{5}} \cdot \frac{2-3\sqrt{5}}{2-3\sqrt{5}}$$

$$= \frac{6-9\sqrt{5}-2\sqrt{5}+3(\sqrt{5})^2}{4-9\cdot 5}$$

$$= \frac{6-11\sqrt{5}+15}{4-45}$$

$$= \frac{21-11\sqrt{5}}{-41} = -\frac{21-11\sqrt{5}}{41}$$

You Try It 7

Simplify: $\dfrac{5+\sqrt{y}}{1-2\sqrt{y}}$

Your solution

Solutions on p. S24

10.3 Exercises

Objective 1

1. Explain in words and then write in symbols the Product Property of Square Roots.

2. Give an example to show that $\sqrt{a^2} \neq a$.

Simplify.

3. $\sqrt{5} \cdot \sqrt{5}$

4. $\sqrt{11} \cdot \sqrt{11}$

5. $\sqrt{3} \cdot \sqrt{12}$

6. $\sqrt{2} \cdot \sqrt{8}$

7. $\sqrt{x} \cdot \sqrt{x}$

8. $\sqrt{y} \cdot \sqrt{y}$

9. $\sqrt{xy^3} \cdot \sqrt{x^5y}$

10. $\sqrt{a^3b^5} \cdot \sqrt{ab^5}$

11. $\sqrt{3a^2b^5} \cdot \sqrt{6ab^7}$

12. $\sqrt{5x^3y} \cdot \sqrt{10x^2y}$

13. $\sqrt{6a^3b^2} \cdot \sqrt{24a^5b}$

14. $\sqrt{8ab^5} \cdot \sqrt{12a^7b}$

15. $\sqrt{2}(\sqrt{2} - \sqrt{3})$

16. $3(\sqrt{12} - \sqrt{3})$

17. $\sqrt{x}(\sqrt{x} - \sqrt{y})$

18. $\sqrt{b}(\sqrt{a} - \sqrt{b})$

19. $\sqrt{5}(\sqrt{10} - \sqrt{x})$

20. $\sqrt{6}(\sqrt{y} - \sqrt{18})$

21. $\sqrt{8}(\sqrt{2} - \sqrt{5})$

22. $\sqrt{10}(\sqrt{20} - \sqrt{a})$

23. $(\sqrt{x} - 3)^2$

24. $(2\sqrt{a} - y)^2$

25. $\sqrt{3a}(\sqrt{3a} - \sqrt{3b})$

26. $\sqrt{5x}(\sqrt{10x} - \sqrt{x})$

27. $\sqrt{2ac} \cdot \sqrt{5ab} \cdot \sqrt{10cb}$

28. $\sqrt{3xy} \cdot \sqrt{6x^3y} \cdot \sqrt{2y^2}$

29. $(\sqrt{5} + 3)(2\sqrt{5} - 4)$

30. $(2 - 3\sqrt{7})(5 + 2\sqrt{7})$

31. $(4 + \sqrt{8})(3 + \sqrt{2})$

32. $(6 - \sqrt{27})(2 + \sqrt{3})$

33. $(2\sqrt{x} + 4)(3\sqrt{x} - 1)$

34. $(5 + \sqrt{y})(6 - 3\sqrt{y})$

35. $(3\sqrt{x} - 2y)(5\sqrt{x} - 4y)$

36. $(5\sqrt{x} + 2\sqrt{y})(3\sqrt{x} - \sqrt{y})$

37. $(\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y})$

[illegible]

38. Why is $\frac{\sqrt{3}}{3}$ in simplest form but $\frac{1}{\sqrt{3}}$ not in simplest form?

39. Why can we multiply $\frac{2}{\sqrt{5}}$ by $\frac{\sqrt{5}}{\sqrt{5}}$ without changing the value of $\frac{2}{\sqrt{5}}$?

Simplify.

40. $\frac{\sqrt{32}}{\sqrt{2}}$

41. $\frac{\sqrt{45}}{\sqrt{5}}$

42. $\frac{\sqrt{98}}{\sqrt{2}}$

43. $\frac{\sqrt{48}}{\sqrt{3}}$

44. $\frac{\sqrt{27a}}{\sqrt{3a}}$

45. $\frac{\sqrt{72x^5}}{\sqrt{2x}}$

46. $\frac{\sqrt{15x^3y}}{\sqrt{3xy}}$

47. $\frac{\sqrt{40x^5y^2}}{\sqrt{5xy}}$

48. $\frac{\sqrt{2a^5b^4}}{\sqrt{98ab^4}}$

49. $\frac{\sqrt{48x^5y^2}}{\sqrt{3x^3y}}$

50. $\frac{\sqrt{9xy^2}}{\sqrt{27x}}$

51. $\frac{\sqrt{4x^2y}}{\sqrt{3xy^3}}$

52. $\frac{\sqrt{16x^3y^2}}{\sqrt{8x^3y}}$

53. $\frac{\sqrt{2}}{\sqrt{8} + 4}$

54. $\frac{1}{\sqrt{2} - 3}$

55. $\frac{5}{\sqrt{7} - 3}$

56. $\frac{3}{5 + \sqrt{5}}$

57. $\frac{\sqrt{3}}{5 - \sqrt{27}}$

58. $\frac{7}{\sqrt{2} - 7}$

59. $\frac{3 - \sqrt{6}}{5 - 2\sqrt{6}}$

60. $\frac{6 - 2\sqrt{3}}{4 + 3\sqrt{3}}$

61. $\frac{-6}{4 + \sqrt{2}}$

62. $\frac{\sqrt{2} + 2\sqrt{6}}{2\sqrt{2} - 3\sqrt{6}}$

63. $\frac{2\sqrt{3} - \sqrt{6}}{5\sqrt{3} + 2\sqrt{6}}$

64. $\frac{3 + \sqrt{x}}{2 - \sqrt{x}}$

65. $\frac{-\sqrt{15}}{3 - \sqrt{12}}$

66. $\frac{\sqrt{a} - 4}{2\sqrt{a} + 2}$

67. $\frac{\sqrt{xy}}{\sqrt{x} - \sqrt{y}}$

68. $\frac{\sqrt{x}}{\sqrt{x} - \sqrt{y}}$

69. $\frac{-12}{\sqrt{6} - 3}$

APPLYING THE CONCEPTS

70. In your own words, describe the process of rationalizing the denominator.

71. Show that $1 + \sqrt{6}$ and $1 - \sqrt{6}$ are solutions of the equation $x^2 - 2x - 5 = 0$.

72. Answer true or false. If the equation is false, correct it.

a. $(\sqrt{y})^4 = y^2$ **b.** $(2\sqrt{x})^3 = 8x\sqrt{x}$ **c.** $(\sqrt{x} + 1)^2 = x + 1$ **d.** $\frac{1}{2 - \sqrt{3}} = 2 + \sqrt{3}$

10.4 Solving Equations Containing Radical Expressions

Objective A To solve an equation containing a radical expression

An equation that contains a variable expression in a radicand is a **radical equation.**

$$\left.\begin{aligned}\sqrt{x} &= 4\\ \sqrt{x+2} &= \sqrt{x-7}\end{aligned}\right\}\ \text{Radical equations}$$

The following property of equality states that if two numbers are equal, the squares of the numbers are equal. This property is used to solve radical equations.

Property of Squaring Both Sides of an Equation

If a and b are real numbers and $a = b$, then $a^2 = b^2$.

The first step when solving a radical equation is to isolate a radical in the equation.

Solve: $\sqrt{x-2} - 7 = 0$

$$\begin{aligned}\sqrt{x-2} - 7 &= 0\\ \sqrt{x-2} &= 7\\ (\sqrt{x-2})^2 &= 7^2\\ x - 2 &= 49\\ x &= 51\end{aligned}$$

- Rewrite the equation with the radical on one side of the equation and the constant on the other side.
- Square both sides of the equation.
- Solve the resulting equation.

Check:
$$\begin{array}{r|l}\multicolumn{2}{c}{\sqrt{x-2} - 7 = 0}\\ \hline \sqrt{51-2} - 7 & 0\\ \sqrt{49} - 7 & 0\\ 7 - 7 & 0\\ 0 = & 0\end{array}$$
A true equation

The solution is 51.

When both sides of an equation are squared, the resulting equation may have a solution that is not a solution of the original equation. Checking a proposed solution of a radical equation, as we did in the preceding example, is a necessary step.

Solve: $\sqrt{2x-5} + 3 = 0$

$$\begin{aligned}\sqrt{2x-5} + 3 &= 0\\ \sqrt{2x-5} &= -3\\ (\sqrt{2x-5})^2 &= (-3)^2\\ 2x - 5 &= 9\\ 2x &= 14\\ x &= 7\end{aligned}$$

- Rewrite the equation with the radical on one side of the equation and the constant on the other side.
- Square both sides of the equation.
- Solve for x.

TAKE NOTE
Any time each side of an equation is squared, you must check the proposed solution of the equation.

Here is the check for the equation on the preceding page.

Check:

$$\begin{array}{r|l}
\sqrt{2x-5}+3 = & 0 \\ \hline
\sqrt{2\cdot 7-5}+3 & 0 \\
\sqrt{14-5}+3 & 0 \\
\sqrt{9}+3 & 0 \\
3+3 & 0 \\
\multicolumn{2}{r}{6 \neq 0}
\end{array}$$

7 does not check as a solution. The equation has no solution.

Example 1
Solve: $\sqrt{3x}+2=5$

Solution

$$\begin{aligned}
\sqrt{3x}+2 &= 5 \\
\sqrt{3x} &= 3 \\
(\sqrt{3x})^2 &= 3^2 \\
3x &= 9 \\
x &= 3
\end{aligned}$$

Check:

$$\begin{array}{r|l}
\sqrt{3x}+2 = & 5 \\ \hline
\sqrt{3\cdot 3}+2 & 5 \\
\sqrt{9}+2 & 5 \\
3+2 & 5 \\
\multicolumn{2}{r}{5 = 5}
\end{array}$$

The solution is 3.

You Try It 1
Solve: $\sqrt{4x}+3=7$

Your solution

Example 2
Solve: $\sqrt{x}-\sqrt{x-5}=1$

Solution

$$\begin{aligned}
\sqrt{x}-\sqrt{x-5} &= 1 \\
\sqrt{x} &= 1+\sqrt{x-5} \\
(\sqrt{x})^2 &= (1+\sqrt{x-5})^2 \\
x &= 1+2\sqrt{x-5}+(x-5) \\
4 &= 2\sqrt{x-5} \\
2 &= \sqrt{x-5} \\
2^2 &= (\sqrt{x-5})^2 \\
4 &= x-5 \\
9 &= x
\end{aligned}$$

Check:

$$\begin{array}{r|l}
\sqrt{x}-\sqrt{x-5} = & 1 \\ \hline
\sqrt{9}-\sqrt{9-5} & 1 \\
\sqrt{9}-\sqrt{4} & 1 \\
3-2 & 1 \\
\multicolumn{2}{r}{1 = 1}
\end{array}$$

The solution is 9.

You Try It 2
Solve: $\sqrt{x}+\sqrt{x+9}=9$

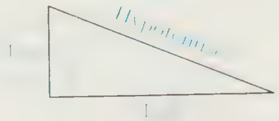
Your solution

Solutions on pp. S24–S25

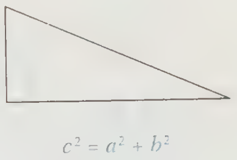
[illegible]

To solve application problems

A right triangle contains one 90° angle. The side opposite the 90° angle is called the **hypotenuse.** The other two sides are called **legs.**



Pythagoras, a Greek mathematician who lived around 550 B.C., is given credit for the Pythagorean Theorem. It states that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs. Actually, this theorem was known to the Babylonians around 1200 B.C.



Point of Interest

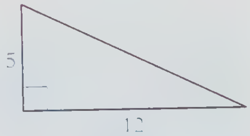
The first known proof of this theorem occurs in a Chinese text, *Arithmetic Classic*, which was first written around 600 B.C. (but there are no existing copies) and revised over a period of 500 years. The earliest known copy of this text dates from approximately 100 B.C.

Pythagorean Theorem

If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $c^2 = a^2 + b^2$.

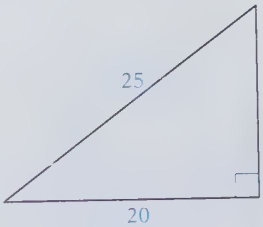
Using this theorem, we can find the hypotenuse of a right triangle when we know the two legs. Use the formula

$$\begin{aligned}\text{Hypotenuse} &= \sqrt{(\text{leg})^2 + (\text{leg})^2}\\ c &= \sqrt{a^2 + b^2}\\ &= \sqrt{(5)^2 + (12)^2}\\ &= \sqrt{25 + 144}\\ &= \sqrt{169}\\ &= 13\end{aligned}$$



The leg of a right triangle can be found when one leg and the hypotenuse are known. Use the formula

$$\begin{aligned}\text{Leg} &= \sqrt{(\text{hypotenuse})^2 - (\text{leg})^2}\\ a &= \sqrt{c^2 - b^2}\\ &= \sqrt{(25)^2 - (20)^2}\\ &= \sqrt{625 - 400}\\ &= \sqrt{225}\\ &= 15\end{aligned}$$



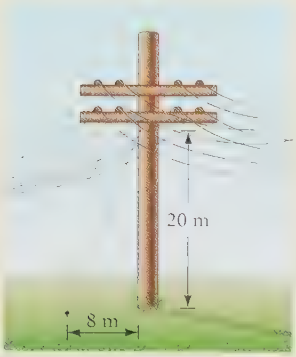
Example 3 and You Try It 3 on the following page illustrate the use of the Pythagorean Theorem. Example 4 and You Try It 4 illustrate other applications of radical equations.

Example 3

A guy wire is attached to a point 20 m above the ground on a telephone pole. The wire is anchored to the ground at a point 8 m from the base of the pole. Find the length of the guy wire. Round to the nearest tenth.

Strategy

To find the length of the guy wire, use the Pythagorean Theorem. One leg is 20 m. The other leg is 8 m. The guy wire is the hypotenuse. Solve the Pythagorean Theorem for the hypotenuse.



Solution

$$c = \sqrt{a^2 + b^2}$$
$$= \sqrt{(20)^2 + (8)^2}$$
$$= \sqrt{400 + 64} = \sqrt{464} \approx 21.5$$

The guy wire has a length of approximately 21.5 m.

You Try It 3

A ladder 8 ft long is resting against a building. How high on the building will the ladder reach when the bottom of the ladder is 3 ft from the building? Round to the nearest hundredth.

Your strategy

Your solution

Example 4

How far would a submarine periscope have to be above the water to locate a ship 4 mi away? The equation for the distance in miles that the lookout can see is $d = \sqrt{1.5h}$, where h is the height in feet above the surface of the water. Round to the nearest hundredth.

Strategy

To find the height above the water, replace d in the equation with the given value and solve for h.

Solution

$$d = \sqrt{1.5h}$$
$$4 = \sqrt{1.5h}$$
$$4^2 = (\sqrt{1.5h})^2$$
$$16 = 1.5h$$
$$10.67 \approx h$$

The periscope must be approximately 10.67 ft above the water.

You Try It 4

Find the length of a pendulum that makes one swing in 2.5 s. The equation for the time for one swing is $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds and L is the length in feet. Use 3.14 for π. Round to the nearest hundredth.

Your strategy

Your solution

Solutions on p. S25

10.4 Exercises

[illegible]

Solve and check.

1. $\sqrt{x} = 5$

2. $\sqrt{y} = 7$

3. $\sqrt{a} = 12$

4. $\sqrt{a} = 9$

5. $\sqrt{5x} = 5$

6. $\sqrt{4x} + 5 = 2$

7. $\sqrt{3x} + 9 = 4$

8. $\sqrt{3x - 2} = 4$

9. $\sqrt{5x + 6} = 1$

10. $\sqrt{2x + 1} = 7$

11. $\sqrt{5x + 4} = 3$

12. $0 = 2 - \sqrt{3 - x}$

13. $0 = 5 - \sqrt{10 + x}$

14. $\sqrt{5x + 2} = 0$

15. $\sqrt{3x - 7} = 0$

16. $\sqrt{3x} - 6 = -4$

17. $\sqrt{5x} + 8 = 23$

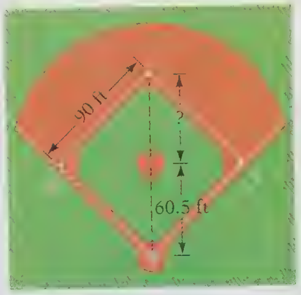
18. $0 = 3 - \sqrt{3x - 9}$

19. $\sqrt{x + 2} = \sqrt{x + 1}$

20. $\sqrt{3x - 2} = \sqrt{5 - 2x}$

[illegible] *Application Problems*

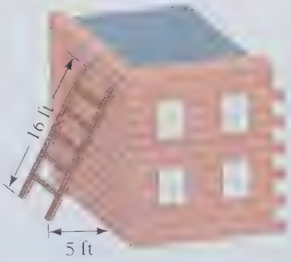
21. The infield of a baseball diamond is a square. The distance between successive bases is 90 ft. The pitcher's mound is on the diagonal between home plate and second base at a distance of 60.5 ft from home plate. (See the figure at the right.) Is the pitcher's mound more or less than halfway between home plate and second base?



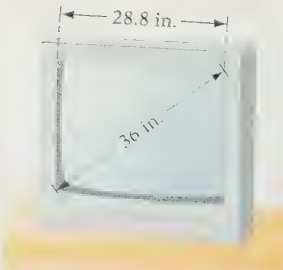
22. The infield of a softball diamond is a square. The distance between successive bases is 60 ft. The pitcher's mound is on the diagonal between home plate and second base at a distance of 46 ft from home plate. Is the pitcher's mound more or less than halfway between home plate and second base?

23. How far would a submarine periscope have to be above the water to locate a ship 5 mi away? The equation for the distance in miles that the lookout can see is $d = \sqrt{1.5h}$, where h is the height in feet above the surface of the water. Round to the nearest hundredth.

24. A 16-foot ladder is leaning against a building. How high on the building will the ladder reach when the bottom of the ladder is 5 ft from the building? (See the figure at the right.) Round to the nearest tenth.



25. The measure of a big-screen television is given by the length of a diagonal across the screen. A 36-inch television has a width of 28.8 in. Find the height of the screen to the nearest tenth of an inch.



26. The measure of a television screen is given by the length of a diagonal across the screen. A 33-inch big-screen television has a width of 26.4 in. Find the height of the screen to the nearest tenth of an inch.

27. The speed of a child riding a merry-go-round at a carnival is given by the equation $v = \sqrt{12r}$, where v is the speed in feet per second and r is the distance in feet from the center of the merry-go-round to the rider. If a child is moving at 15 ft/s, how far is the child from the center of the merry-go-round?

28. Find the length of a pendulum that makes one swing in 1.5 s. The equation for the time of one swing of a pendulum is $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds and L is the length in feet. Use 3.14 for π. Round to the nearest hundredth.

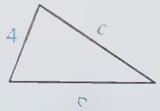
APPLYING THE CONCEPTS

29. In the coordinate plane, a triangle is formed by drawing lines between the points (0, 0) and (5, 0), (5, 0) and (5, 12), and (5, 12) and (0, 0). Find the perimeter of the triangle.

30. The hypotenuse of a right triangle is $5\sqrt{2}$ cm, and one leg is $4\sqrt{2}$ cm.
a. Find the perimeter of the triangle.
b. Find the area of the triangle.

31. If a and b are real numbers and $a^2 = b^2$, does $a = b$? Explain your answer.

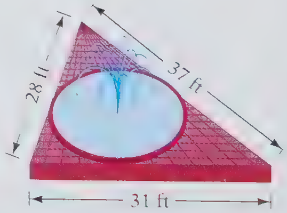
32. Can the Pythagorean Theorem be used to find the length of side c of the triangle at the right? If so, determine c. If not, explain why the theorem cannot be used.



33. A circular fountain is being designed for a triangular plaza in a cultural center. The fountain is placed so that each side of the triangle touches the fountain, as shown in the diagram at the right. Find the area of the fountain. The formula for the radius of the circle is given by

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

where $s = \frac{1}{2}(a + b + c)$ and a, b, and c are the lengths of the sides of the triangle. Round to the nearest hundredth.

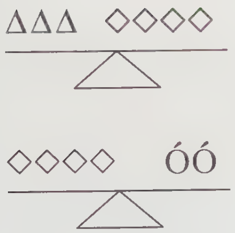


34. Complete each statement using <, =, or >.
a. For an acute triangle with longest side c, $a^2 + b^2$ ________ c^2.
b. For a right triangle with longest side c, $a^2 + b^2$ ________ c^2.
c. For an obtuse triangle with longest side c, $a^2 + b^2$ ________ c^2.

Focus on Problem Solving

Deductive Reasoning

Deductive reasoning uses a rule or statement of fact to reach a conclusion. For instance, if two angles of one triangle are equal to two angles of another triangle, then the two triangles are similar. Thus any time we establish this fact about two triangles, we know that the triangles are similar. Below are two examples of deductive reasoning.



Given that ΔΔΔ = ◇◇◇◇ and ◇◇◇◇ = ÓÓ, then ΔΔΔΔΔΔ is equivalent to how many Ós?

Because 3 Δs = 4 ◇s and 4 ◇s = 2 Ós, 3 Δs = 2 Ós.

6 Δs is twice 3 Δs. We need to find twice 2 Ós, which is 4 Ós.

Therefore, ΔΔΔΔΔΔ = ÓÓÓÓ.

Lomax, Parish, Thorpe, and Wong are neighbors. Each drives a different type of vehicle: a compact car, a sedan, a sports car, or a station wagon. From the following statements, determine which type of vehicle each of the neighbors drives.

> Although the vehicle owned by Lomax has more mileage on it than does either the sedan or the sports car, it does not have the highest mileage of all four cars. (Use X1 in the chart below to eliminate possibilities that this statement rules out.)
>
> Wong and the owner of the sports car live on one side of the street, and Thorpe and the owner of the compact car live on the other side of the street. (Use X2 to eliminate possibilities that this statement rules out.)
>
> Thorpe owns the vehicle with the most mileage on it. (Use X3 to eliminate possibilities that this statement rules out.)

	Compact	*Sedan*	*Sports Car*	*Wagon*
Lomax	√	X1	X1	X2
Parish	X2	X2	√	X2
Thorpe	X2	X3	X2	√
Wong	X2		X2	

TAKE NOTE

To use the chart to solve this problem, write an X in a box to indicate that a possibility has been eliminated. Write a √ to show that a match has been found. When a row or column has 3 X's, a √ is written in the remaining open box in that row or column of the chart.

Lomax drives the compact car, Parish drives the sports car, Thorpe drives the station wagon, and Wong drives the sedan.

1. Given that ‡‡ = ••••• and ••••• = ΛΛ, then ‡‡‡‡‡ = how many Λs?

2. Given that □□□□□□ = ÓÓÓÓ and ÓÓÓÓ = ÎÎ, then □□□ = how many Îs?

3. Given that ¤¤¤¤ = ΩΩΩ and ΩΩΩ = ΔΔ, then ΔΔΔΔ = how many ¤s?

4. Given that ¥¥¥¥¥ = §§ and §§ = ÂÂÂ, then ÂÂÂÂÂÂ = how many ¥s?

5. Anna, Kay, Megan, and Nicole decide to travel together during spring break, but they need to find a destination where each of them will be able to participate in her favorite sport (golf, horseback riding, sailing, or tennis). From the following statements, determine the favorite sport of each student.

a. Anna and the student whose favorite sport is sailing both like to swim, whereas Nicole and the student whose favorite sport is tennis would prefer to scuba dive.

b. Megan and the student whose favorite sport is sailing are roommates. Nicole and the student whose favorite sport is golf each live in a single.

6. Chang, Nick, Pablo, and Saul each take a different form of transportation (bus, car, subway, or taxi) from the office to the airport. From the following statements, determine which form of transportation each takes.

a. Chang spent more on transportation than the fellow who took the bus but less than the fellow who took the taxi.

b. Pablo, who did not travel by bus and who spent the least on transportation, arrived at the airport after Nick but before the fellow who took the subway.

c. Saul spent less on transportation than either Chang or Nick.

Projects and Group Activities

Measurements as Approximations

From arithmetic, you know the rules for rounding decimals.

If the digit to the right of the given place value is less than 5, drop that digit and all digits to its right.

6.31 rounded to the nearest tenth is 6.3.

If the digit to the right of the given place value is greater than or equal to 5, increase the digit in the given place value by 1 and drop all digits to its right.

6.28 rounded to the nearest tenth is 6.3.

Given the rules for rounding numbers, what range of values can the number 6.3 represent? The smallest possible value of 6.3 is 6.25; any number smaller than that would not have been rounded up to 6.3. What about the largest possible value of 6.3? The number 6.34 would be rounded down to 6.3. So would the numbers 6.349, 6.3499, 6.34999, and so on. Therefore, we cannot name the largest possible value of 6.3. We can say that the number must be less than 6.35. Any number less than 6.35 would be rounded down to 6.3. The exact value of 6.3 is greater than or equal to 6.25 and less than 6.35.

The dimensions of a rectangle are given as 4.3 cm by 3.2 cm. Using the smallest and the largest possible values of the length and of the width, we can represent the possible values of the area, A, of the rectangle as follows:

$$4.25(3.15) \le A < 4.35(3.25)$$
$$13.3875 \le A < 14.1375$$

The area is greater than or equal to 13.3875 cm² and less than 14.1375 cm².

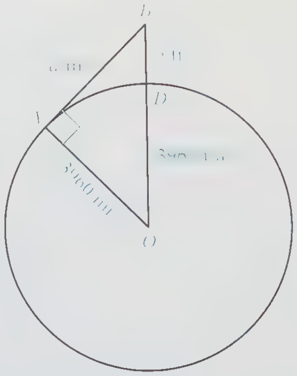
1. The measurements of the three sides of a triangle are given as 8.37 m, 5.42 m, and 9.61 m. Find the possible lengths of the perimeter of the triangle.
2. The length of a side of a square is given as 4.7 cm. Find the possible values for the area of the square.
3. What are the possible values for the area of a rectangle whose dimensions are 6.5 cm by 7.8 cm?
4. The length of a box is 40 cm, the width is 25 cm, and the height is 8 cm. What are the possible values of the volume of the box?

Distance to the Horizon

The formula $d = \sqrt{1.5h}$ can be used to calculate the approximate distance d (in miles) that a person can see who uses a periscope h feet above the water. That formula is derived by using the Pythagorean Theorem.

Consider the diagram (not to scale) at the right, which shows the Earth as a sphere and the periscope as extending h feet above its surface. From geometry, because AB is tangent to the circle and OA is a radius, triangle AOB is a right triangle. Therefore,

$$(OA)^2 + (AB)^2 = (OB)^2$$



Substituting into this formula, we have

$$3960^2 + d^2 = \left(3960 + \frac{h}{5280}\right)^2$$

- Because h is in feet, $\frac{h}{5280}$ is in miles.

$$3960^2 + d^2 = 3960^2 + \frac{2 \cdot 3960}{5280}h + \left(\frac{h}{5280}\right)^2$$
$$d^2 = \frac{3}{2}h + \left(\frac{h}{5280}\right)^2$$
$$d = \sqrt{\frac{3}{2}h + \left(\frac{h}{5280}\right)^2}$$

At this point, an assumption is made that $\sqrt{\frac{3}{2}h + \left(\frac{h}{5280}\right)^2} \approx \sqrt{1.5h}$, where we have written $\frac{3}{2}$ as 1.5. Thus $d \approx \sqrt{1.5h}$ is used to approximate the distance that can be seen using a periscope h feet above the water.

1. Write a paragraph that justifies the assumption that

$$\sqrt{\frac{3}{2}h + \left(\frac{h}{5280}\right)^2} \approx \sqrt{1.5h}$$

(*Suggestion:* Evaluate each expression for various values of h. Because h is the height of a periscope above water, it is unlikely that $h > 25$ ft.)

2. The distance d is the distance from the top of the periscope to A. The distance along the surface of the water is given by arc AD. This distance can be approximated by the equation

$$L \approx \sqrt{1.5h} + 0.306186\left(\sqrt{\frac{h}{5280}}\right)^3$$

Using this formula, calculate L when $h = 10$.

Chapter Summary

Key Words

A *square root* of a positive number x is a number whose square is x. [p. 445]

The *principal square root* of a number is the positive square root. [p. 445]

The symbol $\sqrt{\ }$ is called a *radical sign* and is used to indicate the principal square root of a number. The *radicand* is the number under the radical sign. [p. 445]

The square of an integer is a *perfect square.* [p. 445]

If a whole number is not a perfect square, its square root can only be approximated. Such numbers are *irrational numbers.* Their decimal representations never terminate or repeat. [p. 445]

Conjugates are binomial expressions that differ only in the sign of a term. (The expressions $a + b$ and $a - b$ are conjugates.) [p. 455]

Rationalizing the denominator is the procedure used to remove a radical from the denominator of a fraction. [p. 457]

A *radical equation* is an equation that contains a variable expression in a radicand. [p. 461]

Essential Rules

The Product Property of Square Roots

If a and b are positive real numbers, then $\sqrt{ab} = \sqrt{a}\sqrt{b}$. [p. 445]

The Quotient Property of Square Roots

If a and b are positive real numbers, then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ and $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$. [p. 456]

Property of Squaring Both Sides of an Equation

If a and b are real numbers and $a = b$, then $a^2 = b^2$. [p. 461]

Pythagorean Theorem

If a and b are legs of a right triangle and c is the hypotenuse, then $c^2 = a^2 + b^2$. [p. 463]

Chapter Review

1. Simplify: $\sqrt{3}(\sqrt{12} - \sqrt{3})$

2. Simplify: $3\sqrt{18a^5b}$

3. Simplify: $2\sqrt{36}$

4. Simplify: $\sqrt{6a}(\sqrt{3a} + \sqrt{2a})$

5. Simplify: $\dfrac{2x}{\sqrt{3} - \sqrt{5}}$

6. Simplify: $-3\sqrt{120}$

7. Solve: $\sqrt{5x} = 10$

8. Simplify: $5\sqrt{48}$

9. Simplify: $\dfrac{\sqrt{98x^7y^9}}{\sqrt{2x^3y}}$

10. Solve: $3 - \sqrt{7x} = 5$

11. Simplify: $6a\sqrt{80b} - \sqrt{180a^2b} + 5a\sqrt{b}$

12. Simplify: $4\sqrt{250}$

13. Simplify: $2x\sqrt{60x^3y^3} + 3x^2y\sqrt{15xy}$

14. Simplify: $(4\sqrt{y} - \sqrt{5})(2\sqrt{y} + 3\sqrt{5})$

15. Simplify: $3\sqrt{12x} + 5\sqrt{48x}$

16. Solve: $\sqrt{2x - 3} + 4 = 0$

17. Simplify: $\dfrac{8}{\sqrt{x} - 3}$

18. Simplify: $4y\sqrt{243x^{17}y^9}$

19. Simplify: $y\sqrt{24y^6}$

20. Solve: $\sqrt{5x + 1} = \sqrt{20x - 8}$

21. Simplify:
$2x^2\sqrt{18x^2y^5} + 6y\sqrt{2x^6y^3} - 9xy^2\sqrt{8x^4y}$

22. Simplify: $\dfrac{16}{\sqrt{a}}$

23. The weight of an object is related to the distance the object is above the surface of Earth. An equation for this relationship is $d = 4000\sqrt{\dfrac{W_0}{W_d}} - 4000$, where W_0 is an object's weight on the surface of Earth and W_d is the object's weight at a distance of d miles above Earth's surface. If a space explorer weighs 36 lb at a distance of 4000 mi above the surface of Earth, how much does the explorer weigh on the surface of Earth?

24. A tsunami is a great sea wave produced by underwater earthquakes or volcanic eruption. The velocity of a tsunami as it approaches land depends on the depth of the water and can be approximated by the equation $v = 3\sqrt{d}$, where d is the depth of the water in feet and v is the velocity of the tsunami in feet per second. Find the depth of the water if the velocity is 30 ft/s.

25. A bicycle will overturn if it rounds a corner too sharply or too fast. An equation for the maximum velocity at which a cyclist can turn a corner without tipping over is $v = 4\sqrt{r}$, where v is the velocity of the bicycle in miles per hour and r is the radius of the corner in feet. What is the radius of the sharpest corner that a cyclist can safely turn while riding at 20 mph?

Chapter Test

1. Simplify: $\sqrt{121x^8y^2}$

2. Simplify: $\sqrt{3x^2y}\sqrt{6xy^2}\sqrt{2x}$

3. Simplify: $5\sqrt{8} - 3\sqrt{50}$

4. Simplify: $\sqrt{45}$

5. Simplify: $\dfrac{\sqrt{162}}{\sqrt{2}}$

6. Solve: $\sqrt{9x} + 3 = 18$

7. Simplify: $\sqrt{32a^5b^{11}}$

8. Simplify: $\dfrac{\sqrt{98a^6b^4}}{\sqrt{2a^3b^2}}$

9. Simplify: $\dfrac{2}{\sqrt{3} - 1}$

10. Simplify: $\sqrt{8x^3y}\sqrt{10xy^4}$

11. Solve: $\sqrt{2x - 4} = \sqrt{3x - 5}$

12. Simplify: $3\sqrt{8y} - 2\sqrt{72x} + 5\sqrt{18y}$

13. Simplify: $\sqrt{72x^7y^2}$

14. Simplify: $(\sqrt{y} - 3)(\sqrt{y} + 5)$

15. Simplify: $2x\sqrt{3xy^3} - 2y\sqrt{12x^3y} - 3xy\sqrt{xy}$

16. Simplify: $\dfrac{2 - \sqrt{5}}{6 + \sqrt{5}}$

17. Simplify: $\sqrt{a}(\sqrt{a} - \sqrt{b})$

18. Simplify: $\sqrt{75}$

19. Find the length of a pendulum that makes one swing in 3 s. The equation for the time of one swing of a pendulum is $T = 2\pi\sqrt{\frac{L}{32}}$, where T is the time in seconds and L is the length in feet. Use 3.14 for π. Round to the nearest hundredth.

20. The square root of the sum of two consecutive odd integers is equal to 10. Find the larger integer.

Cumulative Review

1. Simplify:
$$\left(\frac{2}{3}\right)^2 \cdot \left(\frac{3}{4} - \frac{3}{2}\right) + \left(\frac{1}{2}\right)^2$$

2. Simplify:
$-3[x - 2(3 - 2x) - 5x] + 2x$

3. Solve:
$2x - 4[3x - 2(1 - 3x)] = 2(3 - 4x)$

4. Simplify: $(-3x^2y)(-2x^3y^4)$

5. Simplify: $\dfrac{12b^4 - 6b^2 + 2}{-6b^2}$

6. Given $f(x) = \dfrac{2x}{x - 3}$, find $f(-3)$.

7. Factor: $2a^3 - 16a^2 + 30a$

8. Simplify: $\dfrac{3x^3 - 6x^2}{4x^2 + 4x} \cdot \dfrac{3x - 9}{9x^3 - 45x^2 + 54x}$

9. Simplify: $\dfrac{x + 2}{x - 4} - \dfrac{6}{(x - 4)(x - 3)}$

10. Solve: $\dfrac{x}{2x - 5} - 2 = \dfrac{3x}{2x - 5}$

11. Find the equation of the line that contains the point $(-2, -3)$ and has slope $\frac{1}{2}$.

12. Solve by substitution:
$4x - 3y = 1$
$2x + y = 3$

13. Solve by the addition method:
$5x + 4y = 7$
$3x - 2y = 13$

14. Solve: $3(x - 7) \geq 5x - 12$

15. Simplify: $\sqrt{108}$

16. Simplify: $3\sqrt{32} - 2\sqrt{128}$

17. Simplify: $2a\sqrt{2ab^3} + b\sqrt{8a^3b} - 5ab\sqrt{ab}$

18. Simplify: $\sqrt{2a^9b}\sqrt{98ab^3}\sqrt{2a}$

19. Simplify: $\sqrt{3}(\sqrt{6} - \sqrt{x^2})$

20. Simplify: $\frac{\sqrt{320}}{\sqrt{5}}$

21. Simplify: $\frac{3}{2 - \sqrt{5}}$

22. Solve: $\sqrt{3x - 2} - 4 = 0$

23. The selling price of a book is \$29.40. The markup rate used by the bookstore is 20%. Find the cost of the book. Use the formula $S = C + rC$, where S is the selling price, C is the cost, and r is the markup rate.

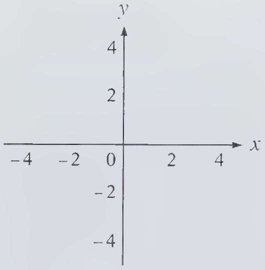
24. How many ounces of pure water must be added to 40 oz of a 12% salt solution to make a salt solution that is 5% salt?

25. The sum of two numbers is twenty-one. The product of the two numbers is one hundred four. Find the two numbers.

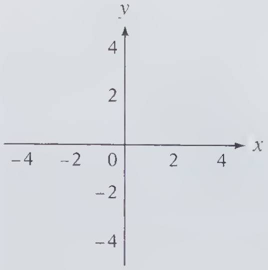
26. A small water pipe takes twice as long to fill a tank as does a larger water pipe. With both pipes open, it takes 16 h to fill the tank. Find the time it would take the small pipe working alone to fill the tank.

27. Solve by graphing:

$$\begin{aligned} 3x - 2y &= 8 \\ 4x + 5y &= 3 \end{aligned}$$



28. Graph the solution set of $3x + y \le 2$.



29. The square root of the sum of two consecutive integers is equal to 9. Find the smaller integer.

30. A stone is dropped from a building and hits the ground 5 s later. How high is the building? The equation for the distance an object falls in T seconds is $T = \sqrt{\frac{d}{16}}$, where d is the distance in feet.

Chapter 11

Quadratic Equations

Objectives

Section 11.1

- **A** To solve a quadratic equation by factoring
- **B** To solve a quadratic equation by taking square roots

Section 11.2

- **A** To solve a quadratic equation by completing the square

Section 11.3

- **A** To solve a quadratic equation by using the quadratic formula

Section 11.4

- **A** To graph a quadratic equation of the form $y = ax^2 + bx + c$

Section 11.5

- **A** To solve application problems

Officers investigating the scene of a traffic accident have a number of factors to analyze. For example, the skid marks, along with the type of street surface and the weather at the time of the accident, can give clues as to what happened. A quadratic equation with variables representing the speed of a car and the length of the skid marks appears in **Exercise 67 on page 484**. Note that the length of the skid marks can potentially reveal whether or not the driver of the car was speeding.

Need help? For online student resources, such as section quizzes, visit this textbook's website at **math.college.hmco.com/students.**

Prep Test

1. Evaluate $b^2 - 4ac$ when $a = 2$, $b = -3$, and $c = -4$.

2. Solve: $5x + 4 = 3$

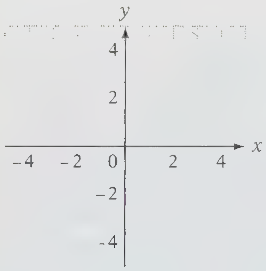
3. Factor: $x^2 + x - 12$

4. Factor: $4x^2 - 12x + 9$

5. Is $x^2 - 10x + 25$ a perfect square trinomial?

6. Solve: $\frac{5}{x-2} = \frac{15}{x}$

7. Graph: $y = -2x + 3$



8. Simplify: $\sqrt{28}$

9. If a is *any* real number, simplify $\sqrt{a^2}$.

10. Walking at a constant speed of 4.5 mph, Lucy and Sam walked from the beginning to the end of hiking trail. When they reached the end, they immediately started back along the same path at a constant speed of 3 mph. If the round-trip took 2 h, what is the length of the hiking trail?

Go Figure

Find the value of $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{8}+\sqrt{9}}$.

11.1 Solving Quadratic Equations by Factoring or by Taking Square Roots

Objective A

To solve a quadratic equation by factoring

An equation of the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$, is a **quadratic equation.**

$4x^2 - 3x + 1 = 0$, $a = 4$, $b = -3$, $c = 1$
$3x^2 - 4 = 0$, $a = 3$, $b = 0$, $c = -4$
$\frac{x^2}{2} - 2x + 4 = 0$, $a = \frac{1}{2}$, $b = -2$, $c = 4$

A quadratic equation is also called a **second-degree equation.**

A quadratic equation is in **standard form** when the polynomial is in descending order and equal to zero.

Recall that the Principle of Zero Products states that if the product of two factors is zero, then at least one of the factors must be zero.

If $a \cdot b = 0$,
then $a = 0$ or $b = 0$.

The Principle of Zero Products can be used in solving quadratic equations.

➡ Solve by factoring: $2x^2 - x = 1$

$$2x^2 - x = 1$$
$$2x^2 - x - 1 = 0$$
$$(2x + 1)(x - 1) = 0$$
$$2x + 1 = 0 \qquad x - 1 = 0$$
$$2x = -1 \qquad x = 1$$
$$x = -\frac{1}{2}$$

- Write the equation in standard form.
- Factor.
- Use the Principle of Zero Products to set each factor equal to zero.
- Rewrite each equation in the form *variable = constant.*

TAKE NOTE
You should always check your solutions by substituting the proposed solutions back into the *original* equation.

Check:

$$\begin{array}{r|l} 2x^2 - x & 1 \\ \hline 2\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) & 1 \\ 2 \cdot \frac{1}{4} + \frac{1}{2} & 1 \\ \frac{1}{2} + \frac{1}{2} & 1 \\ 1 = 1 & \end{array} \qquad \begin{array}{r|l} 2x^2 - x & 1 \\ \hline 2(1)^2 - 1 & 1 \\ 2 \cdot 1 - 1 & 1 \\ 2 - 1 & 1 \\ 1 = 1 & \end{array}$$

The solutions are $-\frac{1}{2}$ and 1.

Solve by factoring: $3x^2 - 4x + 8 = (4x + 1)(x - 2)$

$$3x^2 - 4x + 8 = (4x + 1)(x - 2)$$

$$3x^2 - 4x + 8 = 4x^2 - 7x - 2$$

- Multiply the factors on the right side of the equation.

$$0 = x^2 - 3x - 10$$

- Write the equation in standard form.

$$0 = (x - 5)(x + 2)$$

- Factor.

$$x - 5 = 0 \qquad x + 2 = 0$$

- Use the Principle of Zero Products to set each factor equal to zero.

$$x = 5 \qquad x = -2$$

- Rewrite each equation in the form *variable = constant*.

Check:

$$\begin{array}{r|l} \multicolumn{2}{c}{3x^2 - 4x + 8 = (4x + 1)(x - 2)} \\ \hline 3(5)^2 - 4(5) + 8 & (4[5] + 1)(5 - 2) \\ 3(25) - 20 + 8 & (20 + 1)(3) \\ 75 - 20 + 8 & (21)(3) \\ 63 = & 63 \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{3x^2 - 4x + 8 = (4x + 1)(x - 2)} \\ \hline 3(-2)^2 - 4(-2) + 8 & (4[-2] + 1)(-2 - 2) \\ 3(4) + 8 + 8 & (-8 + 1)(-4) \\ 12 + 8 + 8 & (-7)(-4) \\ 28 = & 28 \end{array}$$

The solutions are 5 and −2.

Solve by factoring: $x^2 - 10x + 25 = 0$

$$x^2 - 10x + 25 = 0$$

$$(x - 5)(x - 5) = 0$$

- Factor.

$$x - 5 = 0 \qquad x - 5 = 0$$

- Use the Principle of Zero Products.

$$x = 5 \qquad x = 5$$

- Solve each equation for *x*.

The solution is 5.

In this last example, 5 is called a **double root** of the quadratic equation.

Example 1

Solve by factoring: $\frac{z^2}{2} - \frac{z}{4} - \frac{1}{4} = 0$

Solution

$$\frac{z^2}{2} - \frac{z}{4} - \frac{1}{4} = 0$$

$$4\left(\frac{z^2}{2} - \frac{z}{4} - \frac{1}{4}\right) = 4(0)$$

- Multiply each side by 4.

$$2z^2 - z - 1 = 0$$

$$(2z + 1)(z - 1) = 0$$

$$2z + 1 = 0 \qquad z - 1 = 0$$

$$2z = -1 \qquad z = 1$$

$$z = -\frac{1}{2}$$

The solutions are $-\frac{1}{2}$ and 1.

You Try It 1

Solve by factoring: $\frac{3y^2}{2} + y - \frac{1}{2} = 0$

Your solution

Solution on p. S25

To solve a quadratic equation by taking square roots

Consider a quadratic equation of the form $x^2 = a$. This equation can be solved by factoring.

$$\begin{aligned} x^2 &= 25 \\ x^2 - 25 &= 0 \\ (x-5)(x+5) &= 0 \\ x = 5 \qquad x &= -5 \end{aligned}$$

The solutions are 5 and -5. The solutions are plus or minus the same number, which is frequently written by using ±; for example, "the solutions are ±5." Because ±5 can be written as $\pm\sqrt{25}$, an alternative method of solving this equation is suggested.

The Square Root Property of an Equality

If $x^2 = a$, then $x = \pm\sqrt{a}$.

⇨ Solve by taking square roots: $x^2 = 25$

$$\begin{aligned} x^2 &= 25 \\ \sqrt{x^2} &= \sqrt{25} \\ x &= \pm\sqrt{25} = \pm 5 \end{aligned}$$

- Take the square root of each side of the equation. Then simplify.

The solutions are 5 and -5.

⇨ Solve by taking square roots: $3x^2 = 36$

$$\begin{aligned} 3x^2 &= 36 \\ x^2 &= 12 \\ \sqrt{x^2} &= \sqrt{12} \\ x &= \pm\sqrt{12} = \pm 2\sqrt{3} \end{aligned}$$

- Solve for x^2.
- Take the square root of each side.
- Simplify.

The solutions are $2\sqrt{3}$ and $-2\sqrt{3}$.

⇨ Solve by taking square roots: $49y^2 - 25 = 0$

$$\begin{aligned} 49y^2 - 25 &= 0 \\ 49y^2 &= 25 \\ y^2 &= \frac{25}{49} \\ \sqrt{y^2} &= \sqrt{\frac{25}{49}} \\ y &= \pm\frac{5}{7} \end{aligned}$$

- Solve for y^2.
- Take the square root of each side.
- Simplify.

The solutions are $\frac{5}{7}$ and $-\frac{5}{7}$.

An equation that contains the square of a binomial can be solved by taking square roots.

Solve by taking square roots: $2(x-1)^2 - 36 = 0$

$$2(x-1)^2 - 36 = 0$$

$$2(x-1)^2 = 36$$

- Solve for $(x-1)^2$.

$$(x-1)^2 = 18$$

$$\sqrt{(x-1)^2} = \sqrt{18}$$

- Take the square root of each side of the equation.

$$x - 1 = \pm\sqrt{18}$$

$$x - 1 = \pm 3\sqrt{2}$$

- Simplify.

$$x - 1 = 3\sqrt{2} \qquad x - 1 = -3\sqrt{2}$$

$$x = 1 + 3\sqrt{2} \qquad x = 1 - 3\sqrt{2}$$

- Solve for x.

The solutions are $1 + 3\sqrt{2}$ and $1 - 3\sqrt{2}$.

Example 2

Solve by taking square roots:
$x^2 + 16 = 0$

Solution

$$x^2 + 16 = 0$$

$$x^2 = -16$$

$$\sqrt{x^2} = \sqrt{-16}$$

$\sqrt{-16}$ is not a real number.

The equation has no real number solution.

You Try It 2

Solve by taking square roots:
$x^2 + 81 = 0$

Your solution

Example 3

Solve by taking square roots:
$5(y-4)^2 = 25$

Solution

$$5(y-4)^2 = 25$$

$$(y-4)^2 = 5$$

$$\sqrt{(y-4)^2} = \sqrt{5}$$

$$y - 4 = \pm\sqrt{5}$$

$$y = 4 \pm \sqrt{5}$$

The solutions are $4 + \sqrt{5}$ and $4 - \sqrt{5}$.

You Try It 3

Solve by taking square roots:
$7(z+2)^2 = 21$

Your solution

Solutions on p. S25

11.1 Exercises

[illegible]

Solve for x.

1. $(x + 3)(x - 5) = 0$

2. $x(x - 7) = 0$

3. $(2x + 5)(3x - 1) = 0$

4. $(x - 4)(2x - 7) = 0$

Solve by factoring.

5. $x^2 + 2x - 15 = 0$

6. $t^2 + 3t - 10 = 0$

7. $z^2 - 4z + 3 = 0$

8. $s^2 - 5s + 4 = 0$

9. $p^2 + 3p + 2 = 0$

10. $v^2 + 6v + 5 = 0$

11. $x^2 - 6x + 9 = 0$

12. $y^2 - 8y + 16 = 0$

13. $12y^2 + 8y = 0$

14. $6x^2 - 9x = 0$

15. $r^2 - 10 = 3r$

16. $t^2 - 12 = 4t$

17. $3v^2 - 5v + 2 = 0$

18. $2p^2 - 3p - 2 = 0$

19. $3s^2 + 8s = 3$

20. $3x^2 + 5x = 12$

21. $\frac{3}{4}z^2 - z = -\frac{1}{3}$

22. $\frac{r^2}{2} = 1 - \frac{r}{12}$

23. $4t^2 = 4t + 3$

24. $5y^2 + 11y = 12$

25. $4v^2 - 4v + 1 = 0$

26. $9s^2 - 6s + 1 = 0$

27. $x^2 - 9 = 0$

28. $t^2 - 16 = 0$

29. $4y^2 - 1 = 0$

30. $9z^2 - 4 = 0$

31. $x + 15 = x(x - 1)$

32. $p + 18 = p(p - 2)$

33. $r^2 - r - 2 = (2r - 1)(r - 3)$

34. $s^2 + 5s - 4 = (2s + 1)(s - 4)$

[illegible]

Solve by taking square roots.

35. $x^2 = 36$

36. $y^2 = 49$

37. $v^2 - 1 = 0$

38. $z^2 - 64 = 0$

39. $4x^2 - 49 = 0$

40. $9w^2 - 64 = 0$

41. $9y^2 = 4$

42. $4z^2 = 25$

43. $16v^2 - 9 = 0$

44. $25x^2 - 64 = 0$

45. $y^2 + 81 = 0$

46. $z^2 + 49 = 0$

47. $w^2 - 24 = 0$

48. $v^2 - 48 = 0$

49. $(x - 1)^2 = 36$

50. $(y + 2)^2 = 49$

51. $2(x + 5)^2 = 8$

52. $4(z - 3)^2 = 100$

53. $9(x - 1)^2 - 16 = 0$

54. $4(y + 3)^2 - 81 = 0$

55. $49(v + 1)^2 - 25 = 0$

56. $81(y - 2)^2 - 64 = 0$

57. $(x - 4)^2 - 20 = 0$

58. $(y + 5)^2 - 50 = 0$

59. $(x + 1)^2 + 36 = 0$

60. $2\left(z - \frac{1}{2}\right)^2 = 12$

61. $3\left(v + \frac{3}{4}\right)^2 = 36$

APPLYING THE CONCEPTS

Solve for x.

62. $(3x^2 - 13)^2 = 4$

63. $(6x^2 - 5)^2 = 1$

64. The value P of an initial investment of A dollars after 2 years is given by $P = A(1 + r)^2$, where r is the annual percentage rate earned by the investment. If an initial investment of \$1500 grew to a value of \$1782.15 in 2 years, what was the annual percentage rate?

65. An initial investment of \$5000 grew to a value of \$5832 in 2 years. Use the formula in Exercise 64 to find the annual percentage rate.

66. The kinetic energy of a moving body is given by $E = \frac{1}{2}mv^2$, where E is the kinetic energy, m is the mass, and v is the velocity in meters per second. What is the velocity of a moving body whose mass is 5 kg and whose kinetic energy is 250 newton-meters?

67. On a certain type of street surface, the equation $d = 0.0074v^2$ can be used to approximate the distance d, in feet, a car traveling v miles per hour will slide when its brakes are applied. After applying the brakes, the owner of a car involved in an accident skidded 40 ft. Did the traffic officer investigating the accident issue the car owner a ticket for speeding if the speed limit is 65 mph?

11.2 Solving Quadratic Equations by Completing the Square

To solve a quadratic equation by completing the square 11

Recall that a perfect-square trinomial is the square of a binomial.

Perfect-Square Trinomial		Square of a Binomial
$x^2 + 6x + 9$	$=$	$(x + 3)^2$
$x^2 - 10x + 25$	$=$	$(x - 5)^2$
$x^2 + 8x + 16$	$=$	$(x + 4)^2$

For each perfect-square trinomial, the square of $\frac{1}{2}$ of the coefficient of x equals the constant term.

$x^2 + 6x + 9$, $\left(\frac{1}{2} \cdot 6\right)^2 = 9$

$x^2 - 10x + 25$, $\left[\frac{1}{2}(-10)\right]^2 = 25$

$x^2 + 8x + 16$, $\left(\frac{1}{2} \cdot 8\right)^2 = 16$

Adding to a binomial the constant term that makes it a perfect-square trinomial is called **completing the square.**

Complete the square of $x^2 - 8x$. Write the resulting perfect-square trinomial as the square of a binomial.

$$\left[\frac{1}{2}(-8)\right]^2 = 16$$

- Find the constant term.

$$x^2 - 8x + 16$$

- Complete the square of $x^2 - 8x$ by adding the constant term.

$$x^2 - 8x + 16 = (x - 4)^2$$

- Write the resulting perfect-square trinomial as the square of a binomial.

Complete the square of $y^2 + 5y$. Write the resulting perfect-square trinomial as the square of a binomial.

$$\left(\frac{1}{2} \cdot 5\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

- Find the constant term.

$$y^2 + 5y + \frac{25}{4}$$

- Complete the square of $y^2 + 5y$ by adding the constant term.

$$y^2 + 5y + \frac{25}{4} = \left(y + \frac{5}{2}\right)^2$$

- Write the resulting perfect-square trinomial as the square of a binomial.

A quadratic equation that cannot be solved by factoring can be solved by completing the square. Add to each side of the equation the term that completes the square. Rewrite the quadratic equation in the form $(x + a)^2 = b$. Take the square root of each side of the equation and then solve for x.

Point of Interest

Early mathematicians solved quadratic equations by literally *completing the square*. For these mathematicians, all equations had geometric interpretations. They found that a quadratic equation could be solved by making certain figures into squares. See the second of the Projects and Group Activities at the end of this chapter for an idea of how this was done.

Solve by completing the square: $x^2 + 8x - 2 = 0$

$$x^2 + 8x - 2 = 0$$

$$x^2 + 8x = 2$$

- Add 2 to each side of the equation.

$$x^2 + 8x + \left(\frac{1}{2} \cdot 8\right)^2 = 2 + \left(\frac{1}{2} \cdot 8\right)^2$$

- Complete the square of $x^2 + 8x$. Add $\left(\frac{1}{2} \cdot 8\right)^2$ to each side of the equation.

$$x^2 + 8x + 16 = 2 + 16$$

- Simplify.

$$(x + 4)^2 = 18$$

- Factor the perfect-square trinomial.

$$\sqrt{(x + 4)^2} = \sqrt{18}$$

- Take the square root of each side of the equation.

$$x + 4 = \pm\sqrt{18} = \pm 3\sqrt{2}$$

- Solve for x.

$$x + 4 = -3\sqrt{2} \qquad x + 4 = 3\sqrt{2}$$

$$x = -4 - 3\sqrt{2} \qquad x = -4 + 3\sqrt{2}$$

Check:

$$\begin{array}{r|l}
\multicolumn{2}{c}{x^2 + 8x - 2 = 0} \\
\hline
(-4 - 3\sqrt{2})^2 + 8(-4 - 3\sqrt{2}) - 2 & 0 \\
16 + 24\sqrt{2} + 18 - 32 - 24\sqrt{2} - 2 & 0 \\
\multicolumn{2}{r}{0 = 0}
\end{array}
\qquad
\begin{array}{r|l}
\multicolumn{2}{c}{x^2 + 8x - 2 = 0} \\
\hline
(-4 + 3\sqrt{2})^2 + 8(-4 + 3\sqrt{2}) - 2 & 0 \\
16 - 24\sqrt{2} + 18 - 32 + 24\sqrt{2} - 2 & 0 \\
\multicolumn{2}{r}{0 = 0}
\end{array}$$

The solutions are $-4 - 3\sqrt{2}$ and $-4 + 3\sqrt{2}$.

If the coefficient of the second-degree term is not 1, a step in completing the square is to multiply each side of the equation by the reciprocal of that coefficient.

Solve by completing the square: $2x^2 - 3x + 1 = 0$

$$2x^2 - 3x + 1 = 0$$

$$2x^2 - 3x = -1$$

- Subtract 1 from each side of the equation.

$$\frac{1}{2}(2x^2 - 3x) = \frac{1}{2} \cdot (-1)$$

- In order for us to complete the square, the coefficient of x^2 must be 1. Multiply each side of the equation by $\frac{1}{2}$.

$$x^2 - \frac{3}{2}x = -\frac{1}{2}$$

$$x^2 - \frac{3}{2}x + \left[\frac{1}{2}\left(-\frac{3}{2}\right)\right]^2 = -\frac{1}{2} + \left[\frac{1}{2}\left(-\frac{3}{2}\right)\right]^2$$

- Complete the square. Add $\left[\frac{1}{2}\left(-\frac{3}{2}\right)\right]^2$ to each side of the equation.

$$x^2 - \frac{3}{2}x + \frac{9}{16} = -\frac{1}{2} + \frac{9}{16}$$

- Simplify.

$$\left(x - \frac{3}{4}\right)^2 = \frac{1}{16}$$

- Factor the perfect-square trinomial.

$$\sqrt{\left(x - \frac{3}{4}\right)^2} = \sqrt{\frac{1}{16}}$$

- Take the square root of each side of the equation.

$$x - \frac{3}{4} = \pm\frac{1}{4}$$

- Solve for x.

$$x - \frac{3}{4} = -\frac{1}{4} \qquad x - \frac{3}{4} = \frac{1}{4}$$

$$x = \frac{1}{2} \qquad x = 1$$

The solutions are $\frac{1}{2}$ and 1.

Example 1

Solve by completing the square:
$2x^2 - 4x - 1 = 0$

Solution

$$\begin{aligned} 2x^2 - 4x - 1 &= 0 \\ 2x^2 - 4x &= 1 \\ \frac{1}{2}(2x^2 - 4x) &= \frac{1}{2} \cdot 1 \\ x^2 - 2x &= \frac{1}{2} \end{aligned}$$

Complete the square.

$$\begin{aligned} x^2 - 2x + 1 &= \frac{1}{2} + 1 \\ (x - 1)^2 &= \frac{3}{2} \\ \sqrt{(x - 1)^2} &= \sqrt{\frac{3}{2}} \\ x - 1 &= \pm\frac{\sqrt{6}}{2} \end{aligned}$$

$$x - 1 = \frac{\sqrt{6}}{2} \qquad x - 1 = -\frac{\sqrt{6}}{2}$$

$$x = 1 + \frac{\sqrt{6}}{2} \qquad x = 1 - \frac{\sqrt{6}}{2}$$

$$= \frac{2 + \sqrt{6}}{2} \qquad = \frac{2 - \sqrt{6}}{2}$$

Check:

$$\begin{array}{r|l} \multicolumn{2}{r}{2x^2 - 4x - 1 = 0} \\ \hline 2\left(\frac{2 + \sqrt{6}}{2}\right)^2 - 4\left(\frac{2 + \sqrt{6}}{2}\right) - 1 & 0 \\ 2\left(\frac{4 + 4\sqrt{6} + 6}{4}\right) - 2(2 + \sqrt{6}) - 1 & 0 \\ 2 + 2\sqrt{6} + 3 - 4 - 2\sqrt{6} - 1 & 0 \\ \multicolumn{2}{r}{0 = 0} \end{array}$$

$$\begin{array}{r|l} \multicolumn{2}{r}{2x^2 - 4x - 1 = 0} \\ \hline 2\left(\frac{2 - \sqrt{6}}{2}\right)^2 - 4\left(\frac{2 - \sqrt{6}}{2}\right) - 1 & 0 \\ 2\left(\frac{4 - 4\sqrt{6} + 6}{4}\right) - 2(2 - \sqrt{6}) - 1 & 0 \\ 2 - 2\sqrt{6} + 3 - 4 + 2\sqrt{6} - 1 & 0 \\ \multicolumn{2}{r}{0 = 0} \end{array}$$

The solutions are $\frac{2 + \sqrt{6}}{2}$ and $\frac{2 - \sqrt{6}}{2}$.

You Try It 1

Solve by completing the square:
$3x^2 - 6x - 2 = 0$

Your solution

Solution on p. S26

Example 2

Solve by completing the square:
$x^2 + 4x + 5 = 0$

Solution

$$x^2 + 4x + 5 = 0$$
$$x^2 + 4x = -5$$

Complete the square.

$$x^2 + 4x + 4 = -5 + 4$$
$$(x + 2)^2 = -1$$
$$\sqrt{(x + 2)^2} = \sqrt{-1}$$

$\sqrt{-1}$ is not a real number.

The quadratic equation has no real number solution.

You Try It 2

Solve by completing the square:
$x^2 + 6x + 12 = 0$

Your solution

Example 3

Solve $\frac{x^2}{4} + \frac{3x}{2} + 1 = 0$ by completing the square. Approximate the solutions to the nearest thousandth.

Solution

$$\frac{x^2}{4} + \frac{3x}{2} + 1 = 0$$
$$4\left(\frac{x^2}{4} + \frac{3x}{2} + 1\right) = 4(0)$$
$$x^2 + 6x + 4 = 0$$
$$x^2 + 6x = -4$$

Complete the square.

$$x^2 + 6x + 9 = -4 + 9$$
$$(x + 3)^2 = 5$$
$$\sqrt{(x + 3)^2} = \sqrt{5}$$
$$x + 3 = \pm\sqrt{5}$$

$$\begin{aligned} x + 3 &= \sqrt{5} & \qquad x + 3 &= -\sqrt{5} \\ x &= -3 + \sqrt{5} & x &= -3 - \sqrt{5} \\ &\approx -3 + 2.236 & &\approx -3 - 2.236 \\ &\approx -0.764 & &\approx -5.236 \end{aligned}$$

The solutions are approximately -0.764 and -5.236.

You Try It 3

Solve $\frac{x^2}{8} + x + 1 = 0$ by completing the square. Approximate the solutions to the nearest thousandth.

Your solution

Solutions on p. S26

11.2 Exercises

Objective A

Complete the square of each binomial. Write the resulting trinomial as the square of a binomial.

1. $x^2 - 8x$

2. $x^2 + 6x$

3. $x^2 + 5x$

4. $x^2 - 3x$

Solve by completing the square.

5. $x^2 + 2x - 3 = 0$

6. $y^2 + 4y - 5 = 0$

7. $z^2 - 6z - 16 = 0$

8. $w^2 + 8w - 9 = 0$

9. $x^2 = 4x - 4$

10. $z^2 = 8z - 16$

11. $v^2 - 6v + 13 = 0$

12. $x^2 + 4x + 13 = 0$

13. $y^2 + 5y + 4 = 0$

14. $v^2 - 5v - 6 = 0$

15. $w^2 + 7w = 8$

16. $y^2 + 5y = -4$

17. $v^2 + 4v + 1 = 0$

18. $y^2 - 2y - 5 = 0$

19. $x^2 + 6x = 5$

20. $w^2 - 8w = 3$

21. $\frac{z^2}{2} = z + \frac{1}{2}$

22. $\frac{y^2}{10} = y - 2$

23. $p^2 + 3p = 1$

24. $r^2 + 5r = 2$

25. $t^2 - 3t = -2$

26. $z^2 - 5z = -3$

27. $v^2 + v - 3 = 0$

28. $x^2 - x = 1$

29. $y^2 = 7 - 10y$

30. $v^2 = 14 + 16v$

31. $r^2 - 3r = 5$

32. $s^2 + 3s = -1$

33. $t^2 - t = 4$

34. $y^2 + y - 4 = 0$

35. $x^2 - 3x + 5 = 0$

36. $z^2 + 5z + 7 = 0$

37. $2t^2 - 3t + 1 = 0$

38. $2x^2 - 7x + 3 = 0$
39. $2r^2 + 5r = 3$
40. $2y^2 - 3y = 9$
41. $2s^2 = 7s - 6$

42. $2x^2 = 3x + 20$
43. $2v^2 = v + 1$
44. $2z^2 = z + 3$
45. $3r^2 + 5r = 2$

46. $3t^2 - 8t = 3$
47. $3y^2 + 8y + 4 = 0$
48. $3z^2 - 10z - 8 = 0$
49. $4x^2 + 4x - 3 = 0$

50. $4v^2 + 4v - 15 = 0$
51. $6s^2 + 7s = 3$
52. $6z^2 = z + 2$
53. $6p^2 = 5p + 4$

Solve by completing the square. Approximate the solutions to the nearest thousandth.

54. $y^2 + 3y = 5$
55. $w^2 + 5w = 2$
56. $2z^2 - 3z = 7$

57. $2x^2 + 3x = 11$
58. $4x^2 + 6x - 1 = 0$
59. $4x^2 + 2x - 3 = 0$

APPLYING THE CONCEPTS

60. Explain why the equation $(x - 2)^2 = -4$ does not have a real number solution.

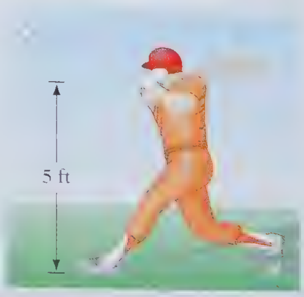
Solve.

61. $\frac{x^2}{6} - \frac{x}{3} = 1$
62. $\sqrt{x + 2} = x - 4$
63. $\sqrt{3x + 4} - x = 2$

64. $\frac{x}{3} + \frac{3}{x} = \frac{8}{3}$
65. $\frac{x + 1}{2} + \frac{3}{x - 1} = 4$
66. $\frac{x - 2}{3} + \frac{2}{x + 2} = 4$

67. $4\sqrt{x + 1} - x = 4$
68. $\sqrt{2x^2 + 7} = x + 2$
69. $3\sqrt{x - 1} + 3 = x$

70. A basketball player shoots at a basket 25 ft away. The height of the ball above the ground at time t is given by $h = -16t^2 + 32t + 6.5$. How many seconds after the ball is released does it hit the basket? *Hint:* When it hits the basket, $h = 10$ ft.



71. A ball player hits a ball. The height of the ball above the ground can be approximated by the equation $h = -16t^2 + 76t + 5$. When will the ball hit the ground? *Hint:* The ball strikes the ground when $h = 0$ ft.

11.3 Solving Quadratic Equations by Using the Quadratic Formula

Objective A

To solve a quadratic equation by using the quadratic formula 11

Any quadratic equation can be solved by completing the square. Applying this method to the standard form of a quadratic equation produces a formula that can be used to solve any quadratic equation.

Solve $ax^2 + bx + c = 0$ by completing the square.

$$ax^2 + bx + c = 0$$

Add the opposite of the constant term to each side of the equation.

$$ax^2 + bx + c + (-c) = 0 + (-c)$$
$$ax^2 + bx = -c$$

Multiply each side of the equation by the reciprocal of a, the coefficient of x^2.

$$\frac{1}{a}(ax^2 + bx) = \frac{1}{a}(-c)$$
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Complete the square by adding $\left(\frac{1}{2} \cdot \frac{b}{a}\right)^2$ to each side of the equation.

$$x^2 + \frac{b}{a}x + \left(\frac{1}{2} \cdot \frac{b}{a}\right)^2 = \left(\frac{1}{2} \cdot \frac{b}{a}\right)^2 - \frac{c}{a}$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

Simplify the right side of the equation.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \left(\frac{c}{a} \cdot \frac{4a}{4a}\right)$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

Factor the perfect-square trinomial on the left side of the equation.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Take the square root of each side of the equation.

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$
$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

Solve for x.

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a} \qquad x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

The Quadratic Formula

The solutions of $ax^2 + bx + c = 0$, $a \neq 0$, are

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve by using the quadratic formula: $2x^2 = 4x - 1$.

First write the equation in standard form. Then use the quadratic formula.

$$2x^2 = 4x - 1$$

$$2x^2 - 4x + 1 = 0$$

- Subtract $4x$ from and add 1 to each side of the equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The quadratic formula

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - (4 \cdot 2 \cdot 1)}}{2 \cdot 2}$$

- $a = 2$, $b = -4$, $c = 1$. Replace a, b, and c by their values.

$$= \frac{4 \pm \sqrt{16 - 8}}{4} = \frac{4 \pm \sqrt{8}}{4}$$

- Simplify.

$$= \frac{4 \pm 2\sqrt{2}}{4} = \frac{2 \pm \sqrt{2}}{2}$$

The solutions are $\frac{2 + \sqrt{2}}{2}$ and $\frac{2 - \sqrt{2}}{2}$.

Example 1

Solve by using the quadratic formula:
$2x^2 - 3x + 1 = 0$

Solution

$$2x^2 - 3x + 1 = 0 \qquad \bullet\ a = 2, b = -3, c = 1$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(1)}}{2 \cdot 2}$$

$$= \frac{3 \pm \sqrt{9 - 8}}{4} = \frac{3 \pm \sqrt{1}}{4} = \frac{3 \pm 1}{4}$$

$$x = \frac{3 + 1}{4} \qquad x = \frac{3 - 1}{4}$$

$$= \frac{4}{4} = 1 \qquad = \frac{2}{4} = \frac{1}{2}$$

The solutions are 1 and $\frac{1}{2}$.

You Try It 1

Solve by using the quadratic formula:
$3x^2 + 4x - 4 = 0$

Your solution

Example 2

Solve by using the quadratic formula:
$\frac{x^2}{2} = 2x - \frac{5}{4}$

Solution

$$\frac{x^2}{2} = 2x - \frac{5}{4}$$

$$4\left(\frac{x^2}{2}\right) = 4\left(2x - \frac{5}{4}\right)$$

$$2x^2 = 8x - 5$$

$$2x^2 - 8x + 5 = 0 \qquad \bullet\ a = 2, b = -8, c = 5$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(5)}}{2 \cdot 2}$$

$$= \frac{8 \pm \sqrt{64 - 40}}{4} = \frac{8 \pm \sqrt{24}}{4}$$

$$= \frac{8 \pm 2\sqrt{6}}{4} = \frac{4 \pm \sqrt{6}}{2}$$

The solutions are $\frac{4 + \sqrt{6}}{2}$ and $\frac{4 - \sqrt{6}}{2}$.

You Try It 2

Solve by using the quadratic formula:
$\frac{x^2}{4} + \frac{x}{2} = \frac{1}{4}$

Your solution

Solutions on p. S26

11.3 Exercises

Objective 1

Solve by using the quadratic formula.

1. $x^2 - 4x - 5 = 0$

2. $y^2 + 3y + 2 = 0$

3. $z^2 - 2z - 15 = 0$

4. $v^2 + 5v + 4 = 0$

5. $y^2 = 2y + 3$

6. $w^2 = 3w + 18$

7. $r^2 = 5 - 4r$

8. $z^2 = 3 - 2z$

9. $2y^2 - y - 1 = 0$

10. $2t^2 - 5t + 3 = 0$

11. $w^2 + 3w + 5 = 0$

12. $x^2 - 2x + 6 = 0$

13. $p^2 - p = 0$

14. $2v^2 + v = 0$

15. $4t^2 - 9 = 0$

16. $4s^2 - 25 = 0$

17. $4y^2 + 4y = 15$

18. $6y^2 + 5y - 4 = 0$

19. $2x^2 + x + 1 = 0$

20. $3r^2 - r + 2 = 0$

21. $\frac{1}{2}t^2 - t = \frac{5}{2}$

22. $y^2 - 4y = 6$

23. $\frac{1}{3}t^2 + 2t - \frac{1}{3} = 0$

24. $z^2 + 4z + 1 = 0$

25. $w^2 = 4w + 9$

26. $y^2 = 8y + 3$

27. $9y^2 + 6y - 1 = 0$

28. $9s^2 - 6s - 2 = 0$

29. $4p^2 + 4p + 1 = 0$

30. $9z^2 + 12z + 4 = 0$

31. $\frac{x^2}{2} = x - \frac{5}{4}$

32. $r^2 = \frac{5}{3}r - 2$

33. $4p^2 + 16p = -11$

34. $4y^2 - 12y = -1$

35. $4x^2 = 4x + 11$

36. $4s^2 + 12s = 3$

37. $9v^2 = -30v - 23$

38. $9t^2 = 30t + 17$

Solve by using the quadratic formula. Approximate the solutions to the nearest thousandth.

39. $x^2 - 2x - 21 = 0$

40. $y^2 + 4y - 11 = 0$

41. $s^2 - 6s - 13 = 0$

42. $w^2 + 8w - 15 = 0$

43. $2p^2 - 7p - 10 = 0$

44. $3t^2 - 8t - 1 = 0$

45. $4z^2 + 8z - 1 = 0$

46. $4x^2 + 7x + 1 = 0$

47. $5v^2 - v - 5 = 0$

APPLYING THE CONCEPTS

48. Factoring, completing the square, and using the quadratic formula are three methods of solving quadratic equations. Describe each method, and cite the advantages and disadvantages of using each.

49. Explain why the equation $0x^2 + 3x + 4 = 0$ cannot be solved by the quadratic formula.

50. Solve $x^2 + ax + b = 0$ for x.

51. True or False?

a. The equations $x = \sqrt{12 - x}$ and $x^2 = 12 - x$ have the same solutions.
b. If $\sqrt{a} + \sqrt{b} = c$, then $a + b = c^2$.
c. $\sqrt{9} = \pm 3$
d. $\sqrt{x^2} = |x|$

Solve.

52. $\sqrt{x + 3} = x - 3$

53. $\sqrt{x + 4} = x + 4$

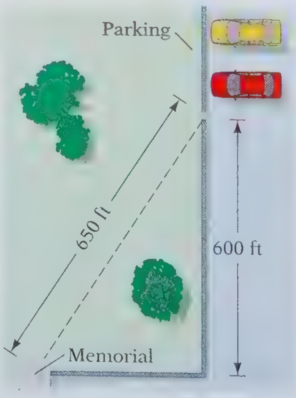
54. $\sqrt{x + 1} = x - 1$

55. $\sqrt{x^2 + 2x + 1} = x - 1$

56. $\frac{x}{4} + \frac{3}{x} = \frac{5}{2}$

57. $\frac{x + 1}{5} - \frac{4}{x - 1} = 2$

58. An L-shaped sidewalk from the parking lot to a memorial is shown in the figure at the right. The distance directly across the grass to the memorial is 650 ft. The distance to the corner is 600 ft. Find the distance from the corner to the memorial.



59. A commuter plane leaves an airport traveling due south at 400 mph. Another plane leaving at the same time travels due east at 300 mph. Find the distance between the two planes after 2 h.

11.4 Graphing Quadratic Equations in Two Variables

Objective A

To graph a quadratic equation of the form $y = ax^2 + bx + c$

11

TAKE NOTE
For the equation $y = 3x^2 - x + 1$, $a = 3$, $b = -1$, and $c = 1$.

An equation of the form $y = ax^2 + bx + c$, $a \neq 0$, is a **quadratic equation in two variables.** Examples of quadratic equations in two variables are shown at the right.

$$y = 3x^2 - x + 1$$
$$y = -x^2 - 3$$
$$y = 2x^2 - 5x$$

For these equations, y is a function of x, and we can write $f(x) = ax^2 + bx + c$. This equation represents a **quadratic function.**

Point of Interest

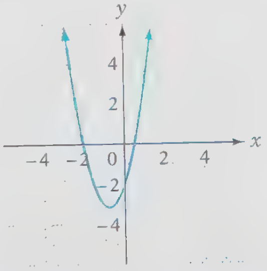
Mirrors in some telescopes are ground into the shape of a parabola. The mirror at the Palomar Mountain Observatory is 2 ft thick at the ends and weighs 14.75 tons. The mirror has been ground to a true paraboloid (the three-dimensional version of a parabola) to within 0.0000015 in. A possible equation of the mirror is $y = 2640x^2$.

⇨ Evaluate $f(x) = 2x^2 - 3x + 4$ when $x = -2$.

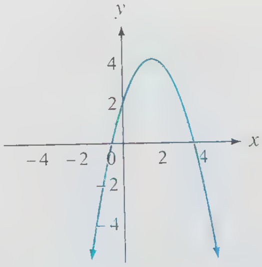
$$f(x) = 2x^2 - 3x + 4$$
$$f(-2) = 2(-2)^2 - 3(-2) + 4 \qquad \text{• Replace } x \text{ by } -2.$$
$$= 2(4) + 6 + 4 = 18 \qquad \text{• Simplify.}$$

The value of the function when $x = -2$ is 18.

The graph of $y = ax^2 + bx + c$ or $f(x) = ax^2 + bx + c$ is a **parabola.** The graph is U-shaped and opens up when a is positive, and down when a is negative. The graphs of two parabolas are shown below.



$y = 2x^2 + 3x - 2$
$a = 2$, a positive number
Parabola opens up



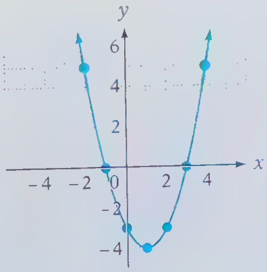
$f(x) = -x^2 + 3x + 2$
$a = -1$, a negative number
Parabola opens down.

TAKE NOTE
One of the equations at the right was written as $y = 2x^2 + 3x - 2$ and the other using functional notation as $f(x) = -x^2 + 3x + 2$. Remember that y and $f(x)$ are different symbols for the same quantity.

⇨ Graph $y = x^2 - 2x - 3$.

x	y
−2	5
−1	0
0	−3
1	−4
2	−3
3	0
4	5

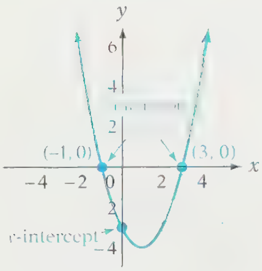
• Find several solutions of the equation. Because the graph is not a straight line, several solutions must be found in order to determine the U-shape. Record the ordered pairs in a table.



• Graph the ordered-pair solutions on a rectangular coordinate system. Draw a parabola through the points.

[illegible]

One of the Projects and Group Activities at the end of this chapter shows how to graph a quadratic equation by using a graphing calculator. You may want to verify the graphs you draw in this section against those drawn by a graphing calculator.



For the graph of $y = x^2 - 2x - 3$, shown here again at the left, note that the graph crosses the x-axis at $(-1, 0)$ and $(3, 0)$. This is also confirmed from the table for the graph (see the preceding page). From the table, note that $y = 0$ when $x = -1$ and when $x = 3$. The x-intercepts of the graph are $(-1, 0)$ and $(3, 0)$.

The y-intercept is the point at which the graph crosses the y-axis. At this point, $x = 0$. From the graph, we can see that the y-intercept is $(0, -3)$.

We can find the x-intercepts algebraically by letting $y = 0$ and solving for x.

$$y = x^2 - 2x - 3$$

$$0 = x^2 - 2x - 3$$

- Replace y by 0 and solve for x.

$$0 = (x + 1)(x - 3)$$

- This equation can be solved by factoring. However, it will be necessary to use the quadratic formula to solve some quadratic equations.

$$x + 1 = 0 \qquad x - 3 = 0$$

$$x = -1 \qquad x = 3$$

The first of the Projects and Group Activities at the end of this chapter shows how to use a graphing calculator to draw the graph of a parabola and how to find the x-intercepts.

We can find the y-intercept algebraically by letting $x = 0$ and solving for y.

$$y = x^2 - 2x - 3$$

$$y = 0^2 - 2(0) - 3$$

- Replace x by 0 and simplify.

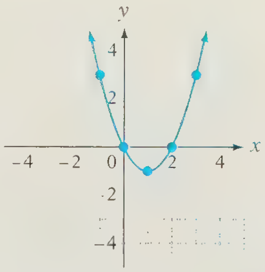
$$= -3$$

The y-intercept is $(0, -3)$.

Example 1 Graph $y = x^2 - 2x$.

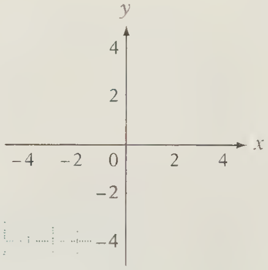
Solution

x	y
−1	3
0	0
1	−1
2	0
3	3



You Try It 1 Graph $y = x^2 + 2$.

Your solution



Example 2 Find the x- and y-intercepts of the graph of $y = x^2 - 2x - 5$.

Solution

To find the x-intercepts, let $y = 0$ and solve for x. This gives the equation $0 = x^2 - 2x - 5$, which is not factorable over the integers. Use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)} = \frac{2 \pm \sqrt{24}}{2}$$

$$= \frac{2 \pm 2\sqrt{6}}{2} = 1 \pm \sqrt{6}$$

The x-intercepts are $(1 - \sqrt{6}, 0)$ and $(1 + \sqrt{6}, 0)$.

To find the y-intercept, let $x = 0$ and solve for y.

$$y = x^2 - 2x - 5$$

$$= 0^2 - 2(0) - 5 = -5$$

The y-intercept is $(0, -5)$.

You Try It 2 Find the x- and y-intercepts of the graph of $f(x) = x^2 - 6x + 9$.

Your solution

Solutions on pp. S26–S27

11.4 Exercises

[illegible]

Determine whether the graph of the equation opens up or down.

1. $y = -\frac{1}{3}x^2$

2. $y = x^2 - 2x - 3$

3. $y = 2x^2 - 4$

4. $f(x) = 3 - 2x - x^2$

Evaluate the function for the given value of x.

5. $f(x) = x^2 - 2x + 1; x = 3$

6. $f(x) = 2x^2 + x - 1; x = -2$

7. $f(x) = 4 - x^2; x = -3$

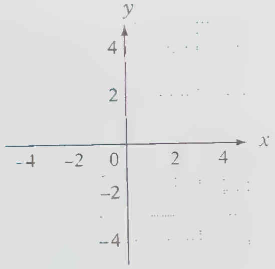
8. $f(x) = x^2 + 6x + 9; x = -3$

9. $f(x) = -x^2 + 5x - 6; x = -4$

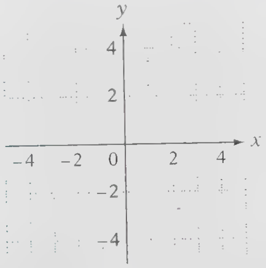
10. $f(x) = -2x^2 + 2x - 1; x = -3$

Graph.

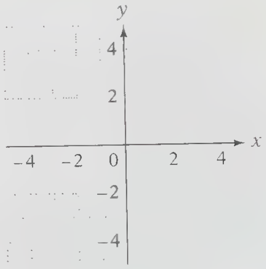
11. $y = x^2$



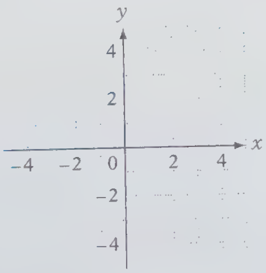
12. $y = -x^2$



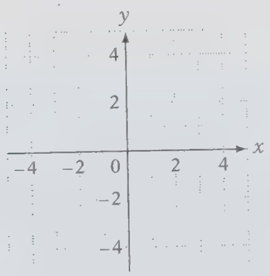
13. $y = -x^2 + 1$



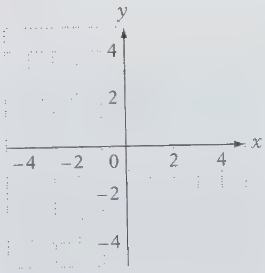
14. $y = x^2 - 1$



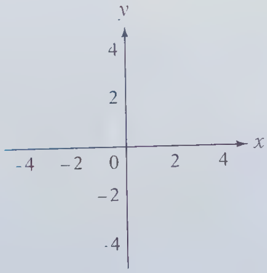
15. $y = 2x^2$



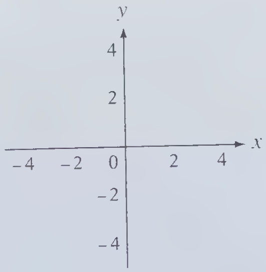
16. $y = \frac{1}{2}x^2$



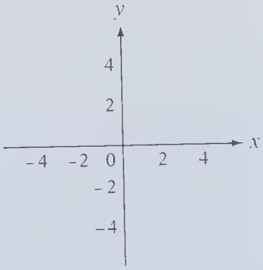
17. $y = -\frac{1}{2}x^2 + 1$



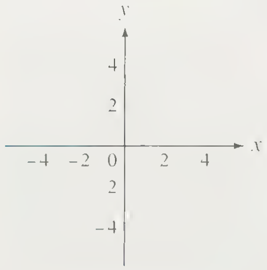
18. $y = 2x^2 - 1$



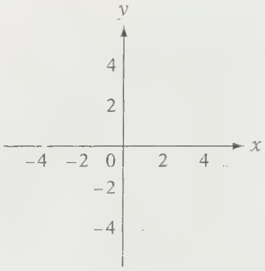
19. $y = x^2 - 4x$



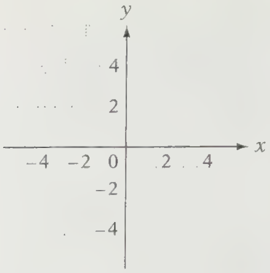
20. $y = x^2 + 4x$



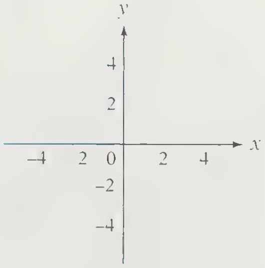
21. $y = x^2 - 2x + 3$



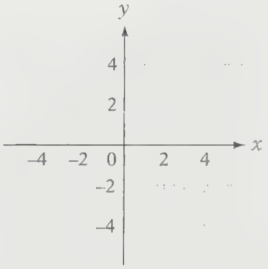
22. $y = x^2 - 4x + 2$



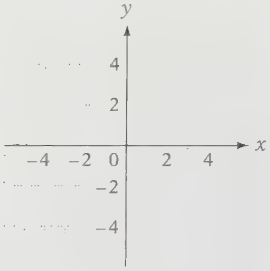
23. $y = -x^2 + 2x + 3$



24. $y = -x^2 - 2x + 3$



25. $y = -x^2 + 4x - 4$



Determine the x- and y-intercepts for each of the following.

26. $y = x^2 - 5x + 6$

27. $y = x^2 + 5x - 6$

28. $f(x) = 9 - x^2$

29. $f(x) = x^2 + 12x + 36$

30. $y = x^2 + 2x - 6$

31. $f(x) = x^2 + 4x - 2$

32. $y = x^2 + 2x + 3$

33. $y = x^2 - x + 1$

34. $f(x) = 2x^2 - x - 3$

35. $f(x) = 2x^2 - 13x + 15$

36. $y = 4 - x - x^2$

37. $y = 2 - 3x - 3x^2$

APPLYING THE CONCEPTS

Show that the equation is a quadratic equation in two variables by writing it in the form $y = ax^2 + bx + c$.

38. $y + 1 = (x - 4)^2$

39. $y - 2 = 3(x + 1)^2$

40. $y - 4 = 2(x - 3)^2$

41. $y + 3 = 3(x - 1)^2$

Find the x-intercepts of each of the following.

42. $y = x^3 - x^2 - 6x$

43. $y = x^3 - 4x^2 - 5x$

44. $y = x^3 + x^2 - 4x - 4$
Hint: Factor by grouping.

45. $y = x^3 + 3x^2 - x - 3$
Hint: Factor by grouping.

11.5 Application Problems

Objective A

To solve application problems

The application problems in this section are varieties of those problems solved earlier in the text. Each of the strategies for the problems in this section will result in a quadratic equation.

In 5 h, two campers rowed 12 mi down a stream and then rowed back to their campsite. The rate of the stream's current was 1 mph. Find the rate at which the campers rowed.

Strategy for Solving an Application Problem

1. Determine the type of problem. For example, is it a distance-rate problem, a geometry problem, or a work problem?

The problem is a distance-rate problem.

2. Choose a variable to represent the unknown quantity. Write numerical or variable expressions for all the remaining quantities. These results can be recorded in a table.

The unknown rate of the campers: r

	Distance	÷	*Rate*	=	*Time*
Downstream	12	÷	$r + 1$	=	$\frac{12}{r+1}$
Upstream	12	÷	$r - 1$	=	$\frac{12}{r-1}$

3. Determine how the quantities are related.

TAKE NOTE
The time going downstream plus the time going upstream is equal to the time of the entire trip.

The total time of the trip was 5 h.

$$\frac{12}{r+1} + \frac{12}{r-1} = 5$$

$$(r+1)(r-1)\left(\frac{12}{r+1} + \frac{12}{r-1}\right) = (r+1)(r-1)5$$

$$(r-1)12 + (r+1)12 = (r^2-1)5$$

$$12r - 12 + 12r + 12 = 5r^2 - 5$$

$$24r = 5r^2 - 5$$

$$0 = 5r^2 - 24r - 5$$

$$0 = (5r+1)(r-5)$$

$$5r + 1 = 0 \qquad r - 5 = 0$$

$$5r = -1 \qquad r = 5$$

$$r = -\frac{1}{5}$$

TAKE NOTE
The solution $r = -\frac{1}{5}$ is not possible, because the rate cannot be a negative number.

The rowing rate was 5 mph.

Example 1

A painter and the painter's apprentice working together can paint a room in 2 h. Working alone, the apprentice requires 3 more hours to paint the room than the painter requires working alone. How long does it take the painter working alone to paint the room?

Strategy

- This is a work problem.
- Time for the painter to paint the room: t
  Time for the apprentice to paint the room: $t + 3$

	Rate	Time	Part
Painter	$\frac{1}{t}$	2	$\frac{2}{t}$
Apprentice	$\frac{1}{t+3}$	2	$\frac{2}{t+3}$

- The sum of the parts of the task completed must equal 1.

Solution

$$\frac{2}{t} + \frac{2}{t+3} = 1$$

$$t(t+3)\left(\frac{2}{t} + \frac{2}{t+3}\right) = t(t+3) \cdot 1$$

$$(t+3)2 + t(2) = t(t+3)$$

$$2t + 6 + 2t = t^2 + 3t$$

$$0 = t^2 - t - 6$$

$$0 = (t-3)(t+2)$$

$$t - 3 = 0 \qquad t + 2 = 0$$

$$t = 3 \qquad t = -2$$

The solution $t = -2$ is not possible.

The time is 3 h.

You Try It 1

The length of a rectangle is 2 m more than the width. The area is 15 m^2. Find the width.

Your strategy

Your solution

Solution on p. S27

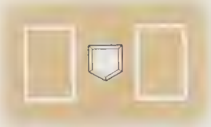
11.5 Exercises

[illegible] *Application Problems*

1. The height of a triangle is 2 m more than twice the length of the base. The area of the triangle is 20 m^2. Find the height of the triangle and the length of the base.

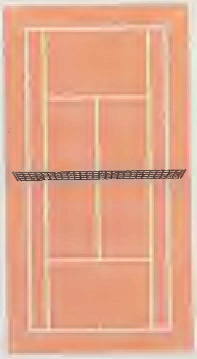
2. The length of a rectangle is 4 ft more than twice the width. The area of the rectangle is 160 ft^2. Find the length and width of the rectangle.

3. The area of the batter's box on a major-league baseball field is 24 ft^2. The length of the batter's box is 2 ft more than the width. Find the length and width of the batter's box.



4. The length of the batter's box on a softball field is 1 ft less than twice the width. The area of the batter's box is 15 ft^2. Find the length and width of the batter's box.

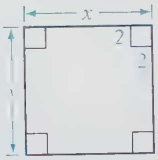
5. The length of a swimming pool is twice the width. The area of the pool is 5000 ft^2. Find the length and width of the pool.

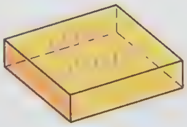


6. The length of a singles tennis court is 24 ft more than twice the width. The area of the tennis court is 2106 ft^2. Find the length and width of the court.

7. The hang time of a football that is kicked on the opening kickoff is given by $s = -16t^2 + 88t + 4$, where s is the height of the football t seconds after leaving the kicker's foot. What is the hang time of a kickoff that hits the ground without being caught? Round to the nearest tenth.

8. A square piece of cardboard is to be formed into a box to transport pizzas. The box is formed by cutting 2-inch square corners from the cardboard and folding them up as shown in the figure at the right. If the volume of the box is 512 in^3, what are the dimensions of the cardboard?





9. The distance, s, a car needs to come to a stop on a certain surface depends on the velocity, v, in feet per second, of the car when the brakes are applied. The equation is given by $s = 0.0344v^2 - 0.758v$. What is the maximum velocity a car can have when the brakes are applied and stop within 150 ft?

10. The perimeter of a rectangular garden is 54 ft. The area of the garden is 180 ft^2. Find the length and width of the garden.

11. The radius of a large pizza is 1 in. less than twice the radius of a small pizza. The difference between the areas of the two pizzas is 33π in^2. Find the radius of the large pizza.

12. The hypotenuse of a right triangle is $\sqrt{13}$ cm. One leg is 1 cm shorter than twice the length of the other leg. Find the lengths of the legs of the right triangle.

13. One computer takes 21 min longer than a second computer to calculate the value of a complex equation. Working together, these computers complete the calculation in 10 min. How long would it take each computer, working separately, to calculate the value?

14. A tank has two drains. One drain takes 16 min longer to empty the tank than does a second drain. With both drains open, the tank is emptied in 6 min. How long would it take each drain, working alone, to empty the tank?

15. Using one engine of a ferryboat, it takes 6 h longer to cross a channel than it does using a second engine alone. With both engines operating, the ferryboat can make the crossing in 4 h. How long would it take each engine, working alone, to power the ferryboat across the channel?

16. An apprentice mason takes 8 h longer to build a small fireplace than an experienced mason. Working together, they can build the fireplace in 3 h. How long would it take each mason, working alone, to complete the fireplace?

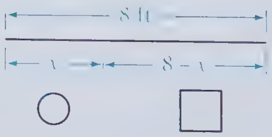
17. It took a small plane 2 h longer to fly 375 mi against the wind than to fly the same distance with the wind. The rate of the wind was 25 mph. Find the rate of the plane in calm air.

18. It took a motorboat 1 h longer to travel 36 mi against the current than to go 36 mi with the current. The rate of the current was 3 mph. Find the rate of the boat in calm water.

APPLYING THE CONCEPTS

19. If a pizza with a diameter of 8 in. costs \$6, what should be the cost of a pizza with a diameter of 16 in. if both pizzas cost the same amount per square inch?

20. A wire 8 ft long is cut into two pieces. A circle is formed from one piece, and a square is formed from the other. The total area of both figures is given by $A = \frac{1}{16}(8 - x)^2 + \frac{x^2}{4\pi}$. What is the length of each piece of wire if the total area is 4.5 ft^2?



Focus on Problem Solving

Algebraic Manipulation and Graphing Techniques

Problem solving is often easier when we have both algebraic manipulation and graphing techniques at our disposal. Solving quadratic equations and graphing quadratic equations in two variables are used here to solve problems involving profit.

A company's revenue, R, is the total amount of money the company earned by selling its products. The cost, C, is the total amount of money the company spent to manufacture and sell its products. A company's profit, P, is the difference between the revenue and the cost: $P = R - C$. A company's revenue and cost may be represented by equations.

A company manufactures and sells woodstoves. The total monthly cost, in dollars, to produce n woodstoves is $C = 30n + 2000$. Write a variable expression for the company's monthly profit if the revenue, in dollars, obtained from selling all n woodstoves is $R = 150n - 0.4n^2$.

$$\begin{aligned} P &= R - C \\ P &= 150n - 0.4n^2 - (30n + 2000) \\ P &= -0.4n^2 + 120n - 2000 \end{aligned}$$

- Replace R by $150n - 0.4n^2$ and C by $30n + 2000$. Then simplify.

How many woodstoves must the company manufacture and sell in order to make a profit of \$6000 a month?

$$\begin{aligned} P &= -0.4n^2 + 120n - 2000 \\ 6000 &= -0.4n^2 + 120n - 2000 \\ 0 &= -0.4n^2 + 120n - 8000 \\ 0 &= n^2 - 300n + 20{,}000 \\ 0 &= (n - 100)(n - 200) \end{aligned}$$

- Substitute 6000 for P.
- Write the equation in standard form.
- Divide each side of the equation by -0.4.
- Factor.

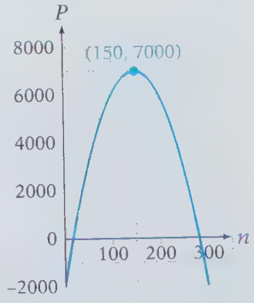
$$\begin{aligned} n - 100 &= 0 & \qquad n - 200 &= 0 \\ n &= 100 & n &= 200 \end{aligned}$$

- Solve for n.

The company will make a monthly profit of \$6000 if either 100 or 200 woodstoves are manufactured and sold.

The graph of $P = -0.4n^2 + 120n - 2000$ is shown at the right. Note that when $P = 6000$, the values of n are 100 and 200.

Also note that the coordinates of the highest point on the graph are (150, 7000). This means that the company makes a *maximum* profit of \$7000 per month when 150 woodstoves are manufactured and sold.



1. The total cost, in dollars, for a company to produce and sell n guitars per month is $C = 240n + 1200$. The company's revenue, in dollars, from selling all n guitars is $R = 400n - 2n^2$.

a. How many guitars must the company produce and sell each month in order to make a monthly profit of \$1200?

b. Graph the profit equation. What is the maximum monthly profit that the company can make?

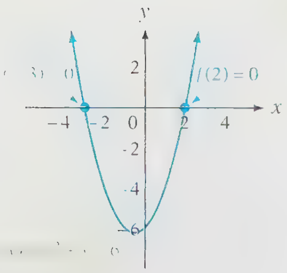
Projects and Group Activities

Graphical Solutions of Quadratic Equations

A real number x is called a **zero of a function** if the function evaluated at x is 0. That is, if $f(x) = 0$, then x is called a zero of the function. For instance, evaluating $f(x) = x^2 + x - 6$ when $x = -3$, we have

$$\begin{aligned} f(x) &= x^2 + x - 6 \\ f(-3) &= (-3)^2 + (-3) - 6 \qquad \bullet \text{ Replace } x \text{ by } -3. \\ f(-3) &= 9 - 3 - 6 = 0 \end{aligned}$$

For this function $f(-3) = 0$, so -3 is a zero of the function.



Verify that 2 is a zero of $f(x) = x^2 + x - 6$ by showing that $f(2) = 0$.

The graph of $f(x) = x^2 + x - 6$ is shown at the left. Note that the graph crosses the x-axis at -3 and 2, the two zeros of the function. The points $(-3, 0)$ and $(2, 0)$ are x-intercepts of the graph.

Or consider the equation $0 = x^2 + x - 6$, which is $f(x) = x^2 + x - 6$ with $f(x)$ replaced by 0. Solving $0 = x^2 + x - 6$, we have

$$\begin{aligned} 0 &= x^2 + x - 6 \\ 0 &= (x + 3)(x - 2) \qquad \bullet \text{ Solve by factoring and using the Principle of Zero Products.} \end{aligned}$$

$$x + 3 = 0 \qquad x - 2 = 0$$

$$x = -3 \qquad x = 2$$

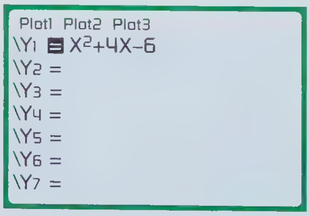
Observe that the solutions of the equation are the zeros of the function. This important connection among the real zeros of a function, the x-intercepts of its graph, and the solutions of the equation is the basis for using a graphing calculator to solve an equation.

The following method of solving a quadratic equation by using a graphing calculator is based on a TI-83 or TI-83 Plus calculator. Other calculators will require a slightly different approach.

Approximate the solutions of $x^2 + 4x = 6$ by using a graphing calculator. Write the equation in standard form.

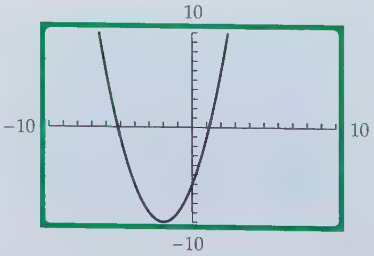
$$x^2 + 4x = 6$$
$$x^2 + 4x - 6 = 0$$

1. Press $\boxed{\text{Y=}}$ and enter $x^2 + 4x - 6$ for Y1.

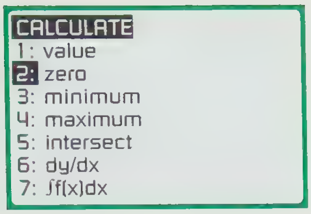


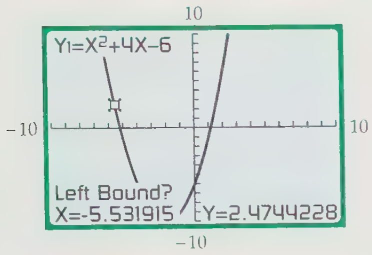
```
Plot1 Plot2 Plot3
\Y1 ■ X²+4X-6
\Y2 =
\Y3 =
\Y4 =
\Y5 =
\Y6 =
\Y7 =
```

2. Press $\boxed{\text{GRAPH}}$. If the graph does not appear on the screen, press $\boxed{\text{ZOOM}}$ 6.

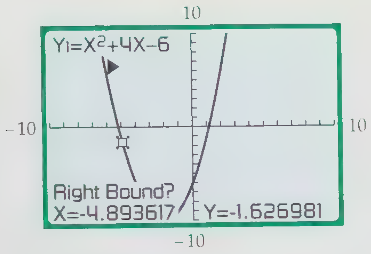


3. Press 2nd CALC 2. Note that the selection for 2 says `zero`. This will begin the calculation of the zeros of the function, which are the solutions of the equation.
4. At the bottom of the screen you will see `LeftBound?` This is asking you to move the blinking cursor so that it is to the *left* of the first x-intercept. Use the left arrow key to move the cursor to the left of the first x-intercept. The values of x and y that appear on your calculator may be different from the ones shown here. Just be sure that you are to the left of the x-intercept. When you are done, press ENTER.
5. At the bottom of the screen you will see `RightBound?` This is asking you to move the blinking cursor so that it is to the *right* of the x-intercept. Use the right arrow key to move the cursor to the right of the x-intercept. The values of x and y that appear on your calculator may be different from the ones shown here. Just be sure that you are to the right of the x-intercept. When you are done, press ENTER.
6. At the bottom of the screen you will see `Guess?` This is asking you to move the blinking cursor to the approximate x-intercept. Use the arrow keys to move the cursor to the approximate x-intercept. The values of x and y that appear on your calculator may be different from the ones shown here. When you are done, press ENTER.
7. The zero of the function is approximately -5.162278. Thus one solution of $x^2 + 4x = 6$ is -5.162278. Also note that the value of y is given as $\text{Y1} = {}^{-}1\text{E}^{-}12$. This is the way the calculator writes a number in scientific notation. We would normally write $\text{Y1} = -1.0 \times 10^{-12}$. This number is very close to zero.

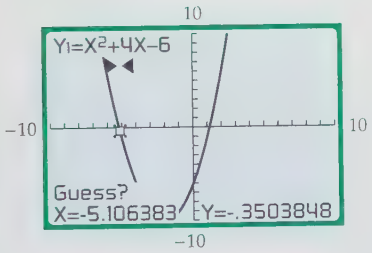




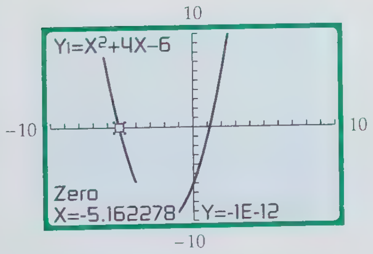
Y1=X²+4X–6
Left Bound?
X=-5.531915 Y=2.4744228



Y1=X²+4X–6
Right Bound?
X=-4.893617 Y=-1.626981

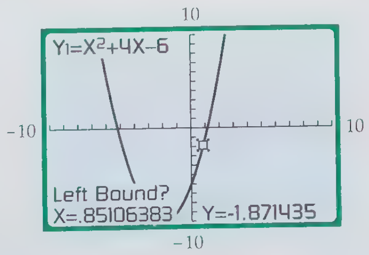


Y1=X²+4X–6
Guess?
X=-5.106383 Y=-.3503848

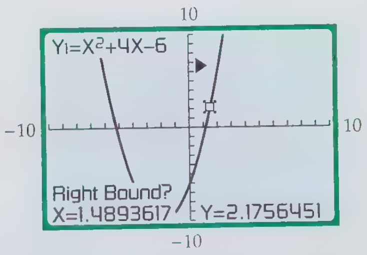


Y1=X²+4X–6
Zero
X=-5.162278 Y=-1E-12

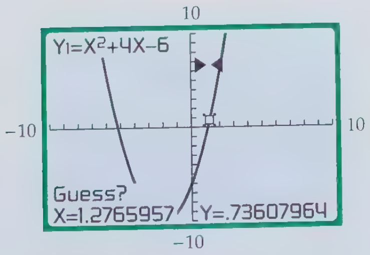
To find the other solution, we repeat Steps 3 through 7. The screens are shown below.



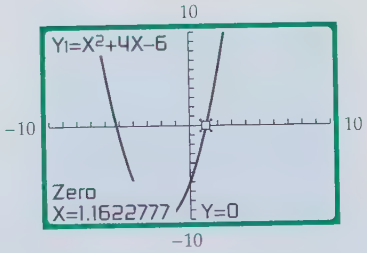
Y1=X²+4X–6
Left Bound?
X=.85106383 Y=-1.871435



Y1=X²+4X–6
Right Bound?
X=1.4893617 Y=2.1756451



Y1=X²+4X–6
Guess?
X=1.2765957 Y=.73607964



Y1=X²+4X–6
Zero
X=1.1622777 Y=0

A second zero of the function is approximately 1.1622777. Thus the two solutions of $x^2 + 4x = 6$ are approximately -5.162278 and 1.1622777.

Use a graphing calculator to approximate the solutions of the following equations.

1. $x^2 + 3x - 4 = 0$

2. $x^2 - 4x - 5 = 0$

3. $x^2 + 3.4x = 4.15$

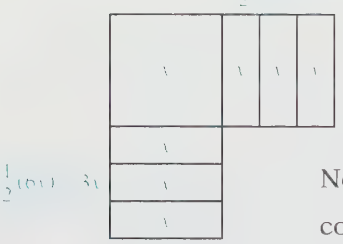
4. $2x^2 - \frac{5}{9}x = \frac{3}{8}$

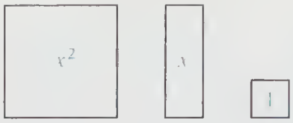
5. $\pi x^2 - \sqrt{17}x - 2 = 0$

6. $\sqrt{2}x^2 + x - \sqrt{7} = 0$

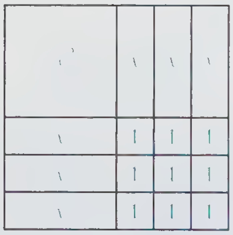
Geometric Construction of Completing the Square

Completing the square as a method for solving a quadratic equation has been known for centuries. The Persian mathematician Al-Khwarismi used this method in a textbook written around 825 A.D. The method was very geometric. That is, Al-Khwarismi literally completed a square. To understand how this method works, consider the following geometric shapes: a square whose area is x^2, a rectangle whose area is x, and another square whose area is 1.





Now consider the expression $x^2 + 6x$. From our discussion in this chapter, to complete the square, we added $\left(\frac{1}{2} \cdot 6\right)^2 = 3^2 = 9$ to the expression. The geometric construction that Al-Khwarismi used is shown at the left.



9 squares were added

Note that it is necessary to add 9 squares to the figure to "complete the square." One of the difficulties of using a geometric method such as this is that it cannot easily be extended to $x^2 - 6x$. There is no way to draw an area of $-6x$! That really did not bother Al-Khwarismi much. Negative numbers were not a significant part of mathematics until well into the 13th century.

1. Show how Al-Khwarismi would have completed the square for $x^2 + 4x$.

2. Show how Al-Khwarismi would have completed the square for $x^2 + 10x$.

Chapter Summary

Key Words

A *quadratic equation* is an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$. A quadratic equation is also called a *second-degree equation*. [p.479]

A quadratic equation is in *standard form* when the polynomial is in descending order and equal to zero. [p.479]

A *quadratic equation in two variables* is given by $y = ax^2 + bx + c$, where $a \neq 0$. [p.495]

A *quadratic function* is given by $f(x) = ax^2 + bx + c$, where $a \neq 0$. The graph of a quadratic function is a *parabola*. [p.495]

Essential Rules

The Quadratic Formula [p.491]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Square Root Property of an Equation [p.481]

If $x^2 = a$, then $x = \pm\sqrt{a}$.

Chapter Review

1. Solve by factoring: $6x^2 + 13x - 28 = 0$

2. Solve by taking square roots: $49x^2 = 25$

3. Solve by completing the square: $x^2 + 2x - 24 = 0$

4. Solve by using the quadratic formula: $x^2 + 5x - 6 = 0$

5. Solve by completing the square: $2x^2 + 5x = 12$

6. Solve by factoring: $12x^2 + 10 = 29x$

7. Solve by taking square roots: $(x + 2)^2 - 24 = 0$

8. Solve by using the quadratic formula: $2x^2 + 3 = 5x$

9. Solve by factoring: $6x(x + 1) = x - 1$

10. Solve by taking square roots: $4y^2 + 9 = 0$

11. Solve by completing the square: $x^2 - 4x + 1 = 0$

12. Solve by using the quadratic formula: $x^2 - 3x - 5 = 0$

13. Solve by completing the square: $x^2 + 6x + 12 = 0$

14. Solve by factoring: $(x + 9)^2 = x + 11$

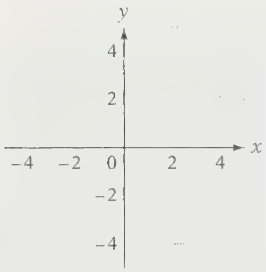
15. Solve by taking square roots: $\left(x - \frac{1}{2}\right)^2 = \frac{9}{4}$

16. Solve by completing the square: $4x^2 + 16x = 7$

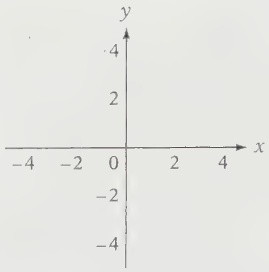
17. Solve by using the quadratic formula:
$x^2 - 4x + 8 = 0$

18. Solve by using the quadratic formula:
$2x^2 + 5x + 2 = 0$

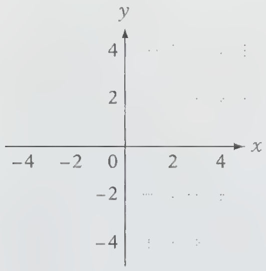
19. Graph $y = -3x^2$.



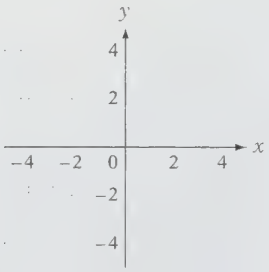
20. Graph $y = -\frac{1}{4}x^2$.



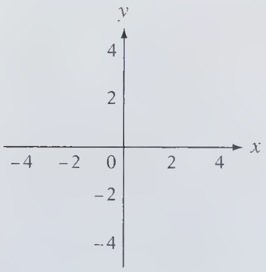
21. Graph $y = 2x^2 + 1$.



22. Graph $y = x^2 - 4x + 3$.



23. Graph $y = -x^2 + 4x - 5$.



24. Find the x- and y-intercepts of the graph of $y = x^2 - 2x - 15$.

25. It took a hawk half an hour longer to fly 70 mi against the wind than to fly 40 mi with the wind. The rate of the wind was 5 mph. Find the rate of the hawk in calm air.

Chapter Test

1. Solve by factoring: $x^2 - 5x - 6 = 0$

2. Solve by factoring: $3x^2 + 7x = 20$

3. Solve by taking square roots: $2(x - 5)^2 - 50 = 0$

4. Solve by taking square roots: $3(x + 4)^2 - 60 = 0$

5. Solve by completing the square: $x^2 + 4x - 16 = 0$

6. Solve by completing the square: $x^2 + 3x = 8$

7. Solve by completing the square: $2x^2 - 6x + 1 = 0$

8. Solve by completing the square: $2x^2 + 8x = 3$

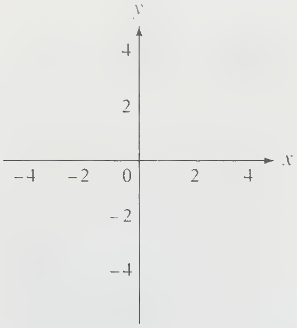
9. Solve by using the quadratic formula: $x^2 + 4x + 2 = 0$

10. Solve by using the quadratic formula: $x^2 - 3x = 6$

11. Solve by using the quadratic formula:
$2x^2 - 5x - 3 = 0$

12. Solve by using the quadratic formula:
$3x^2 - x = 1$

13. Graph $y = x^2 + 2x - 4$.



14. Find the x- and y-intercepts of the graph of $f(x) = x^2 + x - 12$.

15. The length of a rectangle is 2 ft less than twice the width. The area of the rectangle is 40 ft². Find the length and width of the rectangle.

16. It took a motorboat 1 h longer to travel 60 mi against a current than it took the boat to travel 60 mi with the current. The rate of the current was 1 mph. Find the rate of the boat in calm water.

Cumulative Review

1. Simplify: $2x - 3[2x - 4(3 - 2x) + 2] - 3$

2. Solve: $-\frac{3}{5}x = -\frac{9}{10}$

3. Solve: $2x - 3(4x - 5) = -3x - 6$

4. Simplify: $(2a^2b)^2(-3a^4b^2)$

5. Simplify: $(x^2 - 8) \div (x - 2)$

6. Factor: $3x^3 + 2x^2 - 8x$

7. Simplify: $\dfrac{3x^2 - 6x}{4x - 6} \div \dfrac{2x^2 + x - 6}{6x^3 - 24x}$

8. Simplify: $\dfrac{x}{2(x - 1)} - \dfrac{1}{(x - 1)(x + 1)}$

9. Simplify: $\dfrac{1 - \dfrac{7}{x} + \dfrac{12}{x^2}}{2 - \dfrac{1}{x} - \dfrac{15}{x^2}}$

10. Find the x- and y-intercepts for the graph of the line $4x - 3y = 12$.

11. Find the equation of the line that contains the point $(-3, 2)$ and has slope $-\frac{4}{3}$.

12. Solve by substitution:
$3x - y = 5$
$y = 2x - 3$

13. Solve by the addition method:
$3x + 2y = 2$
$5x - 2y = 14$

14. Solve: $2x - 3(2 - 3x) > 2x - 5$

15. Simplify: $(\sqrt{a} - \sqrt{2})(\sqrt{a} + \sqrt{2})$

16. Simplify: $\dfrac{\sqrt{108a^7b^3}}{\sqrt{3a^4b}}$

17. Simplify: $\dfrac{\sqrt{3}}{5 + 2\sqrt{3}}$

18. Solve: $3 = 8 - \sqrt{5x}$

19. Solve by factoring: $6x^2 - 17x = -5$

20. Solve by taking square roots:
$2(x - 5)^2 = 36$

21. Solve by completing the square:
$3x^2 + 7x = -3$

22. Solve by using the quadratic formula:
$2x^2 - 3x - 2 = 0$

23. Find the cost per pound of a mixture made from 20 lb of cashews that cost \$3.50 per pound and 50 lb of peanuts that cost \$1.75 per pound.

24. A stock investment of 100 shares paid a dividend of \$215. At this rate, how many additional shares must the investor own to earn a dividend of \$752.50?

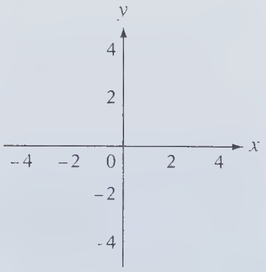
25. A 720-mile trip from one city to another takes 3 h when a plane is flying with the wind. The return trip, against the wind, takes 4.5 h. Find the rate of the plane in still air and the rate of the wind.

26. A student received a 70, a 91, an 85, and a 77 on four tests in a mathematics class. What scores on the last test will enable the student to receive a minimum of 400 points?

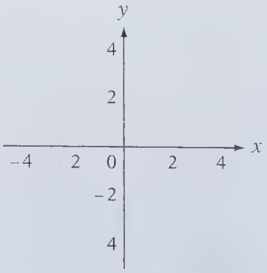
27. The sum of the squares of three consecutive odd integers is 83. Find the middle odd integer.

28. A jogger ran 7 mi at a constant rate and then reduced the rate by 3 mph and ran an additional 8 mi at the reduced rate. The total time spent jogging the 15 mi was 3 h. Find the jogger's rate for the last 8 mi.

29. Graph the solution set of $2x - 3y > 6$.



30. Graph $y = x^2 - 2x - 3$.



Final Exam

1. Evaluate $-|-3|$.

2. Subtract: $-15 - (-12) - 3$

3. Simplify: $-2^4 \cdot (-2)^4$

4. Simplify: $-7 - \dfrac{12 - 15}{2 - (-1)} \cdot (-4)$

5. Evaluate $\dfrac{a^2 - 3b}{2a - 2b^2}$ when $a = 3$ and $b = -2$.

6. Simplify: $6x - (-4y) - (-3x) + 2y$

7. Simplify: $(-15z)\left(-\dfrac{2}{5}\right)$

8. Simplify: $-2[5 - 3(2x - 7) - 2x]$

9. Solve: $20 = -\dfrac{2}{5}x$

10. Solve: $4 - 2(3x + 1) = 3(2 - x) + 5$

11. Write $\dfrac{1}{8}$ as a percent.

12. Find 19% of 80.

13. Simplify: $(2x^2 - 5x + 1) - (5x^2 - 2x - 7)$

14. Simplify: $(-3xy^3)^4$

15. Simplify: $(3x^2 - x - 2)(2x + 3)$

16. Simplify: $\dfrac{(-2x^2y^3)^3}{(-4xy^4)^2}$

17. Simplify: $\dfrac{12x^2y - 16x^3y^2 - 20y^2}{4xy^2}$

18. Simplify: $(5x^2 - 2x - 1) \div (x + 2)$

19. Simplify: $(4x^{-2}y)^2(2xy^{-2})^{-2}$

20. Given $f(t) = \dfrac{t}{t + 1}$, find $f(3)$.

21. Factor: $x^2 - 5x - 6$

22. Factor: $6x^2 - 5x - 6$

23. Factor: $8x^3 - 28x^2 + 12x$

24. Factor: $25x^2 - 16$

25. Factor: $2a(4 - x) - 6(x - 4)$

26. Factor: $75y - 12x^2y$

27. Solve: $2x^2 = 7x - 3$

28. Simplify: $\dfrac{2x^2 - 3x + 1}{4x^2 - 2x} \cdot \dfrac{4x^2 + 4x}{x^2 - 2x + 1}$

29. Simplify: $\dfrac{5}{x + 3} - \dfrac{3x}{2x - 5}$

30. Simplify: $x - \dfrac{1}{1 - \dfrac{1}{x}}$

31. Solve: $\dfrac{5x}{3x - 5} - 3 = \dfrac{7}{3x - 5}$

32. Solve $a = 3a - 2b$ for a.

33. Find the slope of the line that contains the points $(-1, -3)$ and $(2, -1)$.

34. Find the equation of the line that contains the point $(3, -4)$ and has slope $-\dfrac{2}{3}$.

35. Solve by substitution:
$y = 4x - 7$
$y = 2x + 5$

36. Solve by the addition method:
$4x - 3y = 11$
$2x + 5y = -1$

37. Solve: $4 - x \geq 7$

38. Solve: $2 - 2(y - 1) \leq 2y - 6$

39. Simplify: $\sqrt{49x^6}$

40. Simplify: $2\sqrt{27a} + 8\sqrt{48a}$

41. Simplify: $\dfrac{\sqrt{3}}{\sqrt{5} - 2}$

42. Solve: $\sqrt{2x - 3} + 4 = 5$

43. Solve by factoring:
$3x^2 - x = 4$

44. Solve by using the quadratic formula:
$4x^2 - 2x - 1 = 0$

45. Translate and simplify "the sum of twice a number and three times the difference between the number and two."

46. Because of depreciation, the value of an office machine is now $2400. This is 80% of its original value. Find the original value.

47. The manufacturer's cost for a laser printer is $900. The manufacturer then sells the printer for $1485. What is the markup rate?

48. An investment of $3000 is made at an annual simple interest rate of 8%. How much additional money must be invested at 11% so that the total interest earned is 10% of the total investment?

49. A grocer mixes 4 lb of peanuts that cost $2 per pound with 2 lb of walnuts that cost $5 per pound. What is the cost per pound of the resulting mixture?

50. A pharmacist mixes together 20 L of a solution that is 60% acid and 30 L of a solution that is 20% acid. What is the percent concentration of the acid in the mixture?

51. At 2 P.M. a small plane had been flying 1 h when a change of wind direction doubled its average ground speed. The complete 860-kilometer trip took 2.5 h. How far did the plane travel in the first hour?

52. The angles of a triangle are such that the second angle is 10° more than the first angle and the third angle is 10° more than the second angle. Find the measure of each of the three angles.

53. The sum of the squares of three consecutive integers is 50. Find the middle integer.

54. The length of a rectangle is 5 m more than the width. The area of the rectangle is 50 m². Find the dimensions of the rectangle.

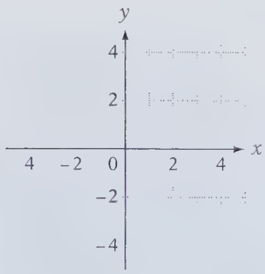
55. A paint formula requires 2 oz of dye for every 15 oz of base paint. How many ounces of dye are required for 120 oz of base paint?

56. It takes a chef 1 h to prepare a dinner. The chef's apprentice can prepare the dinner in 1.5 h. How long would it take the chef and the apprentice, working together, to prepare the dinner?

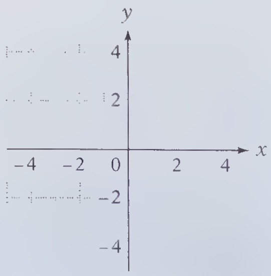
57. With the current, a motorboat travels 50 mi in 2.5 h. Against the current, it takes twice as long to travel 50 mi. Find the rate of the boat in calm water and the rate of the current.

58. Flying against the wind, it took a pilot $\frac{1}{2}$ h longer to travel 500 mi than it took flying with the wind. The rate of the plane in calm air is 225 mph. Find the rate of the wind.

59. Graph the line that has slope $-\frac{1}{2}$ and y-intercept $(0, -3)$.



60. Graph $y = x^2 - 4x + 3$.



Appendix

Guidelines for Using Graphing Calculators

Texas Instruments TI-83 and TI-83Plus

To evaluate an expression

a. Press the Y= key. A menu showing \Y1 = through \Y7 = will be displayed vertically with a blinking cursor to the right of \Y1 =. Press CLEAR, if necessary, to delete an unwanted expression.

b. Input the expression to be evaluated. For example, to input the expression $-3a^2b - 4c$, use the following keystrokes:

(−) 3 ALPHA A X^2 ALPHA B − 4 ALPHA C 2nd QUIT

Note the difference between the keys for a negative sign (−) and a minus sign −.

c. Store the value of each variable that will be used in the expression. For example, to evaluate the expression above when $a = 3$, $b = -2$, and $c = -4$, use the following keystrokes:

3 STO▷ ALPHA A ENTER (−) 2 STO▷ ALPHA B ENTER (−) 4 STO▷ ALPHA C ENTER

These steps store the value of each variable.

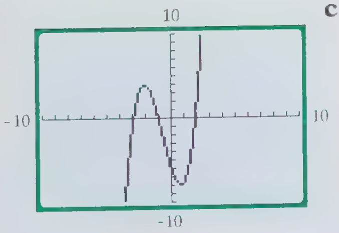
d. Press VARS ▷ 1 1 ENTER. The value for the expression, Y1, for the given values is displayed; in this case, Y1 = 70.

To graph a function

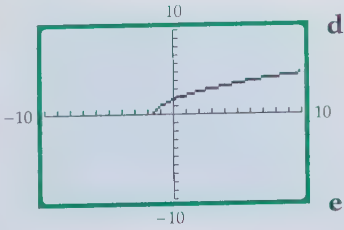
a. Press the Y= key. A menu showing \Y1 = through \Y7 = will be displayed vertically with a blinking cursor to the right of \Y1 =. Press CLEAR, if necessary, to delete an unwanted expression.

b. Input the expression for each function that is to be graphed. Press X,T,θ,*n* to input x. For example, to input $y = x^3 + 2x^2 - 5x - 6$, use the following keystrokes:

X,T,θ,*n* ∧ 3 + 2 X,T,θ,*n* X^2 − 5 X,T,θ,*n* − 6



c. Set the viewing window by pressing WINDOW. Enter the values for the minimum x-value (Xmin), the maximum x-value (Xmax), the distance between tick marks on the x-axis (Xscl), the minimum y-value (Ymin), the maximum y-value (Ymax), and the distance between tick marks on the y-axis (Yscl). Now press GRAPH. For the graph shown at the left, Xmin = −10, Xmax = 10, Xscl = 1, Ymin = −10, Ymax = 10, and Yscl = 1. This is called the standard viewing window. Pressing ZOOM 6 is a quick way to set the calculator to the standard viewing window. *Note:* This will also immediately graph the function in that window.



d. Press the Y= key. The equal sign has a black rectangle around it. This indicates that the function is active and will be graphed when the GRAPH key is pressed. A function is deactivated by using the arrow keys. Move the cursor over the equal sign and press ENTER. When the cursor is moved to the right, the black rectangle will not be present and that equation will not be active.

e. Graphing some radical equations requires special care. To graph the equation $y = \sqrt{2x + 3}$ shown at the left, enter the following keystrokes:

Y= CLEAR 2nd √ 2 X,T,θ,*n* + 3) GRAPH

To display the x-coordinates of rectangular coordinates as integers

a. Set the viewing window as follows: Xmin $= -47$, Xmax $= 47$, Xscl $= 10$, Ymin $= -31$, Ymax $= 31$, Yscl $= 10$.

b. Graph the function and use the TRACE feature. Press TRACE and then move the cursor with the ◁ and ▷ keys. The values of x and $y = f(x)$ displayed on the bottom of the screen are the coordinates of a point on the graph.

To display the x-coordinates of rectangular coordinates in tenths

a. Set the viewing window as follows: ZOOM 4

b. Graph the function. Press TRACE and then move the cursor with the ◁ and ▷ keys. The values of x and $y = f(x)$ displayed on the bottom of the screen are the coordinates of a point on the graph.

To evaluate a function for a given value of x, or to produce ordered pairs of a function

a. Input the equation; for example, input $Y_1 = 2x^3 - 3x + 2$.

b. Press 2nd QUIT.

c. To evaluate the function when $x = 3$, press VARS ▷ 1 1 (3) ENTER. The value for the function for the given x-value is displayed, in this case, $Y_1(3) = 47$. An ordered pair of the function is $(3, 47)$.

d. Repeat step **c.** to produce as many pairs as desired.

The TABLE feature can also be used to determine ordered pairs.

To use a table

a. Press 2nd TBLSET to activate the table setup menu.

b. TblStart is the beginning number for the table; ΔTbl is the difference between any two successive x-values in the table.

c. The portion of the table that appears as
Indpnt: Auto Ask
Depend: Auto Ask
allows you to choose between having the calculator automatically produce the results (Auto) or having the calculator ask you for values of x. You can choose Ask by using the arrow keys.

d. Once a table has been set up, enter an expression for Y_1. Now select TABLE by pressing 2nd TABLE. A table showing ordered pair solutions of the equation will be displayed on the screen.

Zoom Features

To zoom in or out on a graph

Here are two methods of using ZOOM.

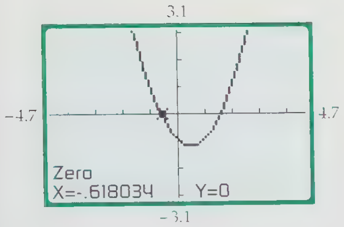
a. The first method uses the built-in features of the calculator. Press TRACE and then move the cursor to a point on the graph that is of interest. Press ZOOM. The ZOOM menu will appear. Press 2 ENTER to zoom in on the graph by the amount shown under the SET FACTORS menu. The center of the new graph is the location at which you placed the cursor. Press ZOOM 3 ENTER to zoom out on the graph by the amount under the SET FACTORS menu. (The SET FACTORS menu is accessed by pressing ZOOM ▷ 4.)

b. The second method uses the ZBOX option under the ZOOM menu. To use this method, press ZOOM 1. A cursor will appear on the graph. Use the arrow keys to move the cursor to a portion of the graph that is of interest. Press ENTER. Now use the arrow keys to draw a box around the portion of the graph you wish to zoom in on. Press ENTER. The portion of the graph defined by the box will be drawn.

c. Pressing ZOOM 6 resets the window to the standard viewing window.

Solving Equations

This discussion is based on the fact that the real number solutions of an equation are related to the x-intercepts of a graph. For instance, the real solutions of the equation $x^2 = x + 1$ are the x-intercepts of the graph of $f(x) = x^2 - x - 1$, which are the zeros of f.



```
Zero
X=-.618034   Y=0
```

To solve $x^2 = x + 1$, rewrite the equation with all terms on one side: $x^2 - x - 1 = 0$. Think of this equation as $Y_1 = x^2 - x - 1$. The x-intercepts of the graph of Y_1 are the solutions of the equation $x^2 = x + 1$.

a. Enter $x^2 - x - 1$ into Y_1.

b. Graph the equation. You may need to adjust the viewing window so that the x-intercepts are visible.

c. Press 2nd CALC 2.

d. Move the cursor to a point on the curve that is to the left of an x-intercept. Press ENTER.

e. Move the cursor to a point on the curve that is to the right of the same x-intercept. Press ENTER ENTER.

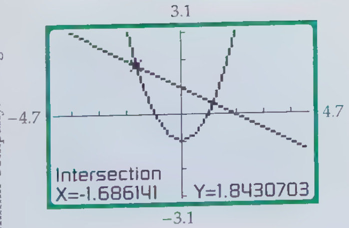
f. The root is shown as the x-coordinate on the bottom of the screen; in this case, the root is approximately -0.618034. To find the next intercept, repeat steps **c.** through **e.** The SOLVER feature under the MATH menu can also be used to find solutions of equations.

Solving Systems of Equations in Two Variables

To solve a system of equations

To solve

$$y = x^2 - 1$$
$$\frac{1}{2}x + y = 1 \quad ,$$



```
Intersection
X=-1.686141   Y=1.8430703
```

a. If necessary, solve one or both of the equations for y.

b. Enter the first equation as Y_1: $Y_1 = x^2 - 1$.

c. Enter the second equation as Y_2: $Y_2 = 1 - .5x$.

d. Graph both equations. (*Note:* The point of intersection must appear on the screen. It may be necessary to adjust the viewing window so that the points of intersection are displayed.)

e. Press 2nd CALC 5.

f. Move the cursor close to the first point of intersection. Press ENTER ENTER ENTER.

g. The first point of intersection is $(-1.686141, 1.8430703)$.

h. Repeat steps **e.** and **f.** for each point of intersection.

Finding Minimum or Maximum Values of a Function

a. Enter the function into Y1. The equation $y = x^2 - x - 1$ is used here.

b. Graph the equation. You may need to adjust the viewing window so that the maximum or minimum points are visible.

c. Press $\boxed{\text{2nd}}$ CALC $\boxed{3}$ to determine a minimum value or press $\boxed{\text{2nd}}$ CALC $\boxed{4}$ to determine a maximum value.

d. Move the cursor to a point on the curve that is to the left of the minimum (maximum). Press $\boxed{\text{ENTER}}$.

e. Move the cursor to a point on the curve that is to the right of the minimum (maximum). Press $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$.

f. The minimum (maximum) is shown as the y-coordinate on the bottom of the screen; in this case the minimum value is -1.25.

Statistics

To calculate a linear regression equation

a. Press $\boxed{\text{STAT}}$ to access the statistics menu. Press 1 to Edit or enter a new list of data. To delete data already in a list, press the up arrow to highlight the list name. For instance, to delete data in L1, highlight L1. Then press $\boxed{\text{CLEAR}}$ and $\boxed{\text{ENTER}}$. Now enter each value of the independent variable in L1. Enter each value of the dependent variable in L2.

b. When all the data has been entered, press $\boxed{\text{STAT}}$ $\boxed{\triangleright}$ 4 $\boxed{\text{ENTER}}$. The values of the slope and y-intercept of the linear regression equation will be displayed on the screen.

c. To graph the linear regression equation, modify step **b.** with these keystrokes: $\boxed{\text{STAT}}$ $\boxed{\triangleright}$ 4 $\boxed{\text{VARS}}$ $\boxed{\triangleright}$ 1 1 $\boxed{\text{ENTER}}$. This will store the regression equation in Y1. Now press $\boxed{\text{GRAPH}}$. It may be necessary to adjust the viewing window.

d. To evaluate the regression equation for a value of x, complete step **c.** but do not graph the equation. Now press $\boxed{\text{VARS}}$ $\boxed{\triangleright}$ 1 1 $\boxed{(}$ A $\boxed{)}$ $\boxed{\text{ENTER}}$, where A is replaced by the number at which you want to evaluate the expression.

Table of Symbols

$+$	add
$-$	subtract
$\cdot, \times, (a)(b)$	multiply
$\frac{a}{b}, \div$	divide
$(\,)$	parentheses, a grouping symbol
$[\,]$	brackets, a grouping symbol
π	pi, a number approximately equal to $\frac{22}{7}$ or 3.14
$-a$	the opposite, or additive inverse, of a
$\frac{1}{a}$	the reciprocal, or multiplicative inverse, of a
$=$	is equal to
$\approx$	is approximately equal to
$\neq$	is not equal to
$<$	is less than
$\leq$	is less than or equal to
$>$	is greater than
$\geq$	is greater than or equal to
(a, b)	an ordered pair whose first component is a and whose second component is b
$^\circ$	degree (for angles)
$\sqrt{a}$	the principal square root of a
$\varnothing, \{\,\}$	the empty set
$\lvert a \rvert$	the absolute value of a
$\cup$	union of two sets
$\cap$	intersection of two sets
$\in$	is an element of (for sets)
$\notin$	is not an element of (for sets)

Table of Measurement Abbreviations

U.S. Customary System

Length

in.	inches
ft	feet
yd	yards
mi	miles

Capacity

oz	fluid ounces
c	cups
qt	quarts
gal	gallons

Weight

oz	ounces
lb	pounds

Area

in^2	square inches
ft^2	square feet

Metric System

Length

mm	millimeter (0.001 m)
cm	centimeter (0.01 m)
dm	decimeter (0.1 m)
m	meter
dam	decameter (10 m)
hm	hectometer (100 m)
km	kilometer (1000 m)

Capacity

ml	milliliter (0.001 L)
cl	centiliter (0.01 L)
dl	deciliter (0.1 L)
L	liter
dal	decaliter (10 L)
hl	hectoliter (100 L)
kl	kiloliter (1000 L)

Weight/Mass

mg	milligram (0.001 g)
cg	centigram (0.01 g)
dg	decigram (0.1 g)
g	gram
dag	decagram (10 g)
hg	hectogram (100 g)
kg	kilogram (1000 g)

Area

cm^2	square centimeters
m^2	square meters

Time

h	hours
min	minutes
s	seconds

Solutions to Chapter 1 "You Try It"

SECTION 1.1

You Try It 1 $A = \{1, 2, 3, 4, 5, 6\}$

You Try It 2

$$\begin{aligned} &-5 < -1 \\ &-1 = -1 \\ &5 > -1 \end{aligned}$$

The element 5 is greater than −1.

You Try It 3

$$\begin{aligned} &|-5| = 5 \\ &-|-23| = -23 \end{aligned}$$

You Try It 4

$$\begin{aligned} &-(-11) = 11 \\ &-0 = 0 \\ &-(8) = -8 \end{aligned}$$

You Try It 5

$$\begin{aligned} &|-37| = 37 \\ &|0| = 0 \\ &|29| = 29 \end{aligned}$$

SECTION 1.2

You Try It 1 $100 + (-43) = 57$

You Try It 2

$$\begin{aligned} &(-51) + 42 + 17 + (-102) \\ &\quad = -9 + 17 + (-102) \\ &\quad = 8 + (-102) \\ &\quad = -94 \end{aligned}$$

You Try It 3

$$\begin{aligned} 19 - (-32) &= 19 + 32 \\ &= 51 \end{aligned}$$

You Try It 4

$$\begin{aligned} &-9 - (-12) - 17 - 4 \\ &\quad = -9 + 12 + (-17) + (-4) \\ &\quad = 3 + (-17) + (-4) \\ &\quad = -14 + (-4) \\ &\quad = -18 \end{aligned}$$

You Try It 5

Strategy To find the difference between the two average temperatures, subtract the smaller number (−130) from the larger number (−17).

Solution $-17 - (-130) = -17 + 130 = 113$

The difference is 113°F.

SECTION 1.3

You Try It 1

$$\begin{aligned} 8(-9)10 &= -72(10) \\ &= -720 \end{aligned}$$

You Try It 2

$$\begin{aligned} (-2)3(-8)7 &= -6(-8)7 \\ &= 48(7) \\ &= 336 \end{aligned}$$

You Try It 3 $(-135) \div (-9) = 15$

You Try It 4 $\dfrac{-72}{4} = -18$

You Try It 5

$$\begin{aligned} -\frac{36}{12} &= -(-3) \\ &= 3 \end{aligned}$$

You Try It 6

Strategy

To find the average daily low temperature:

- Add the seven temperature readings.
- Divide the sum by 7.

Solution

$-6 + (-7) + 0 + (-5) + (-8) + (-1) + (-1) = -28$

$-28 \div 7 = -4$

The average daily low temperature was −4°C.

SECTION 1.4

You Try It 1

$$\begin{array}{r} 0.444 \\ 9\overline{)4.000} \\ \underline{-3\,6} \\ 40 \\ \underline{-36} \\ 40 \\ \underline{-36} \\ 4 \end{array} \qquad \frac{4}{9} = 0.\overline{4}$$

You Try It 2

$$\begin{aligned} &125\% = 125\left(\frac{1}{100}\right) = \frac{125}{100} = \frac{5}{4} \\ &125\% = 125(0.01) = 1.25 \end{aligned}$$

You Try It 3 $16\frac{2}{3}\% = 16\frac{2}{3}\left(\frac{1}{100}\right) = \frac{50}{3}\left(\frac{1}{100}\right) = \frac{1}{6}$

You Try It 4

$$\begin{aligned} &\frac{9}{16} = \frac{9}{16}(100\%) = \frac{900}{16}\% = 56.25\% \\ &\text{or } 56\frac{1}{4}\% \end{aligned}$$

You Try It 5 $0.043 = 0.043(100\%) = 4.3\%$

You Try It 6 The LCM of 9 and 12 is 36.

$$\frac{5}{9} - \frac{11}{12} = \frac{20}{36} - \frac{33}{36} = \frac{20}{36} + \frac{-33}{36}$$
$$= \frac{20 + (-33)}{36} = \frac{-13}{36} = -\frac{13}{36}$$

You Try It 7 The LCM of 8, 6, and 4 is 24.

$$-\frac{7}{8} - \frac{5}{6} + \frac{3}{4} = \frac{21}{24} - \frac{20}{24} + \frac{18}{24}$$
$$= \frac{-21}{24} + \frac{-20}{24} + \frac{18}{24}$$
$$= \frac{-21 + (-20) + 18}{24}$$
$$= \frac{-23}{24} = -\frac{23}{24}$$

You Try It 8 $16.127 - 67.91 = 16.127 + (-67.91)$
$= -51.783$

You Try It 9 The product is negative.

$$-\frac{7}{12} \times \frac{9}{14} = -\frac{7 \cdot 9}{12 \cdot 14}$$
$$= -\frac{\overset{1}{\cancel{7}} \cdot \overset{1}{\cancel{3}} \cdot 3}{2 \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot 2 \cdot \underset{1}{\cancel{7}}}$$
$$= -\frac{3}{8}$$

You Try It 10 The quotient is positive.

$$-\frac{3}{8} \div \left(-\frac{5}{12}\right) = \frac{3}{8} \div \frac{5}{12} = \frac{3}{8} \times \frac{12}{5}$$
$$= \frac{3 \cdot 12}{8 \cdot 5} = \frac{3 \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2 \cdot 5} = \frac{9}{10}$$

You Try It 11 Multiply the absolute values.
The product is negative.
$-5.44 \times 3.8 = -20.672$

You Try It 12 Divide the absolute values.
The quotient is negative.

$$\begin{array}{r} 0.231 \\ 1.7.\overline{)0.3.940} \\ -3\,4 \\ \hline 54 \\ -51 \\ \hline 30 \\ -17 \\ \hline 13 \end{array}$$

$-0.394 \div 1.7 \approx -0.23$

You Try It 13

Strategy To find the difference between projected e-book sales in 2005 and in 2002, subtract the e-book sales in 2002 ($.41) from the projected sales in 2005 ($2.28).

Solution $2.28 - 0.41 = 1.87$

The difference in e-book sales between the two years is $1.87 billion.

SECTION 1.5

You Try It 1 $-6^3 = -(6 \cdot 6 \cdot 6) = -216$

You Try It 2 $(-3)^4 = (-3)(-3)(-3)(-3) = 81$

You Try It 3 $(3^3)(-2)^3 = (3)(3)(3) \cdot (-2)(-2)(-2)$
$= 27(-8) = -216$

You Try It 4 $\left(-\frac{2}{5}\right)^2 = \left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right) = \frac{2 \cdot 2}{5 \cdot 5} = \frac{4}{25}$

You Try It 5 $-3(0.3)^3 = -3(0.3)(0.3)(0.3)$
$= -0.9(0.3)(0.3)$
$= -0.27(0.3) = -0.081$

You Try It 6 $(6.97 - 4.72)^2 \cdot 4.5 \div 0.05$
$= (2.25)^2 \cdot 4.5 \div 0.05$
$= 5.0625 \cdot 4.5 \div 0.05$
$= 22.78125 \div 0.05$
$= 455.625$

You Try It 7 $18 - 5[8 - 2(2 - 5)] \div 10$
$= 18 - 5[8 - 2(-3)] \div 10$
$= 18 - 5[8 + 6] \div 10$
$= 18 - 5[14] \div 10$
$= 18 - 70 \div 10$
$= 18 - 7$
$= 11$

You Try It 8 $36 \div (8 - 5)^2 - (-3)^2 \cdot 2$
$= 36 \div (3)^2 - (-3)^2 \cdot 2$
$= 36 \div 9 - 9 \cdot 2$
$= 4 - 9 \cdot 2$
$= 4 - 18$
$= -14$

You Try It 9

$$\begin{aligned}&\frac{5}{8} \div \left(\frac{1}{3} - \frac{3}{4}\right) + \frac{7}{12}\\ &= \frac{5}{8} \div \left(-\frac{5}{12}\right) + \frac{7}{12}\\ &= \frac{5}{8} \cdot \left(-\frac{12}{5}\right) + \frac{7}{12}\\ &= -\frac{3}{2} + \frac{7}{12}\\ &= -\frac{18}{12} + \frac{7}{12}\\ &= -\frac{11}{12}\end{aligned}$$

Solutions to Chapter 2 "You Try It"

SECTION 2.1

You Try It 1 -4 is the constant term.

You Try It 2

$$\begin{aligned}&2xy + y^2\\ &2(-4)(2) + (2)^2\\ &\quad = 2(-4)(2) + 4\\ &\quad = (-8)(2) + 4\\ &\quad = (-16) + 4\\ &\quad = -12\end{aligned}$$

You Try It 3

$$\frac{a^2 + b^2}{a + b}$$

$$\begin{aligned}\frac{5^2 + (-3)^2}{5 + (-3)} &= \frac{25 + 9}{5 + (-3)}\\ &= \frac{34}{2}\\ &= 17\end{aligned}$$

You Try It 4

$$\begin{aligned}&x^3 - 2(x + y) + z^2\\ &(2)^3 - 2[2 + (-4)] + (-3)^2\\ &\quad = 8 - 2(-2) + 9\\ &\quad = 8 + 4 + 9\\ &\quad = 12 + 9\\ &\quad = 21\end{aligned}$$

SECTION 2.2

You Try It 1 $3a - 2b - 5a + 6b = -2a + 4b$

You Try It 2 $-3y^2 + 7 + 8y^2 - 14 = 5y^2 - 7$

You Try It 3 $-5(4y^2) = -20y^2$

You Try It 4 $-7(-2a) = 14a$

You Try It 5 $(-5x)(-2) = 10x$

You Try It 6 $5(3 + 7b) = 15 + 35b$

You Try It 7 $(3a - 1)5 = 15a - 5$

You Try It 8 $-8(-2a + 7b) = 16a - 56b$

You Try It 9 $3(12x^2 - x + 8) = 36x^2 - 3x + 24$

You Try It 10 $3(-a^2 - 6a + 7) = -3a^2 - 18a + 21$

You Try It 11

$$\begin{aligned}3y - 2(y - 7x) &= 3y - 2y + 14x\\ &= y + 14x\end{aligned}$$

You Try It 12

$$\begin{aligned}-2(x - 2y) - (-x + 3y) &= -2x + 4y + x - 3y\\ &= -x + y\end{aligned}$$

You Try It 13

$$\begin{aligned}3y - 2[x - 4(2 - 3y)] &= 3y - 2[x - 8 + 12y]\\ &= 3y - 2x + 16 - 24y\\ &= -2x - 21y + 16\end{aligned}$$

SECTION 2.3

You Try It 1 the difference between twice n and one-third of n

$$2n - \frac{1}{3}n$$

You Try It 2 the quotient of 7 less than b and 15

$$\frac{b - 7}{15}$$

You Try It 3 the unknown number: n
the cube of the number: n^3
the total of ten and the cube of the number: $10 + n^3$

$-4(10 + n^3)$

You Try It 4 the unknown number: x
the difference between the number and sixty: $x - 60$

$5(x - 60) = 5x - 300$

You Try It 5 the speed of the older model: s
the speed of the new jet plane is twice the speed of the older model: $2s$

You Try It 6 the length of the longer piece: y
the length of the shorter piece: $6 - y$

Solutions to Chapter 3 "You Try It"

SECTION 3.1

You Try It 1

$$\begin{array}{r|l} \multicolumn{2}{c}{5 - 4x = 8x + 2} \\ \hline 5 - 4\left(\frac{1}{4}\right) & 8\left(\frac{1}{4}\right) + 2 \\ 5 - 1 & 2 + 2 \\ \multicolumn{2}{c}{4 = 4} \end{array}$$

Yes, $\frac{1}{4}$ is a solution.

You Try It 2

$$\begin{array}{r|l} \multicolumn{2}{c}{10x - x^2 = 3x - 10} \\ \hline 10(5) - (5)^2 & 3(5) - 10 \\ 50 - 25 & 15 - 10 \\ \multicolumn{2}{c}{25 \neq 5} \end{array}$$

No, 5 is not a solution.

You Try It 3

$$\begin{aligned} \frac{5}{6} &= y - \frac{3}{8} \\ \frac{5}{6} + \frac{3}{8} &= y - \frac{3}{8} + \frac{3}{8} \\ \frac{29}{24} &= y \end{aligned}$$

The solution is $\frac{29}{24}$.

You Try It 4

$$\begin{aligned} -\frac{2}{5}x &= 6 \\ \left(-\frac{5}{2}\right)\left(-\frac{2}{5}x\right) &= \left(-\frac{5}{2}\right)(6) \\ x &= -15 \end{aligned}$$

The solution is -15.

You Try It 5

$$\begin{aligned} 4x - 8x &= 16 \\ -4x &= 16 \\ \frac{-4x}{-4} &= \frac{16}{-4} \\ x &= -4 \end{aligned}$$

The solution is -4.

You Try It 6

$$\begin{aligned} P \cdot B &= A \\ \frac{1}{6}B &= 18 \\ 6 \cdot \frac{1}{6}B &= 6 \cdot 18 \\ B &= 108 \end{aligned}$$

• $16\frac{2}{3}\% = \frac{1}{6}$

18 is $16\frac{2}{3}\%$ of 108.

You Try It 7

Strategy To find the percent, solve the basic percent equation using $B = 158.3$ million and $A = 22.45$ million. The percent is the unknown.

Solution

$$\begin{aligned} P \cdot B &= A \\ P(158.3) &= 22.45 \\ \frac{P(158.3)}{158.3} &= \frac{22.45}{158.3} \\ P &\approx 0.14181933 \end{aligned}$$

The Big Ten conference received 14.2% of the 2000 football bowl revenues.

SECTION 3.2

You Try It 1

$$\begin{aligned} 5x + 7 &= 10 \\ 5x + 7 - 7 &= 10 - 7 \\ 5x &= 3 \\ \frac{5x}{5} &= \frac{3}{5} \\ x &= \frac{3}{5} \end{aligned}$$

The solution is $\frac{3}{5}$.

You Try It 2

$$\begin{aligned} 2 &= 11 + 3x \\ 2 - 11 &= 11 - 11 + 3x \\ -9 &= 3x \\ \frac{-9}{3} &= \frac{3x}{3} \\ -3 &= x \end{aligned}$$

The solution is −3.

You Try It 3

$$\begin{aligned} \frac{5}{8} - \frac{2x}{3} &= \frac{5}{4} \\ \frac{5}{8} - \frac{5}{8} - \frac{2}{3}x &= \frac{5}{4} - \frac{5}{8} \\ -\frac{2}{3}x &= \frac{5}{8} \\ -\frac{3}{2}\left(-\frac{2}{3}x\right) &= -\frac{3}{2}\left(\frac{5}{8}\right) \\ x &= -\frac{15}{16} \end{aligned}$$

The solution is $-\frac{15}{16}$.

- **Recall that** $\frac{2x}{3} = \frac{2}{3}x$

You Try It 4

$$\begin{aligned} \frac{2}{3}x + 3 &= \frac{7}{2} \\ 6\left(\frac{2}{3}x + 3\right) &= 6\left(\frac{7}{2}\right) \\ 6\left(\frac{2}{3}x\right) + 6(3) &= 6\left(\frac{7}{2}\right) \\ 4x + 18 &= 21 \\ 4x + 18 - 18 &= 21 - 18 \\ 4x &= 3 \\ \frac{4x}{4} &= \frac{3}{4} \\ x &= \frac{3}{4} \end{aligned}$$

The solution is $\frac{3}{4}$.

You Try It 5

$$\begin{aligned} x - 5 + 4x &= 25 \\ 5x - 5 &= 25 \\ 5x - 5 + 5 &= 25 + 5 \\ 5x &= 30 \\ \frac{5x}{5} &= \frac{30}{5} \\ x &= 6 \end{aligned}$$

The solution is 6.

You Try It 6

Strategy Given: $S = 986$
$r = 45\% = 0.45$
Unknown: C

Solution

$$\begin{aligned} S &= C + rC \\ 986 &= C + 0.45C \\ 986 &= 1.45C \\ 680 &= C \end{aligned}$$

The cost of the outboard motor is $680.

You Try It 7

Strategy Given: $S = 159$
$r = 25\% = 0.25$
Unknown: R

Solution

$$\begin{aligned} S &= R - rR \\ 159 &= R - 0.25R \\ 159 &= 0.75R \\ 212 &= R \end{aligned}$$

The regular price of the garage door opener is $212.

You Try It 8

Strategy Given: $P = 45$
Unknown: D

Solution

$$\begin{aligned} P &= 15 + \frac{1}{2}D \\ 45 &= 15 + \frac{1}{2}D \\ 45 - 15 &= 15 - 15 + \frac{1}{2}D \\ 30 &= \frac{1}{2}D \\ 2(30) &= 2 \cdot \frac{1}{2}D \\ 60 &= D \end{aligned}$$

The depth is 60 ft.

SECTION 3.3

You Try It 1

$$\begin{aligned} 5x + 4 &= 6 + 10x \\ 5x - 10x + 4 &= 6 + 10x - 10x \\ -5x + 4 &= 6 \\ -5x + 4 - 4 &= 6 - 4 \\ -5x &= 2 \\ \frac{-5x}{-5} &= \frac{2}{-5} \\ x &= -\frac{2}{5} \end{aligned}$$

The solution is $-\frac{2}{5}$.

You Try It 2

$$\begin{aligned} 5x - 10 - 3x &= 6 - 4x \\ 2x - 10 &= 6 - 4x \\ 2x + 4x - 10 &= 6 - 4x + 4x \\ 6x - 10 &= 6 \\ 6x - 10 + 10 &= 6 + 10 \\ 6x &= 16 \\ \frac{6x}{6} &= \frac{16}{6} \\ x &= \frac{8}{3} \end{aligned}$$

The solution is $\frac{8}{3}$.

You Try It 3

$$\begin{aligned} 5x - 4(3 - 2x) &= 2(3x - 2) + 6 \\ 5x - 12 + 8x &= 6x - 4 + 6 \\ 13x - 12 &= 6x + 2 \\ 13x - 6x - 12 &= 6x - 6x + 2 \\ 7x - 12 &= 2 \\ 7x - 12 + 12 &= 2 + 12 \\ 7x &= 14 \\ \frac{7x}{7} &= \frac{14}{7} \\ x &= 2 \end{aligned}$$

The solution is 2.

You Try It 4

$$\begin{aligned} -2[3x - 5(2x - 3)] &= 3x - 8 \\ -2[3x - 10x + 15] &= 3x - 8 \\ -2[-7x + 15] &= 3x - 8 \\ 14x - 30 &= 3x - 8 \\ 14x - 3x - 30 &= 3x - 3x - 8 \\ 11x - 30 &= -8 \\ 11x - 30 + 30 &= -8 + 30 \\ 11x &= 22 \\ \frac{11x}{11} &= \frac{22}{11} \\ x &= 2 \end{aligned}$$

The solution is 2.

You Try It 5

Strategy To find the location of the fulcrum when the system balances, replace the variables F_1, F_2, and d in the lever system equation by the given values and solve for x.

Solution

$$\begin{aligned} F_1x &= F_2(d - x) \\ 45x &= 80(25 - x) \\ 45x &= 2000 - 80x \\ 45x + 80x &= 2000 - 80x + 80x \\ 125x &= 2000 \\ \frac{125x}{125} &= \frac{2000}{125} \\ x &= 16 \end{aligned}$$

The fulcrum is 16 ft from the 45-pound force.

SECTION 3.4

You Try It 1

The smaller number: n
The larger number: $12 - n$

The total of three times the smaller number and six	amounts to	seven less than the product of four and the larger number

$$\begin{aligned} 3n + 6 &= 4(12 - n) - 7 \\ 3n + 6 &= 48 - 4n - 7 \\ 3n + 6 &= 41 - 4n \\ 3n + 4n + 6 &= 41 - 4n + 4n \\ 7n + 6 &= 41 \\ 7n + 6 - 6 &= 41 - 6 \\ 7n &= 35 \\ \frac{7n}{7} &= \frac{35}{7} \\ n &= 5 \end{aligned}$$

$12 - n = 12 - 5 = 7$

The smaller number is 5.
The larger number is 7.

You Try It 2

Strategy

- First integer: n
  Second integer: $n + 1$
  Third integer: $n + 2$
- The sum of the three integers is -6.

Solution

$$\begin{aligned} n + (n + 1) + (n + 2) &= -6 \\ 3n + 3 &= -6 \\ 3n &= -9 \\ n &= -3 \end{aligned}$$

$n + 1 = -3 + 1 = -2$
$n + 2 = -3 + 2 = -1$

The three consecutive integers are -3, -2, and -1.

You Try It 3

Strategy
To find the number of tickets that you are purchasing, write and solve an equation using x to represent the number of tickets purchased.

Solution

\$3.50 plus \$17.50 for each ticket	is	\$161

$$\begin{aligned} 3.50 + 17.50x &= 161 \\ 3.50 - 3.50 + 17.50x &= 161 - 3.50 \\ 17.50x &= 157.50 \\ \frac{17.50x}{17.50} &= \frac{157.50}{17.50} \\ x &= 9 \end{aligned}$$

You are purchasing 9 tickets.

You Try It 4

Strategy

To find the length, write and solve an equation using x to represent the length of the shorter piece and $22 - x$ to represent the length of the longer piece.

Solution

The length of the longer piece	is	4 in. more than twice the length of the shorter piece

$$\begin{aligned} 22 - x &= 2x + 4 \\ 22 - x - 2x &= 2x - 2x + 4 \\ 22 - 3x &= 4 \\ 22 - 22 - 3x &= 4 - 22 \\ -3x &= -18 \\ \frac{-3x}{-3} &= \frac{-18}{-3} \\ x &= 6 \end{aligned}$$

$22 - x = 22 - 6 = 16$

The length of the shorter piece is 6 in.
The length of the longer piece is 16 in.

SECTION 3.5

You Try It 1

Strategy To find the length of each side, use the formula for the perimeter of a square. Substitute 52 for P and solve for s.

Solution

$$\begin{aligned} P &= 4s \\ 52 &= 4s \\ 13 &= s \end{aligned}$$

The length of each side of the patio is 13 ft.

You Try It 2

Strategy To find the supplement, let x represent the supplement of a 107° angle. Use the fact that supplementary angles are two angles whose sum is 180° to write an equation. Solve for x.

Solution

$$\begin{aligned} x + 107^\circ &= 180^\circ \\ x &= 73^\circ \end{aligned}$$

The supplement of a 107° angle is a 73° angle.

You Try It 3

Strategy The angles labeled are adjacent angles of intersecting lines and are therefore supplementary angles. To find x, write an equation and solve for x.

Solution

$$\begin{aligned} x + (3x + 20^\circ) &= 180^\circ \\ 4x + 20^\circ &= 180^\circ \\ 4x &= 160^\circ \\ x &= 40^\circ \end{aligned}$$

You Try It 4

Strategy $2x = y$ because alternate exterior angles have the same measure. $y + (x + 15^\circ) = 180^\circ$ because adjacent angles of intersecting lines are supplementary angles. Substitute $2x$ for y and solve for x.

Solution

$$\begin{aligned} y + (x + 15^\circ) &= 180^\circ \\ 2x + (x + 15^\circ) &= 180^\circ \\ 3x + 15^\circ &= 180^\circ \\ 3x &= 165^\circ \\ x &= 55^\circ \end{aligned}$$

You Try It 5

Strategy

- To find the measure of angle a, use the fact that $\angle a$ and $\angle y$ are vertical angles.
- To find the measure of angle b, use the fact that the sum of the measures of the interior angles of a triangle is 180°.
- To find the measure of angle d, use the fact that the sum of an interior and an exterior angle is 180°.

Solution

$$\angle a = \angle y = 55^\circ$$

$$\begin{aligned} \angle a + \angle b + 90^\circ &= 180^\circ \\ 55^\circ + \angle b + 90^\circ &= 180^\circ \\ \angle b + 145^\circ &= 180^\circ \\ \angle b &= 35^\circ \end{aligned}$$

$$\begin{aligned} \angle d + \angle b &= 180^\circ \\ \angle d + 35^\circ &= 180^\circ \\ \angle d &= 145^\circ \end{aligned}$$

You Try It 6

Strategy To find the measure of the third angle, use the fact that the sum of the measures of the interior angles of a triangle is 180°. Write an equation using x to represent the measure of the third angle. Solve the equation for x.

Solution

$$\begin{aligned} x + 90^\circ + 27^\circ &= 180^\circ \\ x + 117^\circ &= 180^\circ \\ x &= 63^\circ \end{aligned}$$

The measure of the third angle is 63°.

SECTION 3.6

You Try It 1

Strategy

- Pounds of \$.55 fertilizer: x

	Amount	*Cost*	*Value*
\$.80 fertilizer	20	.80	0.80(20)
\$.55 fertilizer	x	.55	$0.55x$
\$.75 fertilizer	$20 + x$	.75	$0.75(20 + x)$

- The sum of the values before mixing equals the value after mixing.

Solution

$$\begin{aligned} 0.80(20) + 0.55x &= 0.75(20 + x) \\ 16 + 0.55x &= 15 + 0.75x \\ 16 - 0.20x &= 15 \\ -0.20x &= -1 \\ x &= 5 \end{aligned}$$

5 lb of the \$.55 fertilizer must be added.

You Try It 2

Strategy

- Liters of 6% solution: x

	Amount	*Percent*	*Quantity*
6% solution	x	0.06	$0.06x$
12% solution	5	0.12	5(0.12)
8% solution	$x + 5$	0.08	$0.08(x + 5)$

- The sum of the quantities before mixing equals the quantity after mixing.

Solution

$$\begin{aligned} 0.06x + 5(0.12) &= 0.08(x + 5) \\ 0.06x + 0.60 &= 0.08x + 0.40 \\ -0.02x + 0.60 &= 0.40 \\ -0.02x &= -0.20 \\ x &= 10 \end{aligned}$$

The pharmacist adds 10 L of the 6% solution to the 12% solution to get an 8% solution.

You Try It 3

Strategy

- Rate of the first train: r
  Rate of the second train: $2r$

	Rate	*Time*	*Distance*
1st train	r	3	$3r$
2nd train	$2r$	3	$3(2r)$

- The sum of the distances traveled by the two trains equals 288 mi.

Solution

$$\begin{aligned} 3r + 3(2r) &= 288 \\ 3r + 6r &= 288 \\ 9r &= 288 \\ r &= 32 \end{aligned}$$

$2r = 2(32) = 64$

The first train is traveling at 32 mph. The second train is traveling at 64 mph.

You Try It 4

Strategy

- Time spent flying out: t
  Time spent flying back: $5 - t$

	Rate	*Time*	*Distance*
Out	150	t	$150t$
Back	100	$5 - t$	$100(5 - t)$

- The distance out equals the distance back.

Solution

$$\begin{aligned} 150t &= 100(5 - t) \\ 150t &= 500 - 100t \\ 250t &= 500 \\ t &= 2 \quad \text{(The time out was 2 h.)} \end{aligned}$$

$$\begin{aligned} \text{The distance out} &= 150t = 150(2) \\ &= 300 \text{ mi} \end{aligned}$$

The parcel of land was 300 mi away.

Solutions to Chapter 4 "You Try It"

SECTION 4.1

You Try It 1

$$\begin{aligned}&(-4x^3 + 2x^2 - 8) + (4x^3 + 6x^2 - 7x + 5)\\&\quad = (-4x^3 + 4x^3) + (2x^2 + 6x^2) + (-7x) + (-8 + 5)\\&\quad = 8x^2 - 7x - 3\end{aligned}$$

You Try It 2

$$\begin{array}{r}6x^3 \qquad\qquad + \;\; 2x + 8\\ \underline{-9x^3 + 2x^2 - 12x - 8}\\ -3x^3 + 2x^2 - 10x \qquad\end{array}$$

You Try It 3

$$\begin{aligned}&(-4w^3 + 8w - 8) - (3w^3 - 4w^2 - 2w - 1)\\&\quad = (-4w^3 + 8w - 8)\\&\qquad + (-3w^3 + 4w^2 + 2w + 1)\\&\quad = -7w^3 + 4w^2 + 10w - 7\end{aligned}$$

You Try It 4

$$\begin{array}{r}13y^3 \qquad\quad - 6y - 7\\ \underline{\qquad - 4y^2 + 6y + 9}\\ 13y^3 - 4y^2 \qquad\quad + 2\end{array}$$

SECTION 4.2

You Try It 1

$$\begin{aligned}(8m^3n)(-3n^5) &= [8(-3)](m^3)(n \cdot n^5)\\&= -24m^3n^6\end{aligned}$$

You Try It 2

$$\begin{aligned}(12p^4q^3)(-3p^5q^2) &= [12(-3)](p^4 \cdot p^5)(q^3 \cdot q^2)\\&= -36p^9q^5\end{aligned}$$

You Try It 3

$$\begin{aligned}(-3a^4bc^2)^3 &= (-3)^{1\cdot 3}a^{4\cdot 3}b^{1\cdot 3}c^{2\cdot 3}\\&= (-3)^3a^{12}b^3c^6\\&= -27a^{12}b^3c^6\end{aligned}$$

You Try It 4

$$\begin{aligned}(-xy^4)(-2x^3y^2)^2 &= (-xy^4)[(-2)^{1\cdot 2}x^{3\cdot 2}y^{2\cdot 2}]\\&= (-xy^4)[(-2)^2x^6y^4]\\&= (-xy^4)(4x^6y^4)\\&= -4x^7y^8\end{aligned}$$

SECTION 4.3

You Try It 1 $(-2y + 3)(-4y) = 8y^2 - 12y$

You Try It 2

$-a^2(3a^2 + 2a - 7) = -3a^4 - 2a^3 + 7a^2$

You Try It 3

$$\begin{array}{r}2y^3 + 2y^2 \qquad - 3\\ \underline{\qquad\qquad 3y - 1}\\ -2y^3 - 2y^2 \qquad + 3\\ \underline{6y^4 + 6y^3 \qquad - 9y \qquad}\\ 6y^4 + 4y^3 - 2y^2 - 9y + 3\end{array}$$

You Try It 4

$$\begin{aligned}(4y - 5)(2y - 3) &= 8y^2 - 12y - 10y + 15\\&= 8y^2 - 22y + 15\end{aligned}$$

You Try It 5

$$\begin{aligned}(3b + 2)(3b - 5) &= 9b^2 - 15b + 6b - 10\\&= 9b^2 - 9b - 10\end{aligned}$$

You Try It 6 $(2a + 5c)(2a - 5c) = 4a^2 - 25c^2$

You Try It 7 $(3x + 2y)^2 = 9x^2 + 12xy + 4y^2$

You Try It 8

Strategy To find the area, replace the variable r in the equation $A = \pi r^2$ by $(x - 4)$ and solve for A.

Solution

$$\begin{aligned}A &= \pi r^2\\A &= \pi(x - 4)^2\\A &= \pi(x^2 - 8x + 16)\\A &= \pi x^2 - 8\pi x + 16\pi\end{aligned}$$

The area of the circle is $(\pi x^2 - 8\pi x + 16\pi)$ ft^2.

SECTION 4.4

You Try It 1

$$\begin{aligned}(-2x^2)(x^{\ 3}y^{\ 4})^{\ 2} &= (-2x^2)(x^6y^8)\\&= -2x^8y^8\end{aligned}$$

You Try It 2

$$\begin{aligned}\frac{(6a^{-2}b^3)^{-1}}{(4a^3b^{-2})^{-2}} &= \frac{6^{-1}a^2b^{-3}}{4^{-2}a^{-6}b^4}\\&= 4^2(6^{-1}a^8b^{\ 7})\\&= \frac{16a^8}{6b^7} = \frac{8a^8}{3b^7}\end{aligned}$$

You Try It 3

$$\begin{aligned}\left[\frac{6r^3s^{-3}}{9r^3s^{-1}}\right]^{-2} &= \left[\frac{2r^0s^{-2}}{3}\right]^{-2}\\&= \frac{2^{\ 2}s^4}{3^{-2}} = \frac{9s^4}{4}\end{aligned}$$

You Try It 4 $0.000000961 = 9.61 \times 10^{-7}$

You Try It 5 $7.329 \times 10^6 = 7{,}329{,}000$

SECTION 4.5

You Try It 1

$$\begin{aligned}\frac{24x^2y^2 - 18xy + 6y}{6xy} &= \frac{24x^2y^2}{6xy} - \frac{18xy}{6xy} + \frac{6y}{6xy}\\&= 4xy - 3 + \frac{1}{x}\end{aligned}$$

You Try It 2

$$\begin{array}{r}
x^2 + 2x - 1 \\
2x - 3\overline{\big)\,2x^3 + x^2 - 8x - 3} \\
\underline{2x^3 - 3x^2} \\
4x^2 - 8x \\
\underline{4x^2 - 6x} \\
-2x - 3 \\
\underline{-2x + 3} \\
-6
\end{array}$$

$$(2x^3 + x^2 - 8x - 3) \div (2x - 3)$$
$$= x^2 + 2x - 1 - \frac{6}{2x - 3}$$

You Try It 3

$$\begin{array}{r}
x^2 + x - 1 \\
x - 1\overline{\big)\,x^3 + 0x^2 - 2x + 1} \\
\underline{x^3 - x^2} \\
x^2 - 2x \\
\underline{x^2 - x} \\
-x + 1 \\
\underline{-x + 1} \\
0
\end{array}$$

$$(x^3 - 2x + 1) \div (x - 1) = x^2 + x - 1$$

Solutions to Chapter 5 "You Try It"

SECTION 5.1

You Try It 1 The GCF is $7a^2$.

$$\begin{aligned} 14a^2 - 21a^4b &= 7a^2(2) + 7a^2(-3a^2b) \\ &= 7a^2(2 - 3a^2b) \end{aligned}$$

You Try It 2 The GCF is 9.

$$\begin{aligned} &27b^2 + 18b + 9 \\ &\quad = 9(3b^2) + 9(2b) + 9(1) \\ &\quad = 9(3b^2 + 2b + 1) \end{aligned}$$

You Try It 3

The GCF is $3x^2y^2$.

$$\begin{aligned} &6x^4y^2 - 9x^3y^2 + 12x^2y^4 \\ &\quad = 3x^2y^2(2x^2) + 3x^2y^2(-3x) + 3x^2y^2(4y^2) \\ &\quad = 3x^2y^2(2x^2 - 3x + 4y^2) \end{aligned}$$

You Try It 4

$$\begin{aligned} &2y(5x - 2) - 3(2 - 5x) \\ &\quad = 2y(5x - 2) + 3(5x - 2) \\ &\quad = (5x - 2)(2y + 3) \end{aligned}$$

You Try It 5

$$\begin{aligned} &a^2 - 3a + 2ab - 6b \\ &\quad = (a^2 - 3a) + (2ab - 6b) \\ &\quad = a(a - 3) + 2b(a - 3) \\ &\quad = (a - 3)(a + 2b) \end{aligned}$$

You Try It 6

$$\begin{aligned} &2mn^2 - n + 8mn - 4 \\ &\quad = (2mn^2 - n) + (8mn - 4) \\ &\quad = n(2mn - 1) + 4(2mn - 1) \\ &\quad = (2mn - 1)(n + 4) \end{aligned}$$

You Try It 7

$$\begin{aligned} &3xy - 9y - 12 + 4x \\ &\quad = (3xy - 9y) - (12 - 4x) \\ &\quad = 3y(x - 3) - 4(3 - x) \\ &\quad = 3y(x - 3) + 4(x - 3) \\ &\quad = (x - 3)(3y + 4) \end{aligned}$$

SECTION 5.2

You Try It 1

Find the positive factors of 20 whose sum is 9.

Factors	*Sum*
1, 20	21
2, 10	12
4, 5	9

$$x^2 + 9x + 20 = (x + 4)(x + 5)$$

You Try It 2

Find the factors of -18 whose sum is 7.

Factors	*Sum*
$+1, -18$	-17
$-1, +18$	17
$+2, -9$	-7
$-2, +9$	7
$+3, -6$	-3
$-3, +6$	3

$$x^2 + 7x - 18 = (x + 9)(x - 2)$$

You Try It 3

The GCF is $-2x$.

$$-2x^3 + 14x^2 - 12x = -2x(x^2 - 7x + 6)$$

Factor the trinomial $x^2 - 7x + 6$. Find two negative factors of 6 whose sum is -7.

Factors	*Sum*
$-1, -6$	-7
$-2, -3$	-5

$$-2x^3 + 14x^2 - 12x = -2x(x - 6)(x - 1)$$

You Try It 4

The GCF is 3.

$$3x^2 - 9xy - 12y^2 = 3(x^2 - 3xy - 4y^2)$$

Factor the trinomial.

Find the factors of -4 whose sum is -3.

Factors	*Sum*
$+1, -4$	-3
$-1, +4$	3
$+2, -2$	0

$$3x^2 - 9xy - 12y^2 = 3(x + y)(x - 4y)$$

SECTION 5.3

You Try It 1

Factor the trinomial $2x^2 - x - 3$.

Positive factors of 2: 1, 2

Factors of -3: $+1, -3$
$-1, +3$

Trial Factors	*Middle Term*
$(x + 1)(2x - 3)$	$-3x + 2x = -x$
$(x - 3)(2x + 1)$	$x - 6x = -5x$
$(x - 1)(2x + 3)$	$3x - 2x = x$
$(x + 3)(2x - 1)$	$-x + 6x = 5x$

$$2x^2 - x - 3 = (x + 1)(2x - 3)$$

You Try It 2

The GCF is $-3y$.

$$-45y^3 + 12y^2 + 12y = -3y(15y^2 - 4y - 4)$$

Factor the trinomial $15y^2 - 4y - 4$.

Positive factors of 15: 1, 15
3, 5

Factors of -4: $1, -4$
$-1, 4$
$2, -2$

Trial Factors	*Middle Term*
$(y + 1)(15y - 4)$	$-4y + 15y = 11y$
$(y - 4)(15y + 1)$	$y - 60y = -59y$
$(y - 1)(15y + 4)$	$4y - 15y = -11y$
$(y + 4)(15y - 1)$	$-y + 60y = 59y$
$(y + 2)(15y - 2)$	$-2y + 30y = 28y$
$(y - 2)(15y + 2)$	$2y - 30y = -28y$
$(3y + 1)(5y - 4)$	$-12y + 5y = -7y$
$(3y - 4)(5y + 1)$	$3y - 20y = -17y$
$(3y - 1)(5y + 4)$	$12y - 5y = 7y$
$(3y + 4)(5y - 1)$	$-3y + 20y = 17y$
$(3y + 2)(5y - 2)$	$-6y + 10y = 4y$
$(3y - 2)(5y + 2)$	$6y - 10y = -4y$

$$-45y^3 + 12y^2 + 12y = -3y(3y - 2)(5y + 2)$$

You Try It 3

Factors of -14 $[2(-7)]$	*Sum*
$+1, -14$	-13
$-1, +14$	13
$+2, -7$	-5
$-2, +7$	5

$$\begin{aligned} 2a^2 + 13a - 7 &= 2a^2 - a + 14a - 7 \\ &= (2a^2 - a) + (14a - 7) \\ &= a(2a - 1) + 7(2a - 1) \\ &= (2a - 1)(a + 7) \end{aligned}$$

$$2a^2 + 13a - 7 = (2a - 1)(a + 7)$$

You Try It 4

The GCF is $5x$.

$$15x^3 + 40x^2 - 80x = 5x(3x^2 + 8x - 16)$$

Factors of -48 $[3(-16)]$	*Sum*
$+1, -48$	-47
$-1, +48$	47
$+2, -24$	-22
$-2, +24$	22
$+3, -16$	-13
$-3, +16$	13
$+4, -12$	-8
$-4, +12$	8

$$\begin{aligned} 3x^2 + 8x - 16 &= 3x^2 - 4x + 12x - 16 \\ &= (3x^2 - 4x) + (12x - 16) \\ &= x(3x - 4) + 4(3x - 4) \\ &= (3x - 4)(x + 4) \end{aligned}$$

$$\begin{aligned} 15x^3 + 40x^2 - 80x &= 5x(3x^2 + 8x - 16) \\ &= 5x(3x - 4)(x + 4) \end{aligned}$$

SECTION 5.4

You Try It 1

$25a^2 - b^2 = (5a)^2 - b^2 = (5a + b)(5a - b)$

You Try It 2

$$\begin{aligned} n^4 - 81 &= (n^2)^2 - 9^2 = (n^2 + 9)(n^2 - 9) \\ &= (n^2 + 9)(n + 3)(n - 3) \end{aligned}$$

You Try It 3 Because $16y^2 = (4y)^2$, $1 = 1^2$, and $8y = 2(4y)(1)$, the trinomial is a perfect-square trinomial.

$16y^2 + 8y + 1 = (4y + 1)^2$

You Try It 4 Because $x^2 = (x)^2$, $36 = 6^2$, and $15x \neq 2(x)(6)$, the trinomial is not a perfect-square trinomial. Try to factor the trinomial by another method.

$x^2 + 15x + 36 = (x + 3)(x + 12)$

You Try It 5

$$\begin{aligned} &(x^2 - 6x + 9) - y^2 \\ &\quad = (x - 3)^2 - y^2 \\ &\quad = (x - 3 + y)(x - 3 - y) \end{aligned}$$

You Try It 6 The GCF is $3x$.

$$\begin{aligned} 12x^3 - 75x &= 3x(4x^2 - 25) \\ &= 3x(2x + 5)(2x - 5) \end{aligned}$$

You Try It 7

Factor by grouping.

$$\begin{aligned} a^2b - 7a^2 - b + 7 &= (a^2b - 7a^2) - (b - 7) \\ &= a^2(b - 7) - (b - 7) \\ &= (b - 7)(a^2 - 1) \\ &= (b - 7)(a + 1)(a - 1) \end{aligned}$$

You Try It 8

The GCF is $4x$.

$$\begin{aligned} 4x^3 + 28x^2 - 120x &= 4x(x^2 + 7x - 30) \\ &= 4x(x + 10)(x - 3) \end{aligned}$$

SECTION 5.5

You Try It 1 $2x(x + 7) = 0$

$$\begin{aligned} 2x &= 0 & \qquad x + 7 &= 0 \\ x &= 0 & x &= -7 \end{aligned}$$

The solutions are 0 and -7.

You Try It 2

$$\begin{aligned} 4x^2 - 9 &= 0 \\ (2x - 3)(2x + 3) &= 0 \end{aligned}$$

$$\begin{aligned} 2x - 3 &= 0 & \qquad 2x + 3 &= 0 \\ 2x &= 3 & 2x &= -3 \\ x &= \frac{3}{2} & x &= -\frac{3}{2} \end{aligned}$$

The solutions are $\frac{3}{2}$ and $-\frac{3}{2}$.

You Try It 3

$$\begin{aligned} (x + 2)(x - 7) &= 52 \\ x^2 - 5x - 14 &= 52 \\ x^2 - 5x - 66 &= 0 \\ (x + 6)(x - 11) &= 0 \end{aligned}$$

$$\begin{aligned} x + 6 &= 0 & \qquad x - 11 &= 0 \\ x &= -6 & x &= 11 \end{aligned}$$

The solutions are -6 and 11.

You Try It 4

Strategy First consecutive positive integer: n

Second consecutive positive integer: $n + 1$

The sum of the squares of the two consecutive positive integers is 61.

Solution

$$\begin{aligned} n^2 + (n + 1)^2 &= 61 \\ n^2 + n^2 + 2n + 1 &= 61 \\ 2n^2 + 2n + 1 &= 61 \\ 2n^2 + 2n - 60 &= 0 \\ 2(n^2 + n - 30) &= 0 \\ 2(n - 5)(n + 6) &= 0 \end{aligned}$$

$$\begin{aligned} n - 5 &= 0 & \qquad n + 6 &= 0 \\ n &= 5 & n &= -6 \end{aligned}$$

Because -6 is not a positive integer, it is not a solution.

$n = 5$

$n + 1 = 5 + 1 = 6$

The two integers are 5 and 6.

You Try It 5

Strategy Width $= x$

Length $= 2x + 4$

The area of the rectangle is 96 in². Use the equation $A = L \cdot W$.

Solution

$$\begin{aligned} A &= L \cdot W \\ 96 &= (2x + 4)x \\ 96 &= 2x^2 + 4x \\ 0 &= 2x^2 + 4x - 96 \\ 0 &= 2(x^2 + 2x - 48) \\ 0 &= 2(x + 8)(x - 6) \end{aligned}$$

$$\begin{aligned} x + 8 &= 0 & \qquad x - 6 &= 0 \\ x &= -8 & x &= 6 \end{aligned}$$

Because the width cannot be a negative number, -8 is not a solution.

$x = 6$

$2x + 4 = 2(6) + 4 = 12 + 4 = 16$

The length is 16 in. The width is 6 in.

Solutions to Chapter 6 "You Try It"

SECTION 6.1

You Try It 1

$$\frac{6x^5y}{12x^2y^3} = \frac{\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{3}}\cdot x^5y}{\underset{1}{\cancel{2}}\cdot 2\cdot\underset{1}{\cancel{3}}\cdot x^2y^3} = \frac{x^3}{2y^2}$$

You Try It 2

$$\frac{x^2+2x-24}{16-x^2} = \frac{\overset{1}{\cancel{(x-4)}}(x+6)}{\underset{1}{\cancel{(4-x)}}(4+x)} = -\frac{x+6}{x+4}$$

You Try It 3

$$\frac{x^2+4x-12}{x^2-3x+2} = \frac{\overset{1}{\cancel{(x-2)}}(x+6)}{(x-1)\underset{1}{\cancel{(x-2)}}} = \frac{x+6}{x-1}$$

You Try It 4

$$\frac{12x^2+3x}{10x-15}\cdot\frac{8x-12}{9x+18} = \frac{3x(4x+1)}{5(2x-3)}\cdot\frac{4(2x-3)}{9(x+2)}$$

$$= \frac{\overset{1}{\cancel{3}}x(4x+1)\cdot 2\cdot 2\overset{1}{\cancel{(2x-3)}}}{5\underset{1}{\cancel{(2x-3)}}\cdot\underset{1}{\cancel{3}}\cdot 3(x+2)}$$

$$= \frac{4x(4x+1)}{15(x+2)}$$

You Try It 5

$$\frac{x^2+2x-15}{9-x^2}\cdot\frac{x^2-3x-18}{x^2-7x+6}$$

$$= \frac{(x-3)(x+5)}{(3-x)(3+x)}\cdot\frac{(x+3)(x-6)}{(x-1)(x-6)}$$

$$= \frac{\overset{-1}{\cancel{(x-3)}}(x+5)\cdot\overset{1}{\cancel{(x+3)}}\overset{1}{\cancel{(x-6)}}}{\underset{1}{\cancel{(3-x)}}\underset{1}{\cancel{(3+x)}}\cdot(x-1)\underset{1}{\cancel{(x-6)}}} = -\frac{x+5}{x-1}$$

You Try It 6

$$\frac{a^2}{4bc^2-2b^2c}\div\frac{a}{6bc-3b^2} = \frac{a^2}{4bc^2-2b^2c}\cdot\frac{6bc-3b^2}{a}$$

$$= \frac{a^2\cdot 3\overset{1}{\cancel{b}}\overset{1}{\cancel{(2c-b)}}}{2\underset{1}{\cancel{b}}c\underset{1}{\cancel{(2c-b)}}\cdot a} = \frac{3a}{2c}$$

You Try It 7

$$\frac{3x^2+26x+16}{3x^2-7x-6}\div\frac{2x^2+9x-5}{x^2+2x-15}$$

$$= \frac{3x^2+26x+16}{3x^2-7x-6}\cdot\frac{x^2+2x-15}{2x^2+9x-5}$$

$$= \frac{\overset{1}{\cancel{(3x+2)}}(x+8)\cdot\overset{1}{\cancel{(x+5)}}\overset{1}{\cancel{(x-3)}}}{\underset{1}{\cancel{(3x+2)}}\underset{1}{\cancel{(x-3)}}\cdot(2x-1)\underset{1}{\cancel{(x+5)}}} = \frac{x+8}{2x-1}$$

SECTION 6.2

You Try It 1

$8uv^2 = 2\cdot 2\cdot 2\cdot u\cdot v\cdot v$

$12uw = 2\cdot 2\cdot 3\cdot u\cdot w$

$\text{LCM} = 2\cdot 2\cdot 2\cdot 3\cdot u\cdot v\cdot v\cdot w = 24uv^2w$

You Try It 2

$m^2-6m+9 = (m-3)(m-3)$

$m^2-2m-3 = (m+1)(m-3)$

$\text{LCM} = (m-3)(m-3)(m+1)$

You Try It 3

The LCM is $36xy^2z$.

$$\frac{x-3}{4xy^2} = \frac{x-3}{4xy^2}\cdot\frac{9z}{9z} = \frac{9xz-27z}{36xy^2z}$$

$$\frac{2x+1}{9y^2z} = \frac{2x+1}{9y^2z}\cdot\frac{4x}{4x} = \frac{8x^2+4x}{36xy^2z}$$

You Try It 4

The LCM is $(x+2)(x-5)(x+5)$.

$$\frac{x+4}{x^2-3x-10} = \frac{x+4}{(x+2)(x-5)}\cdot\frac{x+5}{x+5}$$

$$= \frac{x^2+9x+20}{(x+2)(x-5)(x+5)}$$

$$\frac{2x}{25-x^2} = \frac{2x}{-(x^2-25)} = -\frac{2x}{(x-5)(x+5)}\cdot\frac{x+2}{x+2}$$

$$= -\frac{2x^2+4x}{(x+2)(x-5)(x+5)}$$

SECTION 6.3

You Try It 1

$$\begin{aligned}&\frac{2x^2}{x^2-x-12}-\frac{7x+4}{x^2-x-12}\\&=\frac{2x^2-(7x+4)}{x^2-x-12}=\frac{2x^2-7x-4}{x^2-x-12}\\&=\frac{(2x+1)\overset{1}{\cancel{(x-4)}}}{(x+3)\underset{1}{\cancel{(x-4)}}}=\frac{2x+1}{x+3}\end{aligned}$$

You Try It 2

$$\begin{aligned}&\frac{x^2-1}{x^2-8x+12}-\frac{2x+1}{x^2-8x+12}+\frac{x}{x^2-8x+12}\\&=\frac{(x^2-1)-(2x+1)+x}{x^2-8x+12}=\frac{x^2-1-2x-1+x}{x^2-8x+12}\\&=\frac{x^2-x-2}{x^2-8x+12}=\frac{(x+1)\overset{1}{\cancel{(x-2)}}}{\underset{1}{\cancel{(x-2)}}(x-6)}=\frac{x+1}{x-6}\end{aligned}$$

You Try It 3

The LCM of the denominators is $24y$.

$$\begin{aligned}\frac{z}{8y}-\frac{4z}{3y}+\frac{5z}{4y}&=\frac{z}{8y}\cdot\frac{3}{3}-\frac{4z}{3y}\cdot\frac{8}{8}+\frac{5z}{4y}\cdot\frac{6}{6}\\&=\frac{3z}{24y}-\frac{32z}{24y}+\frac{30z}{24y}\\&=\frac{3z-32z+30z}{24y}=\frac{z}{24y}\end{aligned}$$

You Try It 4

$2-x=-(x-2)$

Therefore, $\dfrac{3}{2-x}=\dfrac{-3}{x-2}$.

The LCM is $x-2$.

$$\begin{aligned}\frac{5x}{x-2}-\frac{3}{2-x}&=\frac{5x}{x-2}-\frac{-3}{x-2}\\&=\frac{5x-(-3)}{x-2}=\frac{5x+3}{x-2}\end{aligned}$$

You Try It 5

The LCM is $(3x-1)(x+4)$.

$$\begin{aligned}\frac{4x}{3x-1}-\frac{9}{x+4}&=\frac{4x}{3x-1}\cdot\frac{x+4}{x+4}-\frac{9}{x+4}\cdot\frac{3x-1}{3x-1}\\&=\frac{4x^2+16x}{(3x-1)(x+4)}-\frac{27x-9}{(3x-1)(x+4)}\\&=\frac{(4x^2+16x)-(27x-9)}{(3x-1)(x+4)}\\&=\frac{4x^2+16x-27x+9}{(3x-1)(x+4)}\\&=\frac{4x^2-11x+9}{(3x-1)(x+4)}\end{aligned}$$

You Try It 6

The LCM is $x-3$.

$$\begin{aligned}2-\frac{1}{x-3}&=2\cdot\frac{x-3}{x-3}-\frac{1}{x-3}\\&=\frac{2x-6}{x-3}-\frac{1}{x-3}\\&=\frac{2x-6-1}{x-3}\\&=\frac{2x-7}{x-3}\end{aligned}$$

You Try It 7

$$\frac{2}{5-x}=\frac{-2}{x-5}$$

The LCM is $(x+5)(x-5)$.

$$\begin{aligned}\frac{2x-1}{x^2-25}+\frac{2}{5-x}&=\frac{2x-1}{(x+5)(x-5)}+\frac{-2}{x-5}\\&=\frac{2x-1}{(x+5)(x-5)}+\frac{-2}{x-5}\cdot\frac{x+5}{x+5}\\&=\frac{2x-1}{(x+5)(x-5)}+\frac{-2(x+5)}{(x+5)(x-5)}\\&=\frac{2x-1+(-2)(x+5)}{(x+5)(x-5)}\\&=\frac{2x-1-2x-10}{(x+5)(x-5)}\\&=\frac{-11}{(x+5)(x-5)}\\&=-\frac{11}{(x+5)(x-5)}\end{aligned}$$

You Try It 8

The LCM is $(3x+2)(x-1)$.

$$\begin{aligned}&\frac{2x-3}{3x^2-x-2}+\frac{5}{3x+2}-\frac{1}{x-1}\\&=\frac{2x-3}{(3x+2)(x-1)}+\frac{5}{3x+2}\cdot\frac{x-1}{x-1}\\&\quad-\frac{1}{x-1}\cdot\frac{3x+2}{3x+2}\\&=\frac{2x-3}{(3x+2)(x-1)}+\frac{5x-5}{(3x+2)(x-1)}\\&\quad-\frac{3x+2}{(3x+2)(x-1)}\\&=\frac{(2x-3)+(5x-5)-(3x+2)}{(3x+2)(x-1)}\\&=\frac{2x-3+5x-5-3x-2}{(3x+2)(x-1)}\\&=\frac{4x-10}{(3x+2)(x-1)}=\frac{2(2x-5)}{(3x+2)(x-1)}\end{aligned}$$

SECTION 6.4

You Try It 1

The LCM of 3, x, 9, and x^2 is $9x^2$.

$$\frac{\frac{1}{3}-\frac{1}{x}}{\frac{1}{9}-\frac{1}{x^2}}=\frac{\frac{1}{3}-\frac{1}{x}}{\frac{1}{9}-\frac{1}{x^2}}\cdot\frac{9x^2}{9x^2}=\frac{\frac{1}{3}\cdot 9x^2-\frac{1}{x}\cdot 9x^2}{\frac{1}{9}\cdot 9x^2-\frac{1}{x^2}\cdot 9x^2}$$

$$=\frac{3x^2-9x}{x^2-9}=\frac{3x\overset{1}{\cancel{(x-3)}}}{\underset{1}{\cancel{(x-3)}}(x+3)}=\frac{3x}{x+3}$$

You Try It 2

The LCM of x and x^2 is x^2.

$$\frac{1+\frac{4}{x}+\frac{3}{x^2}}{1+\frac{10}{x}+\frac{21}{x^2}}=\frac{1+\frac{4}{x}+\frac{3}{x^2}}{1+\frac{10}{x}+\frac{21}{x^2}}\cdot\frac{x^2}{x^2}$$

$$=\frac{1\cdot x^2+\frac{4}{x}\cdot x^2+\frac{3}{x^2}\cdot x^2}{1\cdot x^2+\frac{10}{x}\cdot x^2+\frac{21}{x^2}\cdot x^2}$$

$$=\frac{x^2+4x+3}{x^2+10x+21}=\frac{(x+1)\overset{1}{\cancel{(x+3)}}}{\underset{1}{\cancel{(x+3)}}(x+7)}$$

$$=\frac{x+1}{x+7}$$

You Try It 3

The LCM is $x-5$.

$$\frac{x+3-\frac{20}{x-5}}{x+8+\frac{30}{x-5}}=\frac{x+3-\frac{20}{x-5}}{x+8+\frac{30}{x-5}}\cdot\frac{x-5}{x-5}$$

$$=\frac{(x+3)(x-5)-\frac{20}{x-5}\cdot(x-5)}{(x+8)(x-5)+\frac{30}{x-5}\cdot(x-5)}$$

$$=\frac{x^2-2x-15-20}{x^2+3x-40+30}=\frac{x^2-2x-35}{x^2+3x-10}$$

$$=\frac{\overset{1}{\cancel{(x+5)}}(x-7)}{(x-2)\underset{1}{\cancel{(x+5)}}}=\frac{x-7}{x-2}$$

SECTION 6.5

You Try It 1

$$\frac{x}{x+6}=\frac{3}{x}\qquad\text{The LCM is } x(x+6).$$

$$\frac{x\overset{1}{\cancel{(x+6)}}}{1}\cdot\frac{x}{\underset{1}{\cancel{x+6}}}=\frac{x(x+6)}{1}\cdot\frac{3}{x}$$

$$\begin{aligned}x^2&=(x+6)3\\x^2&=3x+18\\x^2-3x-18&=0\\(x+3)(x-6)&=0\end{aligned}$$

$$\begin{aligned}x+3&=0 & x-6&=0\\x&=-3 & x&=6\end{aligned}$$

Both -3 and 6 check as solutions.
The solutions are -3 and 6.

You Try It 2

$$\frac{5x}{x+2}=3-\frac{10}{x+2}\qquad\text{The LCM is } x+2.$$

$$\frac{(x+2)}{1}\cdot\frac{5x}{x+2}=\frac{(x+2)}{1}\left(3-\frac{10}{x+2}\right)$$

$$\frac{\overset{1}{\cancel{x+2}}}{1}\cdot\frac{5x}{\underset{1}{\cancel{x+2}}}=\frac{x+2}{1}\cdot 3-\frac{\overset{1}{\cancel{x+2}}}{1}\cdot\frac{10}{\underset{1}{\cancel{x+2}}}$$

$$\begin{aligned}5x&=(x+2)3-10\\5x&=3x+6-10\\5x&=3x-4\\2x&=-4\\x&=-2\end{aligned}$$

-2 does not check as a solution.
The equation has no solution.

SECTION 6.6

You Try It 1

$$\frac{2}{x+3}=\frac{6}{5x+5}$$

$$\frac{(x+3)(5x+5)}{1}\cdot\frac{2}{x+3}=\frac{(x+3)(5x+5)}{1}\cdot\frac{6}{5x+5}$$

$$\frac{\overset{1}{\cancel{(x+3)}}(5x+5)}{1}\cdot\frac{2}{\underset{1}{\cancel{x+3}}}=\frac{(x+3)\overset{1}{\cancel{(5x+5)}}}{1}\cdot\frac{6}{\underset{1}{\cancel{5x+5}}}$$

$$\begin{aligned}(5x+5)2&=(x+3)6\\10x+10&=6x+18\\4x+10&=18\\4x&=8\\x&=2\end{aligned}$$

The solution is 2.

You Try It 2

Strategy To find the total area that 256 ceramic tiles will cover, write and solve a proportion using x to represent the number of square feet that 256 tiles will cover.

Solution

$$\begin{aligned} \frac{9}{16} &= \frac{x}{256} \\ 256\left(\frac{9}{16}\right) &= 256\left(\frac{x}{256}\right) \\ 144 &= x \end{aligned}$$

A 144-square-foot area can be tiled using 256 ceramic tiles.

You Try It 3

Strategy To find the area of triangle AOB:

- Solve a proportion to find the length of AO (the height of triangle AOB).
- Use the formula for the area of a triangle. AB is the base and AO is the height.

Solution

$$\begin{aligned} \frac{CD}{AB} &= \frac{DO}{AO} \\ \frac{4}{10} &= \frac{3}{AO} \\ 10 \cdot AO \cdot \frac{4}{10} &= 10 \cdot AO \cdot \frac{3}{AO} \\ 4(AO) &= 30 \\ AO &= 7.5 \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(10)(7.5) \\ &= 37.5 \end{aligned}$$

The area of triangle AOB is 37.5 cm^2.

SECTION 6.7

You Try It 1

$$\begin{aligned} 5x - 2y &= 10 \\ 5x - 5x - 2y &= -5x + 10 \\ -2y &= -5x + 10 \\ \frac{-2y}{-2} &= \frac{-5x + 10}{-2} \\ y &= \frac{5}{2}x - 5 \end{aligned}$$

You Try It 2

$$\begin{aligned} s &= \frac{A + L}{2} \\ 2 \cdot s &= 2\left(\frac{A + L}{2}\right) \\ 2s &= A + L \\ 2s - A &= A - A + L \\ 2s - A &= L \end{aligned}$$

You Try It 3

$$\begin{aligned} S &= a + (n - 1)d \\ S &= a + nd - d \\ S - a &= a - a + nd - d \\ S - a &= nd - d \\ S - a + d &= nd - d + d \\ S - a + d &= nd \\ \frac{S - a + d}{d} &= \frac{nd}{d} \\ \frac{S - a + d}{d} &= n \end{aligned}$$

You Try It 4

$$\begin{aligned} S &= rS + C \\ S - rS &= rS - rS + C \\ S - rS &= C \\ (1 - r)S &= C \\ \frac{(1 - r)S}{1 - r} &= \frac{C}{1 - r} \\ S &= \frac{C}{1 - r} \end{aligned}$$

SECTION 6.8

You Try It 1

Strategy

- Time for one printer to complete the job: t

	Rate	*Time*	*Part*
1st printer	$\frac{1}{t}$	2	$\frac{2}{t}$
2nd printer	$\frac{1}{t}$	5	$\frac{5}{t}$

- The sum of the parts of the task completed must equal 1.

Solution

$$\begin{aligned} \frac{2}{t} + \frac{5}{t} &= 1 \\ t\left(\frac{2}{t} + \frac{5}{t}\right) &= t \cdot 1 \\ 2 + 5 &= t \\ 7 &= t \end{aligned}$$

Working alone, one printer takes 7 h to print the payroll.

You Try It 2

Strategy

- Rate sailing across the lake: r
  Rate sailing back: $3r$

	Distance	*Rate*	*Time*
Across	6	r	$\frac{6}{r}$
Back	6	$3r$	$\frac{6}{3r}$

- The total time for the trip was 2 h.

Solution

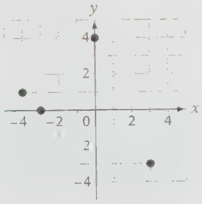
$$\frac{6}{r} + \frac{6}{3r} = 2$$
$$3r\left(\frac{6}{r} + \frac{6}{3r}\right) = 3r(2)$$
$$3r \cdot \frac{6}{r} + 3r \cdot \frac{6}{3r} = 6r$$
$$18 + 6 = 6r$$
$$24 = 6r$$
$$4 = r$$

The rate across the lake was 4 km/h.

Solutions to Chapter 7 "You Try It"

SECTION 7.1

You Try It 1



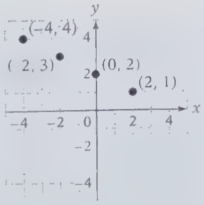
You Try It 2 $A(4, -2)$, $B(-2, 4)$.
The abscissa of D is 0.
The ordinate of C is 0.

You Try It 3

$$\begin{array}{r|l} x - 3y = & -14 \\ \hline -2 - 3(4) & -14 \\ -2 - 12 & -14 \\ -14 = & -14 \end{array}$$

Yes, $(-2, 4)$ is a solution of $x - 3y = -14$.

You Try It 4



You Try It 5
{(145, 140), (140, 125), (150, 130), (165, 150), (140, 130), (165, 160)}

No, the relation is not a function. The two ordered pairs (140, 125) and (140, 130) have the same first coordinate but different second coordinates.

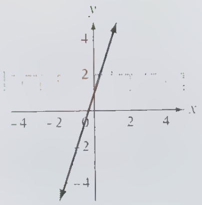
You Try It 6 Determine the ordered pairs defined by the equation. Replace x in $y = \frac{1}{2}x + 1$ by the given values and solve for y. $\{(-4, -1), (0, 1), (2, 2)\}$
Yes, y is a function of x.

You Try It 7

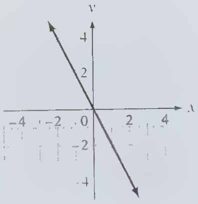
$$H(x) = \frac{x}{x - 4}$$
$$H(8) = \frac{8}{8 - 4}$$
$$H(8) = \frac{8}{4} = 2$$

SECTION 7.2

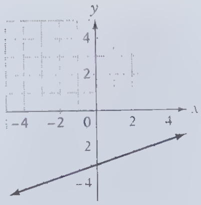
You Try It 1



You Try It 2

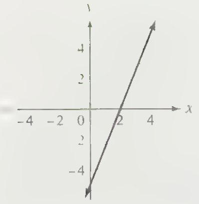


You Try It 3



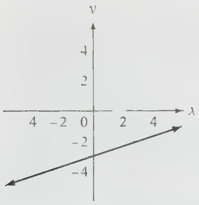
You Try It 4

$$\begin{aligned} 5x - 2y &= 10 \\ -2y &= -5x + 10 \\ y &= \frac{5}{2}x - 5 \end{aligned}$$

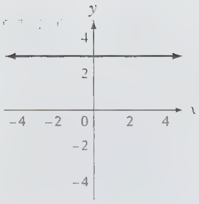


You Try It 5

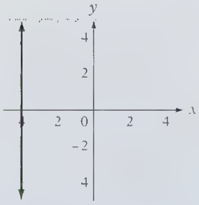
$$\begin{aligned} x - 3y &= 9 \\ -3y &= -x + 9 \\ y &= \frac{1}{3}x - 3 \end{aligned}$$



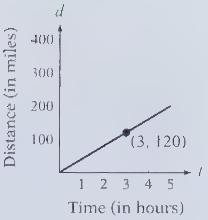
You Try It 6



You Try It 7



You Try It 8



The ordered pair (3, 120) means that in 3 h the car will travel 120 mi.

SECTION 7.3

You Try It 1

x-intercept:

$$\begin{aligned} y &= 2x - 4 \\ 0 &= 2x - 4 \\ -2x &= -4 \\ x &= 2 \end{aligned}$$

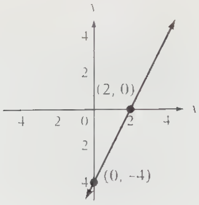
$(2, 0)$

y-intercept:

$(0, b)$

$b = -4$

$(0, -4)$



You Try It 2 Let $P_1 = (1, 4)$ and $P_2 = (-3, 8)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 4}{-3 - 1} = \frac{4}{-4} = -1$$

The slope is -1.

You Try It 3 Let $P_1 = (-1, 2)$ and $P_2 = (4, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{4 - (-1)} = \frac{0}{5} = 0$$

The slope is 0.

You Try It 4

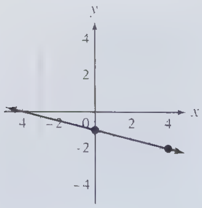
$$m = \frac{8650 - 6100}{1 - 4} = \frac{2550}{-3}$$

$$m = -850$$

A slope of -850 means that the value of the car is decreasing at a rate of \$850 per year.

You Try It 5 y-intercept $= (0, b) = (0, -1)$

$$m = -\frac{1}{4}$$

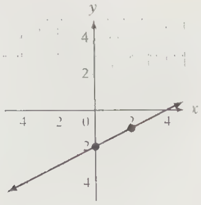


You Try It 6 Solve the equation for y.

$$\begin{aligned} x - 2y &= 4 \\ -2y &= -x + 4 \\ y &= \frac{1}{2}x - 2 \end{aligned}$$

y-intercept $= (0, b) = (0, -2)$

$m = \frac{1}{2}$



SECTION 7.4

You Try It 1 Because the slope and y-intercept are known, use the slope-intercept formula, $y = mx + b$.

$$y = mx + b$$
$$y = \frac{5}{3}x + 2$$

You Try It 2 $m = \frac{3}{4}$ $\quad (x_1, y_1) = (4, -2)$

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = \frac{3}{4}(x - 4)$$
$$y + 2 = \frac{3}{4}x - 3$$
$$y = \frac{3}{4}x - 5$$

The equation of the line is $y = \frac{3}{4}x - 5$.

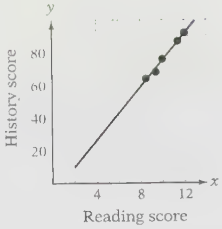
You Try It 3 Find the slope of the line between the two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-1)}{3 - (-6)} = \frac{2}{9}$$

Use the point-slope formula.

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = \frac{2}{9}[x - (-6)]$$
$$y + 1 = \frac{2}{9}x + \frac{4}{3}$$
$$y = \frac{2}{9}x + \frac{1}{3}$$

You Try It 4



The slope of the line means that the grade on the history test increases 8.3 points for each 1-point increase in the grade on the reading test.

Solutions to Chapter 8 "You Try It"

SECTION 8.1

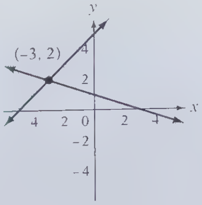
You Try It 1

$2x - 5y = 8$	
$2(-1) - 5(-2)$	8
$-2 + 10$	8
$8 = 8$	

$-x + 3y = -5$	
$-(-1) + 3(-2)$	-5
$1 + (-6)$	-5
$-5 = -5$	

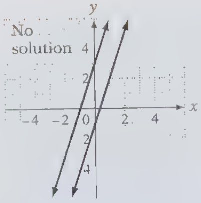
Yes, $(-1, -2)$ is a solution of the system of equations.

You Try It 2



The solution is $(-3, 2)$.

You Try It 3



The lines are parallel. The system of equations is inconsistent and therefore does not have a solution.

SECTION 8.2

You Try It 1
(1) $7x - y = 4$
(2) $3x + 2y = 9$

Solve Equation (1) for y.

$$7x - y = 4$$
$$-y = -7x + 4$$
$$y = 7x - 4$$

Substitute in Equation (2).

$$\begin{aligned} 3x + 2y &= 9 \\ 3x + 2(7x - 4) &= 9 \\ 3x + 14x - 8 &= 9 \\ 17x - 8 &= 9 \\ 17x &= 17 \\ x &= 1 \end{aligned}$$

Substitute in Equation (1).

$$\begin{aligned} 7x - y &= 4 \\ 7(1) - y &= 4 \\ 7 - y &= 4 \\ -y &= -3 \\ y &= 3 \end{aligned}$$

The solution is (1, 3).

You Try It 2

$$\begin{aligned} (1)\quad 3x - y &= 4 \\ (2)\quad y &= 3x + 2 \end{aligned}$$

$$\begin{aligned} 3x - y &= 4 \\ 3x - (3x + 2) &= 4 \\ 3x - 3x - 2 &= 4 \\ -2 &= 4 \end{aligned}$$

This is not a true equation. The system of equations is inconsistent and therefore does not have a solution.

You Try It 3

$$\begin{aligned} (1)\quad y &= -2x + 1 \\ (2)\quad 6x + 3y &= 3 \end{aligned}$$

$$\begin{aligned} 6x + 3y &= 3 \\ 6x + 3(-2x + 1) &= 3 \\ 6x - 6x + 3 &= 3 \\ 3 &= 3 \end{aligned}$$

The system of equations is dependent. The solutions are the ordered pairs that satisfy the equation $y = -2x + 1$.

You Try It 4

Strategy

- Amount invested at 6.5%: x
  Amount invested at 4.5%: y

	Principal	*Rate*	*Interest*
Amount at 6.5%	x	0.065	$0.065x$
Amount at 4.5%	y	0.045	$0.045y$

- The sum of the two investments is \$330,000: $x + y = 330{,}000$.
  The interest earned at 6.5% equals the interest earned at 4.5%: $0.065x = 0.045y$

Solution

$$\begin{aligned} (1)\quad x + y &= 330{,}000 \\ (2)\quad 0.065x &= 0.045y \end{aligned}$$

Solve Equation (2) for y.

$$(3)\quad y = \frac{13}{9}x$$

Replace y by $\frac{13}{9}x$ in Equation (1) and solve for x.

$$\begin{aligned} x + y &= 330{,}000 \\ x + \frac{13}{9}x &= 330{,}000 \\ \frac{22}{9}x &= 330{,}000 \\ x &= 135{,}000 \end{aligned}$$

Replace x by 135,000 in Equation (3) and solve for y.

$$\begin{aligned} y &= \frac{13}{9}x \\ &= \frac{13}{9}(135{,}000) = 195{,}000 \end{aligned}$$

\$135,000 should be invested at 6.5% and \$195,000 should be invested at 4.5%.

SECTION 8.3

You Try It 1

$$\begin{aligned} (1)\quad x - 2y &= 1 \\ (2)\quad 2x + 4y &= 0 \end{aligned}$$

Eliminate y.

$$\begin{aligned} 2(x - 2y) &= 2 \cdot 1 \\ 2x + 4y &= 0 \end{aligned}$$

$$\begin{aligned} 2x - 4y &= 2 \\ 2x + 4y &= 0 \end{aligned}$$

Add the equations.

$$\begin{aligned} 4x &= 2 \\ x &= \frac{2}{4} = \frac{1}{2} \end{aligned}$$

Replace x in Equation (2).

$$\begin{aligned} 2\left(\frac{1}{2}\right) + 4y &= 0 \\ 1 + 4y &= 0 \\ 4y &= -1 \\ y &= -\frac{1}{4} \end{aligned}$$

The solution is $\left(\frac{1}{2}, -\frac{1}{4}\right)$.

You Try It 2

$$\begin{aligned} (1)\quad 2x - 3y &= 4 \\ (2)\quad -4x + 6y &= -8 \end{aligned}$$

Eliminate y.

$$\begin{aligned} 2(2x - 3y) &= 2 \cdot 4 \\ -4x + 6y &= -8 \end{aligned}$$

$$\begin{aligned} 4x - 6y &= 8 \\ -4x + 6y &= -8 \end{aligned}$$

Add the equations.

$$\begin{aligned} 0x + 0y &= 0 \\ 0 &= 0 \end{aligned}$$

The system of equations is dependent. The solutions are the ordered pairs that satisfy the equation $2x - 3y = 4$.

You Try It 3

$$\begin{aligned} (1)\quad 4x + 5y &= 11 \\ (2)\quad 3y &= x + 10 \end{aligned}$$

Write equation (2) in the form $Ax + By = C$.

$$\begin{aligned} 3y &= x + 10 \\ -x + 3y &= 10 \end{aligned}$$

Eliminate x.

$$\begin{aligned} 4x + 5y &= 11 \\ 4(-x + 3y) &= 4 \cdot 10 \end{aligned}$$

$$\begin{aligned} 4x + 5y &= 11 \\ -4x + 12y &= 40 \end{aligned}$$

Add the equations.

$$\begin{aligned} 17y &= 51 \\ y &= 3 \end{aligned}$$

Replace y in Equation (1).

$$\begin{aligned} 4x + 5y &= 11 \\ 4x + 5 \cdot 3 &= 11 \\ 4x + 15 &= 11 \\ 4x &= -4 \\ x &= -1 \end{aligned}$$

The solution is $(-1, 3)$.

SECTION 8.4

You Try It 1

Strategy

- Rate of the current: c
  Rate of the canoeist in calm water: r

	Rate	*Time*	*Distance*
With current	$r + c$	3	$3(r + c)$
Against current	$r - c$	5	$5(r - c)$

- The distance traveled with the current is 15 mi.
  The distance traveled against the current is 15 mi.

Solution

$$\begin{aligned} 3(r + c) &= 15 \qquad \frac{1}{3} \cdot 3(r + c) = \frac{1}{3} \cdot 15 \\ 5(r - c) &= 15 \qquad \frac{1}{5} \cdot 5(r - c) = \frac{1}{5} \cdot 15 \end{aligned}$$

$$\begin{aligned} r + c &= 5 \\ r - c &= 3 \end{aligned}$$

$$\begin{aligned} 2r &= 8 \\ r &= 4 \end{aligned}$$

$$\begin{aligned} r + c &= 5 \\ 4 + c &= 5 \\ c &= 1 \end{aligned}$$

The rate of the current is 1 mph.
The rate of the canoeist in calm water is 4 mph.

You Try It 2

Strategy

- Cost of an orange tree: x
  Cost of a grapefruit tree: y
  First purchase:

	Amount	*Unit Cost*	*Value*
Orange trees	25	x	$25x$
Grapefruit trees	20	y	$20y$

Second purchase:

	Amount	*Unit Cost*	*Value*
Orange trees	20	x	$20x$
Grapefruit trees	30	y	$30y$

- The total of the first purchase was \$290.
  The total of the second purchase was \$330.

Solution

$$\begin{aligned} 25x + 20y &= 290 \qquad 4(25x + 20y) = 4 \cdot 290 \\ 20x + 30y &= 330 \qquad -5(20x + 30y) = -5 \cdot 330 \end{aligned}$$

$$\begin{aligned} 100x + 80y &= 1160 \\ -100x - 150y &= -1650 \\ -70y &= -490 \\ y &= 7 \end{aligned}$$

$$\begin{aligned} 25x + 20y &= 290 \\ 25x + 20(7) &= 290 \\ 25x + 140 &= 290 \\ 25x &= 150 \\ x &= 6 \end{aligned}$$

The cost of an orange tree is \$6.
The cost of a grapefruit tree is \$7.

Solutions to Chapter 9 "You Try It"

SECTION 9.1

You Try It 1 $A = \{-9, -7, -5, -3, -1\}$

You Try It 2 $A = \{1, 3, 5, \ldots\}$

You Try It 3 $A \cup B = \{-2, -1, 0, 1, 2, 3, 4\}$

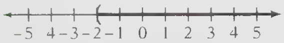
You Try It 4 $C \cap D = \{10, 16\}$

You Try It 5 $A \cap B = \varnothing$

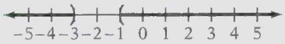
You Try It 6 $\{x \mid x < 59, x \in \text{positive even integers}\}$

You Try It 7 $\{x \mid x > -3, x \in \text{real numbers}\}$

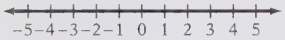
You Try It 8 The solution set is the numbers greater than -2.



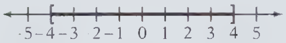
You Try It 9 The solution set is the numbers greater than -1 and the numbers less than -3.



You Try It 10 The solution set is the real numbers.



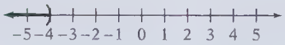
You Try It 11 The solution set is the numbers less than or equal to 4 and greater than or equal to -4.



SECTION 9.2

You Try It 1

$$\begin{aligned} x + 2 &< -2 \\ x + 2 - 2 &< -2 - 2 \\ x &< -4 \end{aligned}$$

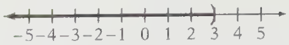


You Try It 2

$$\begin{aligned} 5x + 3 &> 4x + 5 \\ 5x - 4x + 3 &> 4x - 4x + 5 \\ x + 3 &> 5 \\ x + 3 - 3 &> 5 - 3 \\ x &> 2 \end{aligned}$$

You Try It 3

$$\begin{aligned} -3x &> -9 \\ \frac{-3x}{-3} &< \frac{-9}{-3} \\ x &< 3 \end{aligned}$$



You Try It 4

$$\begin{aligned} -\frac{3}{4}x &\ge 18 \\ -\frac{4}{3}\left(-\frac{3}{4}x\right) &\le -\frac{4}{3}(18) \\ x &\le -24 \end{aligned}$$

You Try It 5

Strategy To find the selling prices, write and solve an inequality using p to represent the possible selling prices.

Solution

$$\begin{aligned} 0.70p &> 314 \\ p &> 448.571 \end{aligned}$$

The dealer will make a profit if the selling price is greater than or equal to $448.58.

SECTION 9.3

You Try It 1

$$\begin{aligned} 5 - 4x &> 9 - 8x \\ 5 - 4x + 8x &> 9 - 8x + 8x \\ 5 + 4x &> 9 \\ 5 - 5 + 4x &> 9 - 5 \\ 4x &> 4 \\ \frac{4x}{4} &> \frac{4}{4} \\ x &> 1 \end{aligned}$$

You Try It 2

$$\begin{aligned} 8 - 4(3x + 5) &\le 6(x - 8) \\ 8 - 12x - 20 &\le 6x - 48 \\ -12 - 12x &\le 6x - 48 \\ -12 - 12x - 6x &\le 6x - 6x - 48 \\ -12 - 18x &\le -48 \\ -12 + 12 - 18x &\le -48 + 12 \\ -18x &\le -36 \\ \frac{-18x}{-18} &\ge \frac{-36}{-18} \\ x &\ge 2 \end{aligned}$$

You Try It 3

Strategy To find the maximum number of miles:

- Write an expression for the cost of each car, using x to represent the number of miles driven during the week.
- Write and solve an inequality.

Solution

Cost of a Company A car	is less than	cost of a Company B car

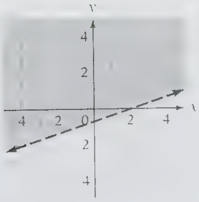
$$\begin{aligned} 8(7) + 0.10x &< 10(7) + 0.08x \\ 56 + 0.10x &< 70 + 0.08x \\ 56 + 0.10x - 0.08x &< 70 + 0.08x - 0.08x \\ 56 + 0.02x &< 70 \\ 56 - 56 + 0.02x &< 70 - 56 \\ 0.02x &< 14 \\ \frac{0.02x}{0.02} &< \frac{14}{0.02} \\ x &< 700 \end{aligned}$$

The maximum number of miles is 699.

SECTION 9.4

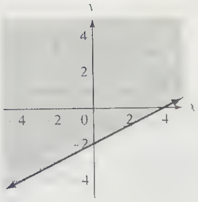
You Try It 1

$$\begin{aligned} x - 3y &< 2 \\ x - x - 3y &< -x + 2 \\ -3y &< -x + 2 \\ \frac{-3y}{-3} &> \frac{-x + 2}{-3} \\ y &> \frac{1}{3}x - \frac{2}{3} \end{aligned}$$

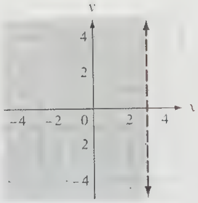


You Try It 2

$$\begin{aligned} 2x - 4y &\le 8 \\ 2x - 2x - 4y &\le -2x + 8 \\ -4y &\le -2x + 8 \\ \frac{-4y}{-4} &\ge \frac{-2x + 8}{-4} \\ y &\ge \frac{1}{2}x - 2 \end{aligned}$$



You Try It 3 $x < 3$



Solutions to Chapter 10 "You Try It"

SECTION 10.1

You Try It 1

$$\begin{aligned} -5\sqrt{32} &= -5\sqrt{16 \cdot 2} = -5\sqrt{16}\sqrt{2} \\ &= -5 \cdot 4\sqrt{2} = -20\sqrt{2} \end{aligned}$$

You Try It 2

$$\begin{aligned} \sqrt{216} &= \sqrt{36 \cdot 6} \\ &= \sqrt{36}\sqrt{6} = 6\sqrt{6} \end{aligned}$$

You Try It 3 $\sqrt{y^{19}} = \sqrt{y^{18} \cdot y} = \sqrt{y^{18}}\sqrt{y} = y^9\sqrt{y}$

You Try It 4

$$\begin{aligned} \sqrt{45b^7} &= \sqrt{9b^6 \cdot 5b} \\ &= \sqrt{9b^6}\sqrt{5b} = 3b^3\sqrt{5b} \end{aligned}$$

You Try It 5

$$\begin{aligned} 3a\sqrt{28a^9b^{18}} &= 3a\sqrt{4a^8b^{18}(7a)} \\ &= 3a\sqrt{4a^8b^{18}}\sqrt{7a} \\ &= 3a \cdot 2a^4b^9\sqrt{7a} = 6a^5b^9\sqrt{7a} \end{aligned}$$

You Try It 6

$$\begin{aligned} \sqrt{25(a + 3)^2} &= 5(a + 3) \\ &= 5a + 15 \end{aligned}$$

You Try It 7 $\sqrt{x^2 + 14x + 49} = \sqrt{(x + 7)^2} = x + 7$

SECTION 10.2

You Try It 1 $9\sqrt{3} + 3\sqrt{3} - 18\sqrt{3} = -6\sqrt{3}$

You Try It 2

$$\begin{aligned}2\sqrt{50} - 5\sqrt{32} &= 2\sqrt{25}\sqrt{2} - 5\sqrt{16}\sqrt{2}\\ &= 2\cdot 5\sqrt{2} - 5\cdot 4\sqrt{2}\\ &= 10\sqrt{2} - 20\sqrt{2}\\ &= -10\sqrt{2}\end{aligned}$$

You Try It 3

$$\begin{aligned}&y\sqrt{28y} + 7\sqrt{63y^3}\\ &\quad= y\sqrt{4\cdot 7y} + 7\sqrt{9y^2\cdot 7y}\\ &\quad= y\sqrt{4}\sqrt{7y} + 7\sqrt{9y^2}\sqrt{7y}\\ &\quad= y\cdot 2\sqrt{7y} + 7\cdot 3y\sqrt{7y}\\ &\quad= 2y\sqrt{7y} + 21y\sqrt{7y}\\ &\quad= 23y\sqrt{7y}\end{aligned}$$

You Try It 4

$$\begin{aligned}&2\sqrt{27a^5} - 4a\sqrt{12a^3} + a^2\sqrt{75a}\\ &\quad= 2\sqrt{9a^4\cdot 3a} - 4a\sqrt{4a^2\cdot 3a} + a^2\sqrt{25\cdot 3a}\\ &\quad= 2\sqrt{9a^4}\sqrt{3a} - 4a\sqrt{4a^2}\sqrt{3a}\\ &\qquad + a^2\sqrt{25}\sqrt{3a}\\ &\quad= 2\cdot 3a^2\sqrt{3a} - 4a\cdot 2a\sqrt{3a} + a^2\cdot 5\sqrt{3a}\\ &\quad= 6a^2\sqrt{3a} - 8a^2\sqrt{3a} + 5a^2\sqrt{3a} = 3a^2\sqrt{3a}\end{aligned}$$

SECTION 10.3

You Try It 1

$$\begin{aligned}&\sqrt{5a}\sqrt{15a^3b^4}\sqrt{20b^5}\\ &\quad= \sqrt{1500a^4b^9} = \sqrt{100a^4b^8\cdot 15b}\\ &\quad= \sqrt{100a^4b^8}\cdot\sqrt{15b}\\ &\quad= 10a^2b^4\sqrt{15b}\end{aligned}$$

You Try It 2

$$\begin{aligned}&\sqrt{5x}(\sqrt{5x} - \sqrt{25y})\\ &\quad= \sqrt{25x^2} - \sqrt{125xy}\\ &\quad= \sqrt{25x^2} - \sqrt{25\cdot 5xy} = \sqrt{25x^2} - \sqrt{25}\sqrt{5xy}\\ &\quad= 5x - 5\sqrt{5xy}\end{aligned}$$

You Try It 3

$$\begin{aligned}(2\sqrt{x} + 7)(2\sqrt{x} - 7) &= 4(\sqrt{x})^2 - 7^2\\ &= 4x - 49\end{aligned}$$

You Try It 4

$$\begin{aligned}&(3\sqrt{x} - \sqrt{y})(5\sqrt{x} - 2\sqrt{y})\\ &\quad= 15(\sqrt{x})^2 - 6\sqrt{xy} - 5\sqrt{xy} + 2(\sqrt{y})^2\\ &\quad= 15(\sqrt{x})^2 - 11\sqrt{xy} + 2(\sqrt{y})^2\\ &\quad= 15x - 11\sqrt{xy} + 2y\end{aligned}$$

You Try It 5

$$\begin{aligned}\frac{\sqrt{15x^6y^7}}{\sqrt{3x^7y^9}} &= \sqrt{\frac{15x^6y^7}{3x^7y^9}} = \sqrt{\frac{5}{xy^2}} = \frac{\sqrt{5}}{\sqrt{xy^2}}\\ &= \frac{\sqrt{5}}{y\sqrt{x}} = \frac{\sqrt{5}}{y\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{5x}}{xy}\end{aligned}$$

You Try It 6

$$\begin{aligned}\frac{\sqrt{3}}{\sqrt{3} - \sqrt{6}} &= \frac{\sqrt{3}}{\sqrt{3} - \sqrt{6}}\cdot\frac{\sqrt{3} + \sqrt{6}}{\sqrt{3} + \sqrt{6}}\\ &= \frac{3 + \sqrt{18}}{3 - 6} = \frac{3 + 3\sqrt{2}}{-3}\\ &= \frac{3(1 + \sqrt{2})}{-3} = -1(1 + \sqrt{2})\\ &= -1 - \sqrt{2}\end{aligned}$$

You Try It 7

$$\begin{aligned}\frac{5 + \sqrt{y}}{1 - 2\sqrt{y}} &= \frac{5 + \sqrt{y}}{1 - 2\sqrt{y}}\cdot\frac{1 + 2\sqrt{y}}{1 + 2\sqrt{y}}\\ &= \frac{5 + 10\sqrt{y} + \sqrt{y} + 2(\sqrt{y})^2}{1 - 4y}\\ &= \frac{5 + 11\sqrt{y} + 2y}{1 - 4y}\end{aligned}$$

SECTION 10.4

You Try It 1

$$\begin{aligned}\sqrt{4x} + 3 &= 7\\ \sqrt{4x} &= 4\\ (\sqrt{4x})^2 &= 4^2\\ 4x &= 16\\ x &= 4\end{aligned}$$

Check:

$$\begin{array}{r|l} \multicolumn{2}{c}{\sqrt{4x} + 3 = 7}\\ \hline \sqrt{4\cdot 4} + 3 & 7\\ \sqrt{16} + 3 & 7\\ 4 + 3 & 7\\ 7 = 7 & \end{array}$$

The solution is 4.

You Try It 2

$$\begin{aligned}\sqrt{x} + \sqrt{x + 9} &= 9\\ \sqrt{x} &= 9 - \sqrt{x + 9}\\ (\sqrt{x})^2 &= (9 - \sqrt{x + 9})^2\\ x &= 81 - 18\sqrt{x + 9} + (x + 9)\\ -90 &= -18\sqrt{x + 9}\\ 5 &= \sqrt{x + 9}\\ 5^2 &= (\sqrt{x + 9})^2\\ 25 &= x + 9\\ 16 &= x\end{aligned}$$

Check:

$$\begin{array}{r|l} \multicolumn{2}{c}{\sqrt{x} + \sqrt{x + 9} = 9} \\ \hline \sqrt{16} + \sqrt{16 + 9} & 9 \\ \sqrt{16} + \sqrt{25} & 9 \\ 4 + 5 & 9 \\ 9 = & 9 \end{array}$$

The solution is 16.

You Try It 3

Strategy To find the distance, use the Pythagorean Theorem. The hypotenuse is the length of the ladder. One leg is the distance from the bottom of the ladder to the base of the building. The distance along the building from the ground to the top of the ladder is the unknown leg.

Solution

$$\begin{aligned} a^2 &= \sqrt{c^2 - b^2} \\ &= \sqrt{(8)^2 - (3)^2} \\ &= \sqrt{64 - 9} \\ &= \sqrt{55} \\ &\approx 7.42 \end{aligned}$$

The distance is approximately 7.42 ft.

You Try It 4

Strategy To find the length of the pendulum, replace T in the equation with the given value and solve for L.

Solution

$$\begin{aligned} T &= 2\pi\sqrt{\frac{L}{32}} \\ 2.5 &= 2(3.14)\sqrt{\frac{L}{32}} \\ 2.5 &= 6.28\sqrt{\frac{L}{32}} \\ \frac{2.5}{6.28} &= \sqrt{\frac{L}{32}} \\ \left(\frac{2.5}{6.28}\right)^2 &= \left(\sqrt{\frac{L}{32}}\right)^2 \\ \frac{6.25}{39.4384} &= \frac{L}{32} \\ (32)\left(\frac{6.25}{39.4384}\right) &= (32)\left(\frac{L}{32}\right) \\ \frac{200}{39.4384} &= L \\ 5.07 &\approx L \end{aligned}$$

The length of the pendulum is approximately 5.07 ft.

Solutions to Chapter 11 "You Try It"

SECTION 11.1

You Try It 1

$$\begin{aligned} \frac{3y^2}{2} + y - \frac{1}{2} &= 0 \\ 2\left(\frac{3y^2}{2} + y - \frac{1}{2}\right) &= 2(0) \\ 3y^2 + 2y - 1 &= 0 \\ (3y - 1)(y + 1) &= 0 \end{aligned}$$

$$\begin{aligned} 3y - 1 &= 0 & \qquad y + 1 &= 0 \\ 3y &= 1 & y &= -1 \\ y &= \frac{1}{3} \end{aligned}$$

The solutions are $\frac{1}{3}$ and -1.

You Try It 2

$$\begin{aligned} x^2 + 81 &= 0 \\ x^2 &= -81 \\ \sqrt{x^2} &= \sqrt{-81} \end{aligned}$$

$\sqrt{-81}$ is not a real number.

The equation has no real number solution.

You Try It 3

$$\begin{aligned} 7(z + 2)^2 &= 21 \\ (z + 2)^2 &= 3 \\ \sqrt{(z + 2)^2} &= \sqrt{3} \\ z + 2 &= \pm\sqrt{3} \\ z &= -2 \pm \sqrt{3} \end{aligned}$$

The solutions are $-2 + \sqrt{3}$ and $-2 - \sqrt{3}$.

SECTION 11.2

You Try It 1

$$3x^2 - 6x - 2 = 0$$
$$3x^2 - 6x = 2$$
$$\frac{1}{3}(3x^2 - 6x) = \frac{1}{3} \cdot 2$$
$$x^2 - 2x = \frac{2}{3}$$

Complete the square.

$$x^2 - 2x + 1 = \frac{2}{3} + 1$$
$$(x - 1)^2 = \frac{5}{3}$$
$$\sqrt{(x-1)^2} = \sqrt{\frac{5}{3}}$$
$$x - 1 = \pm\sqrt{\frac{5}{3}} = \pm\frac{\sqrt{15}}{3}$$

$$x - 1 = \frac{\sqrt{15}}{3} \qquad x - 1 = -\frac{\sqrt{15}}{3}$$
$$x = 1 + \frac{\sqrt{15}}{3} \qquad x = 1 - \frac{\sqrt{15}}{3}$$
$$= \frac{3 + \sqrt{15}}{3} \qquad = \frac{3 - \sqrt{15}}{3}$$

The solutions are $\frac{3+\sqrt{15}}{3}$ and $\frac{3-\sqrt{15}}{3}$.

You Try It 2

$$x^2 + 6x + 12 = 0$$
$$x^2 + 6x = -12$$
$$x^2 + 6x + 9 = -12 + 9$$
$$(x + 3)^2 = -3$$
$$\sqrt{(x+3)^2} = \sqrt{-3}$$

$\sqrt{-3}$ is not a real number.

The quadratic equation has no real number solution.

You Try It 3

$$\frac{x^2}{8} + x + 1 = 0$$
$$8\left(\frac{x^2}{8} + x + 1\right) = 8(0)$$
$$x^2 + 8x + 8 = 0$$
$$x^2 + 8x = -8$$
$$x^2 + 8x + 16 = -8 + 16$$
$$(x + 4)^2 = 8$$
$$\sqrt{(x+4)^2} = \sqrt{8}$$
$$x + 4 = \pm\sqrt{8} = \pm 2\sqrt{2}$$

$$x + 4 = 2\sqrt{2} \qquad x + 4 = -2\sqrt{2}$$
$$x = -4 + 2\sqrt{2} \qquad x = -4 - 2\sqrt{2}$$
$$\approx -4 + 2(1.414) \qquad \approx -4 - 2(1.414)$$
$$\approx -4 + 2.828 \qquad \approx -4 - 2.828$$
$$\approx -1.172 \qquad \approx -6.828$$

The solutions are approximately -1.172 and -6.828.

SECTION 11.3

You Try It 1

$$3x^2 + 4x - 4 = 0$$
$$a = 3, b = 4, c = -4$$
$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(3)(-4)}}{2 \cdot 3}$$
$$= \frac{-4 \pm \sqrt{16 + 48}}{6}$$
$$= \frac{-4 \pm \sqrt{64}}{6} = \frac{-4 \pm 8}{6}$$
$$x = \frac{-4 + 8}{6} \qquad x = \frac{-4 - 8}{6}$$
$$= \frac{4}{6} = \frac{2}{3} \qquad = \frac{-12}{6} = -2$$

The solutions are $\frac{2}{3}$ and -2.

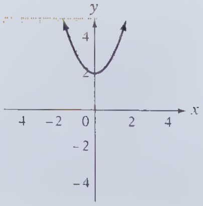
You Try It 2

$$\frac{x^2}{4} + \frac{x}{2} = \frac{1}{4}$$
$$4\left(\frac{x^2}{4} + \frac{x}{2}\right) = 4\left(\frac{1}{4}\right)$$
$$x^2 + 2x = 1$$
$$x^2 + 2x - 1 = 0$$
$$a = 1, b = 2, c = -1$$
$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-1)}}{2 \cdot 1}$$
$$= \frac{-2 \pm \sqrt{4 + 4}}{2} = \frac{-2 \pm \sqrt{8}}{2}$$
$$= \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

The solutions are $-1 + \sqrt{2}$ and $-1 - \sqrt{2}$.

SECTION 11.4

You Try It 1



You Try It 2 To find the x-intercept, let $f(x) = 0$ and solve for x.

$$\begin{aligned} f(x) &= x^2 - 6x + 9 \\ 0 &= x^2 - 6x + 9 \\ 0 &= (x-3)(x-3) \end{aligned}$$

$$\begin{aligned} x - 3 &= 0 & \qquad x - 3 &= 0 \\ x &= 3 & x &= 3 \end{aligned}$$

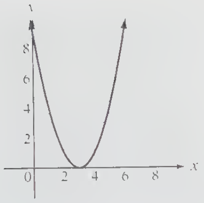
The x-intercept is (3, 0).

Note from the graph that there is only one x-intercept. The equation has a double root.

To find the y-intercept, evaluate the function at $x = 0$.

$$\begin{aligned} f(x) &= x^2 - 6x + 9 \\ f(0) &= 0^2 - 6(0) + 9 = 9 \end{aligned}$$

The y-intercept is (0, 9).



SECTION 11.5

You Try It 1

Strategy

- This is a geometry problem.
- Width of the rectangle: W
  Length of the rectangle: $W + 2$
- Use the equation $A = L \cdot W$.

Solution

$$\begin{aligned} A &= L \cdot W \\ 15 &= (W + 2)W \\ 15 &= W^2 + 2W \\ 0 &= W^2 + 2W - 15 \\ 0 &= (W + 5)(W - 3) \end{aligned}$$

$$\begin{aligned} W + 5 &= 0 & \qquad W - 3 &= 0 \\ W &= -5 & W &= 3 \end{aligned}$$

The solution -5 is not possible. The width is 3 m.

Answers to Chapter 1 Odd-Numbered Exercises

PREP TEST

1. 127.16 **2.** 55.107 **3.** 4517 **4.** 11,396 **5.** 24 **6.** 24 **7.** 4 **8.** $3 \cdot 7$ **9.** $\frac{2}{5}$ **10.** d

SECTION 1.1

3. $8 > -6$ **5.** $-12 < 1$ **7.** $42 > 19$ **9.** $0 > -31$ **11.** $53 > -46$ **13.** false **15.** true **17.** false
19. true **21.** true **23.** $\{1, 2, 3, 4, 5, 6, 7, 8\}$ **25.** $\{1, 2, 3, 4, 5, 6, 7, 8\}$ **27.** $\{-6, -5, -4, -3, -2, -1\}$ **29.** 5
31. $-23, -18$ **33.** 21, 37 **35.** $-52, -46, 0$ **37.** $-17, 0, 4, 29$ **39.** 5, 6, 7, 8, 9 **41.** $-10, -9, -8, -7, -6, -5$
43. -4 **45.** 9 **47.** 36 **49.** 40 **51.** -39 **53.** 74 **55.** -82 **57.** -81 **59.** $|22| > |-19|$
61. $|-71| < |-92|$ **63.** $|12| < |-31|$ **65.** $|-28| < |43|$ **67a.** $11, 7, 3, -1, -5$ **b.** 11, 7, 3, 1, 5 **69.** never true

SECTION 1.2

3. -11 **5.** -5 **7.** -83 **9.** -46 **11.** 0 **13.** -5 **15.** 9 **17.** 1 **19.** -10 **21.** -18 **23.** -41
25. -12 **27.** 0 **29.** -34 **31.** 0 **33.** -61 **37.** 8 **39.** -7 **41.** -9 **43.** 9 **45.** -3 **47.** 18
49. -9 **51.** 11 **53.** -18 **55.** 0 **57.** 2 **59.** -138 **61.** -8 **63.** -12 **65.** -20 **67.** The difference is 399°C. **69.** The difference is 7046 m. **71.** The difference between the highest and lowest elevations is greatest in Asia. **73.** The difference is 5182°C. **75.** The total net loss was \$9 million. **79.** The difference is 16°F.
81. No. For example, $10 - (-8) = 18$.

SECTION 1.3

3. 42 **5.** -28 **7.** 60 **9.** -253 **11.** -238 **13.** -114 **15.** -105 **17.** 252 **19.** -216
21. -315 **23.** 420 **25.** 2880 **27.** -2772 **29.** 0 **31.** -2 **33.** 8 **35.** -7 **37.** -12
39. -6 **41.** -7 **43.** 11 **45.** -14 **47.** 15 **49.** -16 **51.** 0 **53.** -29 **55.** undefined **57.** -11
59. undefined **61.** The average daily high temperature was -26°F. **63.** The 5-day moving average was $-142, -58, -56, -12, -22, 180$. **65.** The score for the exam was 74. **67.** -6

SECTION 1.4

1. $\frac{1}{3}$ **3.** $\frac{4}{11}$ **5.** $\frac{2}{3}$ **7.** $\frac{3}{2}$ **9.** 0 **11.** $\frac{3}{5}$ **13.** $\frac{11}{15}$ **15.** $\frac{1}{2}$ **17.** $0.1\overline{6}$ **19.** 0.125 **21.** $0.\overline{2}$ **23.** $0.\overline{45}$
25. $0.91\overline{6}$ **27.** 0.9375 **29.** $0.9\overline{4}$ **33.** $\frac{3}{4}$; 0.75 **35.** $\frac{16}{25}$; 0.64 **37.** $\frac{7}{4}$; 1.75 **39.** $\frac{19}{100}$; 0.19 **41.** $\frac{1}{20}$; 0.05
43. $\frac{1}{9}$ **45.** $\frac{1}{8}$ **47.** $\frac{2}{3}$ **49.** $\frac{1}{200}$ **51.** $\frac{5}{6}$ **53.** 0.073 **55.** 0.158 **57.** 0.003 **59.** 0.099 **61.** 1.212
63. 15% **65.** 5% **67.** 17.5% **69.** 115% **71.** 0.8% **73.** 54% **75.** $33\frac{1}{3}\%$ **77.** $45\frac{5}{11}\%$ **79.** 87.5%
81. $166\frac{2}{3}\%$ **83.** $\frac{13}{12}$ **85.** $-\frac{5}{24}$ **87.** $-\frac{19}{24}$ **89.** $\frac{5}{26}$ **91.** $\frac{11}{8}$ **93.** $\frac{1}{12}$ **95.** $\frac{7}{24}$ **97.** 0 **99.** $-\frac{7}{16}$
101. $\frac{11}{24}$ **103.** 1 **105.** $\frac{11}{8}$ **107.** 8.022 **109.** -38.8 **111.** -6.192 **113.** 13.355 **115.** 4.676
117. -10.03 **119.** -60.03 **121.** 11.56 **123.** -0.88 **125.** -4.73 **127.** $-\frac{3}{8}$ **129.** $\frac{1}{10}$ **131.** $-\frac{4}{9}$
133. $-\frac{7}{30}$ **135.** $\frac{15}{64}$ **137.** $-\frac{10}{9}$ **139.** $-\frac{147}{32}$ **141.** $\frac{25}{8}$ **143.** $\frac{2}{3}$ **145.** 4.164 **147.** 4.347 **149.** -4.028
151. -2.22 **153.** -1.104 **155.** 0.506 **157.** -0.2376 **159.** $-274.\overline{4}$ **161.** -2.59 **163.** -5.11
165. -2060.55 **167.** 31% responded that the Internet helped business. **169a.** There was a 28.6% increase in software piracy in Western Europe. **b.** There was a 7.7% decrease in piracy in North America. **c.** Eastern Europe had more than a 50% decrease in piracy.

171.

$\frac{2}{3}$	$-\frac{1}{6}$	0
$\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$
$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$

173. Answers will vary. **175.** $a = 2, b = 3, c = 6$

SECTION 1.5

1. 36 **3.** -49 **5.** 9 **7.** 81 **9.** $\frac{1}{4}$ **11.** 0.09 **13.** 12 **15.** 0.216 **17.** -12 **19.** 16 **21.** -864
23. -1008 **25.** 3 **27.** $-77{,}760$ **31.** 9 **33.** 12 **35.** 1 **37.** 8 **39.** -16 **41.** 12 **43.** 13
45. -36 **47.** 13 **49.** 4 **51.** 15 **53.** -1 **55.** 4 **57.** 0.51 **59.** 1.7 **61.** $-\frac{1}{16}$

CHAPTER REVIEW*

1. -6 [1.2A] **2.** 0.28 [1.4A] **3.** -25 [1.5A] **4.** 10 [1.5B] **5.** $-4 < 2$ [1.1A] **6.** 0.062 [1.4B]
7. -42 [1.3A] **8.** $\frac{7}{12}$ [1.4C] **9.** 60 [1.5B] **10.** -4 [1.1A] **11.** -1.068 [1.4C] **12.** 62.5% [1.4B]
13. $0.1\overline{3}$ [1.4A] **14.** -4 [1.2B] **15.** 4 [1.1B] **16.** 8 [1.3B] **17.** -20 [1.3B] **18.** $\frac{159}{200}$ [1.4B]
19. 31 [1.5B] **20.** -13 [1.2A] **21.** $54\frac{2}{7}\%$ [1.4B] **22.** $\frac{16}{81}$ [1.5A] **23.** -4.6224 [1.4D] **24.** -5 [1.1B]
25. 1 [1.2B] **26.** $-\frac{8}{15}$ [1.4D] **27.** 7, 0, 9 [1.1B] **28.** 81 [1.3A] **29.** $-|6| < |-10|$ [1.1B]
30. 1 [1.5B] **31.** The score for the exam was 98. [1.3C] **32.** IBM accounted for 7.4% of worldwide shipments of personal computers. [1.4E] **33.** The difference is 396°C. [1.2C]

CHAPTER TEST*

1. 17 [1.3B] **2.** $83\frac{1}{3}\%$ [1.4B] **3.** 12 [1.5A] **4.** -5.3578 [1.4D] **5.** -14 [1.2B] **6.** $\frac{3}{8}$ [1.4B]
7. $\frac{1}{15}$ [1.4C] **8.** 8 [1.5B] **9.** 90 [1.3A] **10.** 4 [1.1B] **11.** -108 [1.5A] **12.** 4 [1.1A]
13. $-2 > -40$ [1.1A] **14.** -4 [1.1B] **15.** $\frac{9}{20}$; 0.45 [1.4B] **16.** -16 [1.2A] **17.** -48 [1.3A]
18. 17 [1.5B] **19.** 4 [1.2B] **20.** $-\frac{1}{2}$ [1.4D] **21.** 102.5% [1.4B] **22.** 9 [1.5B] **23.** $0.\overline{7}$ [1.4A]
24a. The annual earnings would be $-\$103.6$ million. **b.** The average monthly earnings was $-\$6.1$ million. [1.4E]
25. The value of 100 shares was \$11,375. [1.4E]

Answers to Chapter 2 Odd-Numbered Exercises

PREP TEST

1. 3 [1.2B] **2.** 4 [1.3D] **3.** $\frac{1}{12}$ [1.4C] **4.** $-\frac{4}{9}$ [1.4D] **5.** $\frac{3}{10}$ [1.4D] **6.** -16 [1.5A] **7.** $\frac{8}{27}$ [1.5A]
8. 48 [1.5B] **9.** 1 [1.5B] **10.** 12 [1.5B]

*Note: The numbers in brackets following the answers in the Chapter Review indicate the objective that corresponds to that problem. For example, the reference [1.2A] stands for Section 1.2, Objective A. This notation will be used for all Pretests, Chapter Reviews, Chapter Tests, and Cumulative Reviews throughout the text.

SECTION 2.1

1. $2x^2, 5x, \underline{-8}$ **3.** $-a^4, \underline{6}$ **5.** $7\underline{x^2y}, 6\underline{xy^2}$ **7.** $1, -9$ **9.** $1, -4, -1$ **13.** 10 **15.** 32 **17.** 21 **19.** 16
21. -9 **23.** 41 **25.** -7 **27.** 13 **29.** -15 **31.** 41 **33.** 1 **35.** 5 **37.** 1 **39.** 57 **41.** 5
43. 8 **45.** -3 **47.** -2 **49.** -4 **51.** 225 **53.** 60 **55.** 4 **57.** 81 **59.** $n^x > x^n$ if $x \geq n + 1$

SECTION 2.2

3. $14x$ **5.** $5a$ **7.** $-6y$ **9.** $7 - 3b$ **11.** $5a$ **13.** $-2ab$ **15.** $5xy$ **17.** 0 **19.** $-\frac{5}{6}x$ **21.** $6.5x$
23. $0.45x$ **25.** $7a$ **27.** $-14x^2$ **29.** $-x + 3y$ **31.** $17x - 3y$ **33.** $-2a - 6b$ **35.** $-3x - 8y$ **37.** $-4x^2 - 2x$
39. $12x$ **41.** $-21a$ **43.** $6y$ **45.** $8x$ **47.** $-6a$ **49.** $12b$ **51.** $-15x^2$ **53.** x^2 **55.** a **57.** x **59.** n
61. x **63.** y **65.** $3x$ **67.** $-2x$ **69.** $-8a^2$ **71.** $8y$ **73.** $4y$ **75.** $-2x$ **77.** $6a$ **79.** $-x - 2$
81. $8x - 6$ **83.** $-2a - 14$ **85.** $-6y + 24$ **87.** $35 - 21b$ **89.** $2 - 5y$ **91.** $15x^2 + 6x$ **93.** $2y - 18$
95. $-15x - 30$ **97.** $-6x^2 - 28$ **99.** $-6y^2 + 21$ **101.** $3x^2 - 3y^2$ **103.** $-4x + 12y$ **105.** $-6a^2 + 7b^2$
107. $4x^2 - 12x + 20$ **109.** $x - 3y + 4$ **111.** $-12a^2 - 20a + 28$ **113.** $12x^2 - 9x + 12$ **115.** $10x^2 - 20xy - 5y^2$
117. $-8b^2 + 6b - 9$ **119.** $a - 7$ **121.** $-11x + 13$ **123.** $-4y - 4$ **125.** $-2x - 16$ **127.** $14y - 45$
129. $a + 7b$ **131.** $6x + 28$ **133.** $5x - 75$ **135.** $4x - 4$ **137.** $2x - 9$ **139a.** False. $8 \div 4 \neq 4 \div 8$
b. False. $(8 \div 4) \div 2 \neq 8 \div (4 \div 2)$ **c.** False. $(7 - 5) - 1 \neq 7 - (5 - 1)$ **d.** False. $6 - 3 \neq 3 - 6$
141. No. 0 does not have a multiplicative inverse.

SECTION 2.3

1. $8 + y$ **3.** $t + 10$ **5.** $z + 14$ **7.** $x^2 - 20$ **9.** $\frac{3}{4}n + 12$ **11.** $8 + \frac{n}{4}$ **13.** $3(y + 7)$ **15.** $t(t + 16)$
17. $\frac{1}{2}x^2 + 15$ **19.** $5n^3 + n^2$ **21.** $r - \frac{r}{3}$ **23.** $x^2 - (x + 17)$ **25.** $9(z + 4)$ **27.** $12 - x$ **29.** $\frac{2}{3}x$ **31.** $\frac{2x}{9}$
33. $11x - 8$ **35.** $(x + 2) - 9; x - 7$ **37.** $\frac{7}{5 + x}$ **39.** $5 + \frac{1}{2}(x + 3); \frac{1}{2}x + \frac{13}{2}$ **41.** $(2x - 4) + x; 3x - 4$
43. $(x - 5)7; 7x - 35$ **45.** $\frac{2x + 5}{x}$ **47.** $x - (3x - 8); -2x + 8$ **49.** $3x + x; 4x$ **51.** $(x + 6) + 5; x + 11$
53. $x - (x + 10); -10$ **55.** $\frac{1}{6}x + \frac{4}{9}x; \frac{11}{18}x$ **57.** $\frac{x}{3} + x; \frac{4}{3}x$ **59.** number of near collisions in 1999: C; number of near collisions in 2000: $C + 91$ **61.** length of one piece: S; length of second piece: $12 - S$ **63.** distance traveled by the first car: x; distance traveled by the second car: $200 - x$ **65.** number of bones in your body: N; number of bones in your foot: $\frac{1}{4}N$ **67.** salary needed in San Diego: S; salary needed in New York City: $2S$ **69.** $2x$

CHAPTER REVIEW

1. $3x^2 - 24x - 21$ [2.2C] **2.** $11x$ [2.2A] **3.** $8a - 4b$ [2.2A] **4.** $-5n$ [2.2B] **5.** 79 [2.1A]
6. $10x - 35$ [2.2C] **7.** $12y^2 + 8y - 10$ [2.2C] **8.** $-6a$ [2.2B] **9.** $-42x^2$ [2.2B] **10.** $-63 - 36x$ [2.2C]
11. $-5y$ [2.2A] **12.** -4 [2.1A] **13.** $24y + 30$ [2.2D] **14.** $9c - 5d$ [2.2A] **15.** $20x$ [2.2B]
16. $7x + 46$ [2.2D] **17.** 29 [2.1A] **18.** $-9r + 8s$ [2.2A] **19.** $-4x^2 + 6x$ [2.2A] **20.** $-90x + 25$ [2.2D]
21. $28a^2 - 8a + 12$ [2.2C] **22.** $-4x + 20$ [2.2D] **23.** -7 [2.1A] **24.** $36y$ [2.2B] **25.** $\frac{2}{3}(x + 10)$ [2.3A]
26. number of calories in an apple: a; number of calories in a candy bar: $2a + 8$ [2.3C]
27. $3x + 5(x - 1); 8x - 5$ [2.3B] **28.** $4x$ [2.3A] **29.** number of ten-dollar bills: T; number of five-dollar bills: $35 - T$ [2.3C] **30.** $2x - \frac{1}{2}x; \frac{3}{2}x$ [2.3B] **31.** number of American League cards: A; number of National League cards: $5A$ [2.3C] **32.** $x - 6$ [2.3A] **33.** $x + 2x; 3x$ [2.3B]

CHAPTER TEST

1. $5x$ [2.2A] **2.** $-6x^2 + 21y^2$ [2.2C] **3.** $-x + 6$ [2.2D] **4.** $-7x + 33$ [2.2D] **5.** $-9x - 7y$ [2.2A]

6. 22 [2.1A] **7.** $2x$ [2.2B] **8.** $7x + 38$ [2.2D] **9.** $-10x^2 + 15x - 30$ [2.2C] **10.** $-2x - 5y$ [2.2A]

11. 3 [2.1A] **12.** $3x$ [2.2B] **13.** y^2 [2.2A] **14.** $-4x + 8$ [2.2C] **15.** $-10a$ [2.2B]

16. $2x + y$ [2.2D] **17.** $36y$ [2.2B] **18.** $15 - 35b$ [2.2C] **19.** $a^2 - b^2$ [2.3A]

20. $10(x - 3)$; $10x - 30$ [2.3B] **21.** $x + 2x^2$ [2.3B] **22.** speed of return throw: s; speed of fastball: $2s$ [2.3C]

23. $\frac{6}{x} - 3$ [2.3B] **24.** $b - 7b$ [2.3A] **25.** shorter piece: x; longer piece: $4x - 3$ [2.3C]

CUMULATIVE REVIEW

1. -7 [1.2A] **2.** 5 [1.2B] **3.** 24 [1.3A] **4.** -5 [1.3B] **5.** 1.25 [1.4A] **6.** $\frac{11}{48}$ [1.4C] **7.** $\frac{1}{6}$ [1.4D]

8. $\frac{1}{4}$ [1.4D] **9.** 75% [1.4B] **10.** -5 [1.5B] **11.** $\frac{53}{48}$ [1.5B] **12.** 16 [2.1A] **13.** $5x^2$ [2.2A]

14. $-7a - 10b$ [2.2A] **15.** $6a$ [2.2B] **16.** $30b$ [2.2B] **17.** $24 - 6x$ [2.2C] **18.** $6y - 18$ [2.2C]

19. $\frac{3}{8}$ [1.4B] **20.** 0.0105 [1.4B] **21.** $-8x^2 + 12y^2$ [2.2C] **22.** $-9y^2 + 9y + 21$ [2.2C]

23. $-7x + 14$ [2.2D] **24.** $5x - 43$ [2.2D] **25.** $17x - 24$ [2.2D] **26.** $-3x + 21y$ [2.2D]

27. $\frac{1}{2}b + b$ [2.3A] **28.** $\frac{10}{y - 2}$ [2.3A] **29.** $8 - \frac{x}{12}$ [2.3B] **30.** $x + (x + 2)$; $2x + 2$ [2.3B]

31. $(3 + x)5 + 12$; $27 + 5x$ [2.3B] **32.** speed of dial-up connection: s; speed of DSL connection: $10s$ [2.3C]

Answers to Chapter 3 Odd-Numbered Exercises

PREP TEST

1. 0.09 [1.4B] **2.** 75% [1.4B] **3.** 63 [2.1A] **4.** $0.65R$ [2.2A] **5.** $\frac{7}{6}x$ [2.2A] **6.** $9x - 18$ [2.2C]

7. $1.66x + 1.32$ [2.2C] **8.** $5 - 2n$ [2.3B] **9.** $5x$ [2.3C] **10.** $5 - x$ [2.3C]

SECTION 3.1

3. yes **5.** no **7.** no **9.** yes **11.** no **13.** yes **15.** no **17.** yes **19.** yes **21.** yes **23.** no

27. 6 **29.** 16 **31.** 7 **33.** -2 **35.** 1 **37.** 0 **39.** 3 **41.** -10 **43.** -3 **45.** -14 **47.** 2

49. 11 **51.** -9 **53.** -1 **55.** -14 **57.** -5 **59.** -1 **61.** 1 **63.** $-\frac{1}{2}$ **65.** $-\frac{3}{4}$ **67.** $\frac{1}{12}$ **69.** $-\frac{7}{12}$

71. 0.6529 **73.** -0.283 **75.** 9.257 **79.** -3 **81.** 0 **83.** -2 **85.** 9 **87.** 80 **89.** -4 **91.** 0

93. 8 **95.** -7 **97.** 12 **99.** -18 **101.** 15 **103.** -20 **105.** 0 **107.** $\frac{8}{3}$ **109.** $\frac{1}{3}$ **111.** $-\frac{1}{2}$

113. $-\frac{3}{2}$ **115.** $\frac{15}{7}$ **117.** 4 **119.** 3 **121.** 4.745 **123.** 2.06 **125.** -2.13 **127.** equal to **129.** 28

131. 0.72 **133.** 64 **135.** 24% **137.** 7.2 **139.** 400 **141.** 9 **143.** 25% **145.** 5 **147.** 200%

149. 400 **151.** 7.7 **153.** 200 **155.** 400 **157.** 20 **159.** 80.34% **161.** 19% of the students are in the fine arts college. **163.** Western Europe's share is $675 billion more than Latin America's share. **165.** There is insufficient information to answer the question. **167.** 35.8% of the total average healthcare costs were out-of-pocket costs. **169a.** 44.4% of the people watching would have to buy one can. **b.** 7.3% of the people watching would have to buy one 6-pack. **171a.** Answers will vary. **b.** Answers will vary.

SECTION 3.2

1. 3 **3.** 6 **5.** −1 **7.** −3 **9.** 2 **11.** 2 **13.** 5 **15.** −3 **17.** 6 **19.** 3 **21.** 1 **23.** 6 **25.** −7 **27.** 0 **29.** $\frac{3}{4}$ **31.** $\frac{4}{9}$ **33.** $\frac{1}{3}$ **35.** $-\frac{1}{2}$ **37.** $-\frac{3}{4}$ **39.** $\frac{1}{3}$ **41.** $-\frac{1}{6}$ **43.** 1 **45.** 1 **47.** 0 **49.** $\frac{13}{10}$ **51.** $\frac{2}{5}$ **53.** $-\frac{4}{3}$ **55.** $-\frac{3}{2}$ **57.** 18 **59.** 8 **61.** −16 **63.** 25 **65.** $\frac{3}{4}$ **67.** $\frac{3}{8}$ **69.** $\frac{16}{9}$ **71.** $\frac{1}{18}$ **73.** $\frac{15}{2}$ **75.** $-\frac{18}{5}$ **77.** 2 **79.** 3 **81.** 7 **83.** 3 **85.** 19 **87.** −1 **89.** −11 **91.** The markup rate is 60%. **93.** The cost is \$59. **95.** The markup rate is 44.4%. **97.** The cost is \$8.50. **99.** The discount rate is 23.2%. **101.** The regular price is \$300. **103.** The markdown rate is 38%. **105.** The regular price is \$275. **107.** The initial velocity is 8 ft/s. **109.** The depreciated value will be \$38,000 in 2 years. **111.** The length is approximately 31.8 in. **113.** The distance is 168 ft. **115.** The population is approximately 51,000 people. **117.** The year is 1952. **119.** The markup is \$18. **121.** The regular price is \$317.65. **123.** 385

SECTION 3.3

1. 2 **3.** 3 **5.** −1 **7.** 2 **9.** −2 **11.** −3 **13.** 0 **15.** −1 **17.** −3 **19.** −1 **21.** 4 **23.** $\frac{2}{3}$ **25.** $\frac{5}{6}$ **27.** $\frac{3}{4}$ **29.** −17 **31.** 41 **33.** 8 **35.** 1 **37.** 4 **39.** −1 **41.** −1 **43.** 24 **45.** 495 **47.** $\frac{1}{2}$ **49.** $-\frac{1}{3}$ **51.** $\frac{10}{3}$ **53.** $-\frac{1}{4}$ **55.** 0 **57.** −1 **59.** The force applied must be 25 lb. **61.** The fulcrum must be 6 ft from the 180-pound person. **63.** The fulcrum must be 10 ft from the 128-pound acrobat. **65.** The minimum force is 34.6 lb. **67.** The breakeven point is 260 units. **69.** The breakeven point is 520 units. **71.** 3000 bats must be sold to break even. **73.** no solution **75.** 0

SECTION 3.4

1. $x - 15 = 7; 22$ **3.** $7x = -21; -3$ **5.** $9 - x = 7; 2$ **7.** $5 - 2x = 1; 2$ **9.** $2x + 5 = 15; 5$ **11.** $4x - 6 = 22; 7$ **13.** $3(4x - 7) = 15; 3$ **15.** $2(x - 25) = 3x; -50$ **17.** $3x - 1 = 15 - x; 4, 11$ **19.** $3x + 2(18 - x) = 44; 8, 10$ **21.** The integers are −8, −6, and −4. **23.** The integers are 11, 13, and 15. **25.** The integers are 6 and 8. **27.** The integers are 2, 4, and 6. **29.** The storage capacity is 2 gigabytes. **31.** The customer used the service for 11 min. **33.** The executive used the phone for 162 min. **35.** The family used 515 kWh of electricity. **37.** The pieces measure 6 yd and 8 yd. **39.** The value of the stock account is \$6000. The value of the mutual fund account is \$4000. **43.** Contradiction **45.** Conditional; $-\frac{5}{16}$ **47.** Conditional; 0

SECTION 3.5

3. The sides measure 50 ft, 50 ft, and 25 ft. **5.** The length is 13 m. The width is 8 m. **7.** The length is 40 ft. The width is 20 ft. **9.** The sides measure 40 cm, 20 cm, and 50 cm. **11.** The length is 130 ft. The width is 39 ft. **13.** The width is 12 ft. **15.** Each side measures 12 in. **17.** complementary, supplementary **19.** The complement of a 28° angle is 62°. **21.** The supplement of a 73° angle is 107°. **23.** 35° **25.** 20° **27.** 53° **29.** 121° **31.** 15° **33.** 18° **35.** 45° **37.** 49° **39.** 12° **41.** $\angle a = 122^\circ; \angle b = 58^\circ$ **43.** $\angle a = 44^\circ; \angle b = 136^\circ$ **45.** 20° **47.** 40° **49.** 128° **51.** $\angle x = 160^\circ, \angle y = 145^\circ$ **53.** $\angle a = 40^\circ, \angle b = 140^\circ$ **55.** $75^\circ - x$ **57.** The measure of the third angle is 45°. **59.** The measure of the third angle is 73°. **61.** The measure of the third angle is 43°. **63.** The length is 9 cm. The width is 4 cm.

SECTION 3.6

1. 20 oz of herbs should be used. **3.** The cost of the mixture is \$1.84 per pound. **5.** 3 lb of caramel is needed. **7.** 2 c of olive oil and 8 c of vinegar are used. **9.** The cost is \$3.00 per ounce. **11.** 16 oz of the \$400 alloy should be used. **13.** 37 lb of almonds and 63 lb of walnuts were used. **15.** 228 adult tickets were sold.

17. The cost is \$.70 per pound. **19.** The resulting gold alloy is 24% gold. **21.** 20 gal of the 15% acid solution are used. **23.** 30 lb of the yarn that is 25% wool is used. **25.** 6.25 gal of the plant food that is 9% nitrogen are used. **27.** The resulting mixture is 19% sugar. **29.** 20 lb of the coffee that is 40% java beans are used. **31.** 100 ml of the 7% solution and 200 ml of the 4% solution are used. **33.** 150 oz of pure chocolate must be added. **35.** The resulting alloy is 50% silver. **37.** The rate of the first plane is 105 mph. The rate of the second plane is 130 mph. **39.** They will be 3000 km apart at 11 A.M. **41.** The cabin cruiser will be alongside the motorboat 2 h after the cabin cruiser leaves. **43.** The distance from the airport to the corporate offices is 120 mi. **45.** The rate of the car is 68 mph. **47.** The distance between the two airports is 300 mi. **49.** The planes will pass each other 2.5 h after the plane leaves Seattle. **51.** They will meet 1.5 h after they begin. **53.** The bus overtakes the car 180 mi from the starting point. **55.** 75 g of pure water must be added. **57.** 3.75 gal must be drained from the radiator and replaced by pure antifreeze. **59.** The cyclist's average speed for the trip was $13\frac{1}{3}$ mph.

CHAPTER REVIEW

1. 21 [3.1B] **2.** 10 [3.3B] **3.** 7 [3.2A] **4.** No [3.1A] **5.** 20 [3.1C] **6.** -2 [3.3B] **7.** 30 is 250% of 12. [3.1D] **8.** 4 [3.3A] **9.** -1 [3.3B] **10.** 4 [3.3A] **11.** The cost is \$671.25. [3.2B] **12.** 68° [3.5B] **13.** A force of 24 lb must be applied to the other end. [3.3C] **14.** The average speed was 32 mph. [3.6C] **15.** The discount rate is $33\frac{1}{3}\%$. [3.2B] **16.** $\angle x = 22°$, $\angle y = 158°$ [3.5B] **17.** 7 qt of cranberry juice and 3 qt of apple juice were used. [3.6A] **18.** The integers are -1, 0, and 1. [3.4A] **19.** The angles measure 75°, 60°, and 45°. [3.5C] **20.** $5n - 4 = 16$; 4 [3.4A] **21.** The height of the Eiffel Tower is 1063 ft. [3.4B] **22.** The length is 80 ft. The width is 20 ft. [3.5A] **23.** The jet overtakes the propeller-driven plane 600 mi from the starting point. [3.6C] **24.** The numbers are 8 and 13. [3.4A] **25.** The resulting mixture is 14% butterfat. [3.6B]

CHAPTER TEST

1. -5 [3.3A] **2.** -5 [3.1B] **3.** -3 [3.2A] **4.** 2 [3.3B] **5.** No [3.1A] **6.** 5 [3.2A] **7.** 0.5% of 8 is 0.04. [3.1D] **8.** $-\frac{1}{3}$ [3.3B] **9.** 2 [3.3A] **10.** -12 [3.1C] **11.** 10 lb of the \$.70 rye flour and 5 lb of the \$.40 wheat flour should be used. [3.6A] **12.** 19° [3.5B] **13.** The discount rate is 20%. [3.2B] **14.** 200 calculators were produced. [3.2B] **15.** The measure of one of the equal angles is 70°. [3.5C] **16.** The integers are 10, 12, and 14. [3.4A] **17.** 1.25 gal of water must be used. [3.6B] **18.** $\angle a = 138°$, $\angle b = 42°$ [3.5B] **19.** $3x - 15 = 27$; 14 [3.4A] **20.** The rate of the snowmobile was 6 mph. [3.6C] **21.** There are 110 color TVs made each day. [3.4B] **22.** The two numbers are 8 and 10. [3.4A] **23.** The distance between the two airports is 360 mi. [3.6C] **24.** The sides measure 10 ft, 5 ft, and 8 ft. [3.5A] **25.** The temperature of the water after mixing is 60°C. [3.3C]

CUMULATIVE REVIEW

1. 6 [1.2B] **2.** -48 [1.3A] **3.** $-\frac{19}{48}$ [1.4C] **4.** -2 [1.4D] **5.** 54 [1.5A] **6.** 24 [1.5B] **7.** 6 [2.1A] **8.** $-17x$ [2.2A] **9.** $-5a - 2b$ [2.2A] **10.** $2x$ [2.2B] **11.** $36y$ [2.2B] **12.** $2x^2 + 6x - 4$ [2.2C] **13.** $-4x + 14$ [2.2D] **14.** $6x - 34$ [2.2D] **15.** Yes [3.1A] **16.** No [3.1A] **17.** 19.2 [3.1D] **18.** -25 [3.1C] **19.** -3 [3.2A] **20.** 3 [3.2A] **21.** 13 [3.3B] **22.** 2 [3.3B] **23.** -3 [3.3A] **24.** $\frac{1}{2}$ [3.3A] **25.** The final temperature of the water is 60°C. [3.3C] **26.** $12 - 5x = -18$; 6 [3.4A] **27.** The area of the garage is 600 ft². [3.4B] **28.** 20 lb of oat flour must be used. [3.6A] **29.** 25 g of pure gold must be added. [3.6B] **30.** The length is 12 ft. The width is 10 ft. [3.5A] **31.** 131° [3.5B] **32.** The measure of one of the angles is 60°. [3.5C] **33.** The length of the track is 120 m. [3.6C]

Answers to Chapter 4 Odd-Numbered Exercises

PREP TEST

1. 1 [1.2B] **2.** -18 [1.3A] **3.** $\frac{2}{3}$ [1.4A] **4.** 48 [2.1A] **5.** 0 [1.4A] **6.** No [2.2A]
7. $5x^2 - 9x - 6$ [2.2A] **8.** 0 [2.2A] **9.** $-6x + 24$ [2.2C] **10.** $-6xy + 10y$ [2.2D]

SECTION 4.1

1. Yes **3.** No **5.** Yes **7.** Yes **9.** Binomial **11.** Trinomial **13.** None **15.** Binomial **17.** $-2x^2 + 3x$
19. $y^2 - 8$ **21.** $5x^2 + 7x + 20$ **23.** $x^3 + 2x^2 - 6x - 6$ **25.** $2a^3 - 3a^2 - 11a + 2$ **27.** $5x^2 + 8x$
29. $7x^2 + xy - 4y^2$ **31.** $3a^2 - 3a + 17$ **33.** $5x^3 + 10x^2 - x - 4$ **35.** $3r^3 + 2r^2 - 11r + 7$ **37.** $4x$
39. $3y^2 - 4y - 2$ **41.** $-7x - 7$ **43.** $4x^3 + 3x^2 + 3x + 1$ **45.** $y^3 + 5y^2 - 2y - 4$ **47.** $-y^2 - 13xy$
49. $2x^2 - 3x - 1$ **51.** $-2x^3 + x^2 + 2$ **53.** $3a^3 - 2$ **55.** $4y^3 + 2y^2 + 2y - 4$ **57.** $x^2 + 9x - 11$

SECTION 4.2

3. $30x^3$ **5.** $-42c^6$ **7.** $9a^7$ **9.** x^3y^4 **11.** $-10x^9y$ **13.** $12x^7y^8$ **15.** $-6x^3y^5$ **17.** x^4y^5z **19.** $a^3b^5c^4$
21. $-30a^5b^8$ **23.** $6a^5b$ **25.** $40y^{10}z^6$ **27.** $x^3y^3z^2$ **29.** $-24a^3b^3c^3$ **31.** $8x^7yz^6$ **33.** $30x^6y^8$ **35.** $-36a^3b^2c^3$
37. x^{15} **39.** x^{14} **41.** x^8 **43.** y^{12} **45.** $-8x^6$ **47.** x^4y^6 **49.** $9x^4y^2$ **51.** $-243x^{15}y^{10}$ **53.** $-8x^7$
55. $24x^8y^7$ **57.** a^4b^6 **59.** $64x^{12}y^3$ **61.** $-18x^3y^4$ **63.** $-8a^7b^5$ **65.** $-54a^9b^3$ **67.** $12x^2$ **69.** $2x^6y^2 + 9x^4y^2$
71. 0 **73.** $17x^4y^8$ **75.** True **77.** False. $(x^2)^5 = x^{2 \cdot 5} = x^{10}$ **79.** No. $2^{(3^2)}$ is larger.

SECTION 4.3

1. 3, 3, $12x - 15$ **3.** $x^2 - 2x$ **5.** $-x^2 - 7x$ **7.** $3a^3 - 6a^2$ **9.** $-5x^4 + 5x^3$ **11.** $-3x^5 + 7x^3$ **13.** $12x^3 - 6x^2$
15. $6x^2 - 12x$ **17.** $3x^2 + 4x$ **19.** $-x^3y + xy^3$ **21.** $2x^4 - 3x^2 + 2x$ **23.** $2a^3 + 3a^2 + 2a$ **25.** $3x^6 - 3x^4 - 2x^2$
27. $-6y^4 - 12y^3 + 14y^2$ **29.** $-2a^3 - 6a^2 + 8a$ **31.** $6y^4 - 3y^3 + 6y^2$ **33.** $x^3y - 3x^2y^2 + xy^3$ **35.** $x^3 + 4x^2 + 5x + 2$
37. $a^3 - 6a^2 + 13a - 12$ **39.** $-2b^3 + 7b^2 + 19b - 20$ **41.** $-6x^3 + 31x^2 - 41x + 10$ **43.** $x^3 - 3x^2 + 5x - 15$
45. $x^4 - 4x^3 - 3x^2 + 14x - 8$ **47.** $15y^3 - 16y^2 - 70y + 16$ **49.** $5a^4 - 20a^3 - 5a^2 + 22a - 8$
51. $y^4 + 4y^3 + y^2 - 5y + 2$ **53.** $x^2 + 4x + 3$ **55.** $a^2 + a - 12$ **57.** $y^2 - 5y - 24$ **59.** $y^2 - 10y + 21$
61. $2x^2 + 15x + 7$ **63.** $3x^2 + 11x - 4$ **65.** $4x^2 - 31x + 21$ **67.** $3y^2 - 2y - 16$ **69.** $9x^2 + 54x + 77$
71. $21a^2 - 83a + 80$ **73.** $6a^2 - 25ab + 14b^2$ **75.** $2a^2 - 11ab - 63b^2$ **77.** $100a^2 - 100ab + 21b^2$
79. $15x^2 + 56xy + 48y^2$ **81.** $14x^2 - 97xy - 60y^2$ **83.** $56x^2 - 61xy + 15y^2$ **85.** $y^2 - 25$ **87.** $4x^2 - 9$
89. $x^2 + 2x + 1$ **91.** $9a^2 - 30a + 25$ **93.** $9x^2 - 49$ **95.** $4a^2 + 4ab + b^2$ **97.** $x^2 - 4xy + 4y^2$ **99.** $16 - 9y^2$
101. $25x^2 + 20xy + 4y^2$ **103.** The area is $(10x^2 - 35x)$ ft². **105.** The area is $(4x^2 + 4x + 1)$ km².
107. The area is $(4x^2 + 10x)$ m². **109.** The total area is $(60w + 3000)$ yd². **111.** $x^4 + 2x^3 - 5x^2 - 6x + 9$
113. $12x^2 - x - 20$ **115.** $x^3 - 7x^2 - 7$

SECTION 4.4

1. 7, 5, 2 **3.** $\frac{1}{25}$ **5.** 64 **7.** $\frac{1}{27}$ **9.** 2 **11.** $\frac{1}{x^2}$ **13.** a^6 **15.** $\frac{4}{x^7}$ **17.** $\frac{2}{3z^2}$ **19.** $5b^8$ **21.** $\frac{x^2}{3}$ **23.** 1
25. -1 **27.** y^4 **29.** a^3 **31.** p^4 **33.** $2x^3$ **35.** $2k$ **37.** m^5n^2 **39.** $\frac{3r^2}{2}$ **41.** $-\frac{2a}{3}$ **43.** $\frac{1}{y^5}$ **45.** $\frac{1}{a^6}$
47. $\frac{1}{3x^3}$ **49.** $\frac{2}{3x^5}$ **51.** $\frac{y^4}{x^2}$ **53.** $\frac{2}{5m^3n^8}$ **55.** $\frac{1}{p^3q}$ **57.** $\frac{1}{2y^3}$ **59.** $\frac{7xz}{8y^3}$ **61.** $\frac{p^2}{2m^3}$ **63.** $-\frac{8x^3}{y^6}$ **65.** $\frac{9}{x^2y^4}$
67. $\frac{2}{x^4}$ **69.** $-\frac{5}{a^8}$ **71.** $-\frac{a^5}{8b^4}$ **73.** $\frac{10y^3}{x^4}$ **75.** $\frac{1}{2x^3}$ **77.** $\frac{3}{x^3}$ **79.** $\frac{1}{2x^2y^6}$ **81.** $\frac{1}{x^6y}$ **83.** $\frac{a^4}{y^{10}}$ **85.** $-\frac{1}{6x^3}$
87. $-\frac{a^2b}{6c^2}$ **89.** $-\frac{7b^6}{a^2}$ **91.** $\frac{s^8t^4}{4r^{12}}$ **93.** $\frac{125p^3}{27m^{15}n^6}$ **97.** 3.24×10^{-9} **99.** 3×10^{-18} **101.** 3.2×10^{16}
103. 1.22×10^{-19} **105.** 5.47×10^8 **107.** 0.000167 **109.** 68,000,000 **111.** 0.0000305 **113.** 0.00000000102

115. 6.023×10^{23} **117.** 3.7×10^{-6} **119.** 1×10^{-9} **121.** 1.6×10^{-19} **123.** $\frac{1}{4}, \frac{1}{2}, 1, 2, 4$ **125.** $4, 2, 1, \frac{1}{2}, \frac{1}{4}$

127. False. $(2a)^{-3} = \frac{1}{(2a)^3} = \frac{1}{8a^3}$ **129.** False. $(2 + 3)^{-1} = (5)^{-1} = \frac{1}{5}$

SECTION 4.5

1. $2a - 5$ **3.** $6y + 4$ **5.** $x - 2$ **7.** $-x + 2$ **9.** $x^2 + 3x - 5$ **11.** $x^4 - 3x^2 - 1$ **13.** $xy + 2$ **15.** $-3y^3 + 5$

17. $3x - 2 + \frac{1}{x}$ **19.** $-3x + 7 - \frac{6}{x}$ **21.** $4a - 5 + 6b$ **23.** $9x + 6 - 3y$ **25.** $(x + 2)(x - 3)$ **27.** $b - 7$

29. $y - 5$ **31.** $2y - 7$ **33.** $2y + 6 + \frac{25}{y - 3}$ **35.** $x - 2 + \frac{8}{x + 2}$ **37.** $3y - 5 + \frac{20}{2y + 4}$ **39.** $6x - 12 + \frac{19}{x + 2}$

41. $b - 5 - \frac{24}{b - 3}$ **43.** $3x + 17 + \frac{64}{x - 4}$ **45.** $5y + 3 + \frac{1}{2y + 3}$ **47.** $4a + 1$ **49.** $2a + 9 + \frac{33}{3a - 1}$

51. $x^2 - 5x + 2$ **53.** $x^2 + 5$ **55.** $3ab$

CHAPTER REVIEW

1. $8b^2 - 2b - 15$ [4.3C] **2.** $21y^2 + 4y - 1$ [4.1A] **3.** $b^2 + 5b + 2 + \frac{7}{b - 7}$ [4.5B] **4.** $x^4y^8z^4$ [4.2A]

5. $-8x^3 - 14x^2 + 18x$ [4.3A] **6.** b^2 [4.4A] **7.** 64 [4.2B] **8.** $2x^2 + 3x - 8$ [4.1B] **9.** $\frac{2x^3}{3}$ [4.4A]

10. $25y^2 - 70y + 49$ [4.3D] **11.** $100a^{15}b^{13}$ [4.2B] **12.** $4b^4 + 12b^2 - 1$ [4.5A] **13.** $13y^3 - 12y^2 - 5y - 1$ [4.1B]

14. $10a^2 + 12a - 22$ [4.1A] **15.** $6y^3 + 17y^2 - 2y - 21$ [4.3B] **16.** $-\frac{2a^3}{3b^3}$ [4.4A]

17. $8a^3b^3 - 4a^2b^4 + 6ab^5$ [4.3A] **18.** $-2y^2 + y - 5$ [4.1B] **19.** $9x^4y^6$ [4.2B]

20. $12b^5 - 4b^4 - 6b^3 - 8b^2 + 5$ [4.3B] **21.** $2x^3 + 9x^2 - 3x - 12$ [4.1A] **22.** $-4y + 8$ [4.5A]

23. $a^2 - 49$ [4.3D] **24.** $-54a^{13}b^5c^7$ [4.2A] **25.** $2y - 9$ [4.5B] **26.** $\frac{x^4y^6}{9}$ [4.4A] **27.** $-20x^3y^5$ [4.2A]

28. $10a^2 + 31a - 63$ [4.3C] **29.** 1.27×10^{-7} [4.4B] **30.** 0.0000000000032 [4.4B]

31. The area is $(2w^2 - w)$ ft². [4.3E] **32.** The area is $(9x^2 - 12x + 4)$ in². [4.3E]

CHAPTER TEST

1. $4x^3 - 6x^2$ [4.3A] **2.** $4x - 1 + \frac{3}{x^2}$ [4.5A] **3.** $-\frac{4}{x^6}$ [4.4A] **4.** $-6x^3y^6$ [4.2A] **5.** $x - 1 + \frac{2}{x + 1}$ [4.5B]

6. $x^3 - 7x^2 + 17x - 15$ [4.3B] **7.** $-8a^6b^3$ [4.2B] **8.** $\frac{9y^{10}}{x^{10}}$ [4.4A] **9.** $a^2 + 3ab - 10b^2$ [4.3C]

10. $4x^4 - 2x^2 + 5$ [4.5A] **11.** $x + 7$ [4.5B] **12.** $6y^4 - 9y^3 + 18y^2$ [4.3A] **13.** $-4x^4 + 8x^3 - 3x^2 - 14x + 21$ [4.3B] **14.** $16y^2 - 9$ [4.3D] **15.** a^4b^7 [4.2A] **16.** $8ab^4$ [4.4A] **17.** $4a - 7$ [4.5A]

18. $-5a^3 + 3a^2 - 4a + 3$ [4.1B] **19.** $4x^2 - 20x + 25$ [4.3D] **20.** $2x + 3 + \frac{2}{2x - 3}$ [4.5B]

21. $-2x^3$ [4.4A] **22.** $10x^2 - 43xy + 28y^2$ [4.3C] **23.** $3x^3 + 6x^2 - 8x + 3$ [4.1A] **24.** 3.02×10^{9} [4.4B]

25. The area is $(\pi x^2 - 10\pi x + 25\pi)$ m². [4.3E]

CUMULATIVE REVIEW

1. $\frac{5}{144}$ [1.4C] **2.** $\frac{5}{3}$ [1.5A] **3.** $\frac{25}{11}$ [1.5B] **4.** $-\frac{22}{9}$ [2.1A] **5.** $5x - 3xy$ [2.2A] **6.** $-9x$ [2.2B]

7. $-18x + 12$ [2.2D] **8.** -16 [3.1C] **9.** -16 [3.3A] **10.** 15 [3.3B] **11.** 22% [3.1D]

12. $4b^3 - 4b^2 - 8b - 4$ [4.1A] **13.** $3y^3 + 2y^2 - 10y$ [4.1B] **14.** a^9b^{15} [4.2B] **15.** $-8x^3y^6$ [4.2A]

16. $6y^4 + 8y^3 - 16y^2$ [4.3A] **17.** $10a^3 - 39a^2 + 20a - 21$ [4.3B] **18.** $15b^2 - 31b + 14$ [4.3C] **19.** $\frac{1}{2b^2}$ [4.4A]

20. $a - 7$ [4.5B] **21.** 0.0000609 [4.4B] **22.** $8x - 2x = 18; 3$ [3.4B] **23.** The resulting mixture is 28% orange juice. [3.6B] **24.** The car overtakes the cyclist 25 mi from the starting point. [3.6C]

25. The length is 15 m. The width is 6 m. [3.5A]

Answers to Chapter 5 Odd-Numbered Exercises

PREP TEST

1. $2 \cdot 3 \cdot 5$ [1.4A] **2.** $-12y + 15$ [2.2C] **3.** $-a + b$ [2.2C] **4.** $-3a + 3b$ [2.2D] **5.** 0 [3.1C] **6.** $-\frac{1}{2}$ [3.2A] **7.** $x^2 - 2x - 24$ [4.3C] **8.** $6x^2 - 11x - 10$ [4.3C] **9.** x^3 [4.4A] **10.** $3x^3y$ [4.4A]

SECTION 5.1

3. $5(a + 1)$ **5.** $8(2 - a^2)$ **7.** $4(2x + 3)$ **9.** $6(5a - 1)$ **11.** $x(7x - 3)$ **13.** $a^2(3 + 5a^3)$ **15.** $y(14y + 11)$ **17.** $2x(x^3 - 2)$ **19.** $2x^2(5x^2 - 6)$ **21.** $4a^5(2a^3 - 1)$ **23.** $xy(xy - 1)$ **25.** $3xy(xy^3 - 2)$ **27.** $xy(x - y^2)$ **29.** $5y(y^2 - 4y + 1)$ **31.** $3y^2(y^2 - 3y - 2)$ **33.** $3y(y^2 - 3y + 8)$ **35.** $a^2(6a^3 - 3a - 2)$ **37.** $ab(2a - 5ab + 7b)$ **39.** $2b(2b^4 + 3b^2 - 6)$ **41.** $x^2(8y^2 - 4y + 1)$ **43.** $(y + 7)(a + z)$ **45.** $(3r + s)(a - b)$ **47.** $(t - 7)(m - 7)$ **49.** $(2y + 1)(4a - b)$ **51.** $(x + 2)(x + 2y)$ **53.** $(p - 2)(p - 3r)$ **55.** $(b - 4)(a + 6)$ **57.** $(z + y)(2z - 1)$ **59.** $(4v + 7)(2v - 3y)$ **61.** $(2x - 5)(x - 3y)$ **63.** $(3y - a)(y - 2)$ **65.** $(y + 1)(3x - y)$ **67.** $(t - 2)(3s + t)$ **69.** 28 is the one perfect number between 20 and 30. **71.** In the equation for the perimeter of a rectangle, when $L + W$ doubles, the perimeter doubles.

SECTION 5.2

1. the same **3.** $(x + 1)(x + 2)$ **5.** $(x - 2)(x + 1)$ **7.** $(a + 4)(a - 3)$ **9.** $(a - 1)(a - 2)$ **11.** $(a + 2)(a - 1)$ **13.** $(b - 3)(b - 3)$ **15.** $(b + 8)(b - 1)$ **17.** $(y + 11)(y - 5)$ **19.** $(y - 3)(y - 2)$ **21.** $(z - 5)(z - 9)$ **23.** $(z - 20)(z + 8)$ **25.** $(p + 3)(p + 9)$ **27.** $(x + 10)(x + 10)$ **29.** $(b + 4)(b + 5)$ **31.** $(x + 3)(x - 14)$ **33.** $(b - 5)(b + 4)$ **35.** $(y - 17)(y + 3)$ **37.** $(p - 7)(p + 3)$ **39.** Nonfactorable over the integers **41.** $(x - 5)(x - 15)$ **43.** $(p + 3)(p + 21)$ **45.** $(x + 2)(x + 19)$ **47.** $(x + 9)(x - 4)$ **49.** $(a + 4)(a - 11)$ **51.** $(a - 3)(a - 18)$ **53.** $(z + 21)(z - 7)$ **55.** $(c + 12)(c - 15)$ **57.** $(p + 9)(p + 15)$ **59.** $(c + 2)(c + 9)$ **61.** $(x + 15)(x - 5)$ **63.** $(x + 25)(x - 4)$ **65.** $(b - 4)(b - 18)$ **67.** $(a + 45)(a - 3)$ **69.** $(b - 7)(b - 18)$ **71.** $(z + 12)(z + 12)$ **73.** $(x - 4)(x - 25)$ **75.** $(x + 16)(x - 7)$ **77.** $3(x + 2)(x + 3)$ **79.** $-(x + 6)(x - 2)$ **81.** $a(b + 8)(b - 1)$ **83.** $x(y + 3)(y + 5)$ **85.** $-2a(a + 2)(a + 1)$ **87.** $4y(y + 6)(y - 3)$ **89.** $2x(x^2 - x + 2)$ **91.** $6(z + 5)(z - 3)$ **93.** $3a(a + 3)(a - 6)$ **95.** $(x + 7y)(x - 3y)$ **97.** $(a - 5b)(a - 10b)$ **99.** $(s + 8t)(s - 6t)$ **101.** Nonfactorable over the integers **103.** $z^2(z + 10)(z - 8)$ **105.** $b^2(b + 2)(b - 5)$ **107.** $3y^2(y + 3)(y + 15)$ **109.** $-x^2(x - 12)(x + 1)$ **111.** $3y(x + 3)(x - 5)$ **113.** $-3x(x - 3)(x - 9)$ **115.** $(x - 3y)(x - 5y)$ **117.** $(a - 6b)(a - 7b)$ **119.** $(y + z)(y + 7z)$ **121.** $3y(x + 21)(x - 1)$ **123.** $3x(x + 4)(x - 3)$ **125.** $4z(z + 11)(z - 3)$ **127.** $4x(x + 3)(x - 1)$ **129.** $5(p + 12)(p - 7)$ **131.** $p^2(p + 12)(p - 3)$ **133.** $(t - 5s)(t - 7s)$ **135.** $(a + 3b)(a - 11b)$ **137.** $y(x + 6)(x - 9)$ **139.** $-36, -12, 12, 36$ **141.** $-22, -10, 10, 22$ **143.** 6, 10, 12 **145.** 6, 10, 12 **147.** 4, 6

SECTION 5.3

1. $(2x + 1)(x + 1)$ **3.** $(2y + 1)(y + 3)$ **5.** $(2a - 1)(a - 1)$ **7.** $(2b - 1)(b - 5)$ **9.** $(2x - 1)(x + 1)$ **11.** $(2x + 1)(x - 3)$ **13.** $(2t - 5)(t + 2)$ **15.** $(3p - 1)(p - 5)$ **17.** $(4y - 1)(3y - 1)$ **19.** Nonfactorable over the integers **21.** $(2t - 1)(3t - 4)$ **23.** $(8x + 1)(x + 4)$ **25.** Nonfactorable over the integers **27.** $(4y + 5)(3y + 1)$ **29.** $(7a - 2)(a + 7)$ **31.** $(3b - 4)(b - 4)$ **33.** $(2z + 1)(z - 14)$ **35.** $(3p - 2)(p + 8)$ **37.** $2(2x + 1)(x + 1)$ **39.** $5(3y - 7)(y - 1)$ **41.** $x(2x - 1)(x - 5)$ **43.** $b(3a - 4)(a - 4)$ **45.** Nonfactorable over the integers **47.** $-3x(x + 4)(x - 3)$ **49.** $4(4y - 1)(5y - 1)$ **51.** $z(4z + 1)(2z + 3)$ **53.** $y(3x + 2)(2x - 5)$ **55.** $5(2t - 5)(t + 2)$ **57.** $p(3p - 1)(p - 5)$ **59.** $2(13z - 3)(z + 4)$ **61.** $2y(5y - 2)(y - 4)$ **63.** $yz(4z - 3)(z + 2)$ **65.** $3a(2a + 3)(7a - 3)$ **67.** $y(3x - 5y)(3x - 5y)$ **69.** $xy(3x - 4y)(3x - 4y)$ **71.** $(3x - 4)(2x - 3)$ **73.** $(5b - 2)(b + 7)$ **75.** $(3a + 8)(2a - 3)$ **77.** $(4z + 3)(z + 2)$ **79.** $(2p + 5)(11p - 2)$ **81.** $(8y + 9)(y + 1)$ **83.** $(3t + 1)(6t - 5)$ **85.** $(6b - 1)(b + 12)$ **87.** $(3x + 2)(3x + 2)$ **89.** $(3b - 2)(2b - 3)$ **91.** $(11b - 7)(3b + 5)$ **93.** $(3y - 4)(6y - 5)$ **95.** $(3a + 7)(5a - 3)$ **97.** $(2y - 5)(4y - 3)$ **99.** $(4z - 5)(2z + 3)$ **101.** Nonfactorable over the integers **103.** $(2z - 5)(5z - 2)$ **105.** $(6z + 7)(6z + 5)$ **107.** $(3x - 2y)(x + y)$ **109.** $(3a - b)(a + 2b)$

111. $(4y - 3z)(y - 2z)$ **113.** $-(z - 7)(z + 4)$ **115.** $-(x - 1)(x + 8)$ **117.** $3(3x - 4)(x + 5)$ **119.** $4(2x - 3)(3x - 2)$ **121.** $a^2(7a - 1)(5a + 2)$ **123.** $5(3b - 2)(b - 7)$ **125.** $(3x - 5y)(x - 7y)$ **127.** $3(8y - 1)(9y + 1)$ **129.** $-(x - 1)(x + 21)$ **133.** $x(x - 1)$ **135.** $(2y + 1)(y + 3)$ **137.** $(4y - 3)(y - 3)$ **139.** $-1, 1, -5, 5$ **141.** $-1, 1, -5, 5$ **143.** $-3, 3, -9, 9$

SECTION 5.4

1. Answers will vary. For instance, **(a)** $x^2 - 25$; **(b)** $x^2 + 6x + 9$ **3.** $(x + 2)(x - 2)$ **5.** $(a + 9)(a - 9)$ **7.** $(y + 1)^2$ **9.** $(a - 1)^2$ **11.** $(2x + 1)(2x - 1)$ **13.** $(x^3 + 3)(x^3 - 3)$ **15.** Nonfactorable over the integers **17.** $(x + y)^2$ **19.** $(2a + 1)^2$ **21.** $(3x + 1)(3x - 1)$ **23.** $(1 + 8x)(1 - 8x)$ **25.** Nonfactorable over the integers **27.** $(3a + 1)^2$ **29.** $(b^2 + 4a)(b^2 - 4a)$ **31.** $(2a - 5)^2$ **33.** $(3a - 7)^2$ **35.** $(5z + y)(5z - y)$ **37.** $(ab + 5)(ab - 5)$ **39.** $(5x + 1)(5x - 1)$ **41.** $(2a - 3b)^2$ **43.** $(2y - 9z)^2$ **45.** $\left(\frac{1}{x} + 2\right)\left(\frac{1}{x} - 2\right)$ **47.** $(3ab - 1)^2$ **49.** $2(2y + 1)(2y - 1)$ **51.** $3a(a + 1)^2$ **53.** $(m^2 + 16)(m + 4)(m - 4)$ **55.** $(9x + 4)(x + 1)$ **57.** $4y^2(2y + 3)^2$ **59.** $(y^4 + 9)(y^2 + 3)(y^2 - 3)$ **61.** $(5 - 2p)^2$ **63.** $(4x - 3 + y)(4x - 3 - y)$ **65.** $(x - 2 + y)(x - 2 - y)$ **67.** $5(x + 1)(x - 1)$ **69.** $x(x + 2)^2$ **71.** $x^2(x + 7)(x - 5)$ **73.** $5(b + 3)(b + 12)$ **75.** Nonfactorable over the integers **77.** $2y(x + 11)(x - 3)$ **79.** $x(x^2 - 6x - 5)$ **81.** $3(y^2 - 12)$ **83.** $(2a + 1)(10a + 1)$ **85.** $y^2(x - 8)(x + 1)$ **87.** $5(2a - 3b)(a + b)$ **89.** $-2(x + 5)(x - 5)$ **91.** $b^2(a - 5)^2$ **93.** $ab(4a + b)(3a - b)$ **95.** $3a(2a - 1)^2$ **97.** $3(81 + a^2)$ **99.** $2a(2a - 5)(3a - 4)$ **101.** $a(2a + 5)^2$ **103.** $3b(3a - 1)^2$ **105.** $-6(x + 4)(x - 2)$ **107.** $x^2(x + y)(x - y)$ **109.** $2a(3a + 2)^2$ **111.** $-b(3a - 2)(2a + 1)$ **113.** $2x^2(2x - 3)(x - 8)$ **115.** $x^2(x + 5)(x - 5)$ **117.** $(a^2 + 4)(a + 2)(a - 2)$ **119.** $-3y^2(2y + 5)(4y - 3)$ **121.** $2(x - 3)(2a - b)$ **123.** $(y + 1)(y - 1)(a - b)$ **125.** $(x - y)(a + b)(a - b)$ **127.** $-12, 12$ **129.** $-16, 16$ **131.** $-10, 10$

SECTION 5.5

3. -3 and -2 **5.** 7 and 3 **7.** 0 and 5 **9.** 0 and 9 **11.** 0 and $-\frac{3}{2}$ **13.** 0 and $\frac{2}{3}$ **15.** -2 and 5 **17.** 9 and -9 **19.** $\frac{7}{2}$ and $-\frac{7}{2}$ **21.** $\frac{1}{3}$ and $-\frac{1}{3}$ **23.** -2 and -4 **25.** -7 and 2 **27.** $-\frac{1}{2}$ and 5 **29.** $-\frac{1}{3}$ and $-\frac{1}{2}$ **31.** 0 and 3 **33.** 0 and 7 **35.** -1 and -4 **37.** 2 and 3 **39.** $\frac{1}{2}$ and -4 **41.** $\frac{1}{3}$ and 4 **43.** 3 and 9 **45.** 9 and -2 **47.** -1 and -2 **49.** -9 and 5 **51.** -7 and 4 **53.** -2 and -3 **55.** -8 and 9 **57.** 1 and 4 **59.** -5 and 2 **61.** The number is 6. **63.** The numbers are 2 and 4. **65.** The numbers are 4 and 5. **67.** The numbers are 3 and 7. **69.** There will be 12 consecutive natural numbers. **71.** There are 6 teams. **73.** The object will hit the ground 3 s later. **75.** The ball will return to the ground 3.75 s later. **77.** The length is 15 in. and the width is 5 in. **79.** The height of the triangle is 14 m. **81.** The dimensions of the type area are 4 in. by 7 in. **83.** The radius of the original circle is approximately 3.81 in. **87.** 3, 48 **89.** $-\frac{3}{2}, -5$ **91.** 0, 9

CHAPTER REVIEW

1. $(b - 3)(b - 10)$ [5.2A] **2.** $(x - 3)(4x + 5)$ [5.1B] **3.** Nonfactorable over the integers [5.3A] **4.** $5x(x^2 + 2x + 7)$ [5.1A] **5.** $7y^3(2y^6 - 7y^3 + 1)$ [5.1A] **6.** $(y - 4)(y + 9)$ [5.2A] **7.** $(2x - 7)(3x - 4)$ [5.3A] **8.** $3ab(4a + b)$ [5.1A] **9.** $(a^3 + 10)(a^3 - 10)$ [5.4A] **10.** $n^2(n + 1)(n - 3)$ [5.2B] **11.** $(6y - 1)(2y + 3)$ [5.3A] **12.** $2b(3b - 4)(2b - 7)$ [5.4B] **13.** $(3y^2 + 5z)(3y^2 - 5z)$ [5.4A] **14.** $(c + 6)(c + 2)$ [5.2A] **15.** $(6a - 5)(3a + 2)$ [5.3B] **16.** $\frac{1}{4}$ and -7 [5.5A] **17.** $4x(x - 6)(x + 1)$ [5.2B] **18.** $3(a + 2)(a - 7)$ [5.2B] **19.** $(2a + 5)(a - 12)$ [5.3B] **20.** -3 and 7 [5.5A] **21.** $(3a - 5b)(7x + 2y)$ [5.1B] **22.** $(ab + 1)(ab - 1)$ [5.4A] **23.** $(2x + 5)(5x + 2y)$ [5.1B] **24.** $5(x + 2)(x - 3)$ [5.2B] **25.** $3(x + 6)^2$ [5.4B] **26.** $(3x - 2)(x - 5)$ [5.3B] **27.** The length is 100 yd. The width is 60 yd. [5.5B] **28.** The distance between the screen and the projector is 20 ft. [5.5B] **29.** The width of the frame is 1.5 in., or $1\frac{1}{2}$ in. [5.5B] **30.** The length of a side is 20 ft. [5.5B]

CHAPTER TEST

1. $(b + 6)(a - 3)$ [5.1B] **2.** $2y^2(y + 1)(y - 8)$ [5.2B] **3.** $4(x + 4)(2x - 3)$ [5.3B] **4.** $(2x + 1)(3x + 8)$ [5.3A]
5. $(a - 3)(a - 16)$ [5.2A] **6.** $2x(3x^2 - 4x + 5)$ [5.1A] **7.** $(x + 5)(x - 3)$ [5.2A] **8.** $\frac{1}{2}$ and $-\frac{1}{2}$ [5.5A]
9. $5(x^2 - 9x - 3)$ [5.1A] **10.** $(p + 6)^2$ [5.4A] **11.** 3 and 5 [5.5A] **12.** $3(x + 2y)^2$ [5.4B]
13. $(b + 4)(b - 4)$ [5.4A] **14.** $3y^2(2x + 1)(x + 1)$ [5.3B] **15.** $(p + 2)(p + 3)$ [5.2A] **16.** $(x - 2)(a + b)$ [5.1B]
17. $(p + 1)(x - 1)$ [5.1B] **18.** $3(a + 5)(a - 5)$ [5.4B] **19.** Nonfactorable over the integers [5.3A]
20. $(x + 3)(x - 12)$ [5.2A] **21.** $(2a - 3b)^2$ [5.4A] **22.** $(2x + 7y)(2x - 7y)$ [5.4A] **23.** $\frac{3}{2}$ and -7 [5.5A]
24. The two numbers are 3 and 7. [5.5B] **25.** The length is 15 cm. The width is 6 cm. [5.5B]

CUMULATIVE REVIEW

1. 7 [1.2B] **2.** 4 [1.5B] **3.** -7 [2.1A] **4.** $15x^2$ [2.2B] **5.** 12 [2.2D] **6.** $\frac{2}{3}$ [3.1C] **7.** $\frac{7}{4}$ [3.3A]
8. 3 [3.3B] **9.** 45 [3.1D] **10.** $9a^6b^4$ [4.2B] **11.** $x^3 - 3x^2 - 6x + 8$ [4.3B] **12.** $4x + 8 + \frac{21}{2x - 3}$ [4.5B]
13. $\frac{y^6}{x^8}$ [4.4A] **14.** $(a - b)(3 - x)$ [5.1B] **15.** $5xy^2(3 - 4y^2)$ [5.1A] **16.** $(x - 7y)(x + 2y)$ [5.2A]
17. $(p - 10)(p + 1)$ [5.2A] **18.** $3a(2a + 5)(3a + 2)$ [5.3A/5.3B] **19.** $(6a - 7b)(6a + 7b)$ [5.4A]
20. $(2x + 7y)^2$ [5.4A] **21.** $(3x - 2)(3x + 7)$ [5.3A] **22.** $2(3x - 4y)^2$ [5.4B] **23.** $(x - 3)(3y - 2)$ [5.1B]
24. $\frac{2}{3}$ and -7 [5.5A] **25.** The pieces measure 4 ft and 6 ft. [3.4B] **26.** The discount rate is 40%. [3.2B]
27. The measure of angle a is 72° and the measure of angle b is 108°. [3.5B] **28.** The distance to the resort was 168 mi. [3.6C] **29.** The integers are 10, 12, and 14. [3.4A] **30.** The length of the base is 12 in. [5.5B]

Answers to Chapter 6 Odd-Numbered Exercises

PREP TEST

1. 36 [1.4C] **2.** $\frac{3x}{y^3}$ [4.4A] **3.** $-\frac{5}{36}$ [1.4C] **4.** $-\frac{10}{11}$ [1.4D] **5.** No. $\frac{0}{a} = 0$ but $\frac{a}{0}$ is undefined. [1.4D]
6. $\frac{19}{8}$ [3.2A] **7.** 130° [3.5B] **8.** $(x - 6)(x + 2)$ [5.2A] **9.** $(2x - 3)(x + 1)$ [5.3A] **10.** 9:40 A.M. [3.6C]

SECTION 6.1

3. $\frac{3}{4x}$ **5.** $\frac{1}{x + 3}$ **7.** -1 **9.** $\frac{2}{3y}$ **11.** $-\frac{3}{4x}$ **13.** $\frac{a}{b}$ **15.** $-\frac{2}{x}$ **17.** $\frac{y - 2}{y - 3}$ **19.** $\frac{x + 5}{x + 4}$ **21.** $\frac{x + 4}{x - 3}$
23. $-\frac{x + 2}{x + 5}$ **25.** $\frac{2(x + 2)}{x + 3}$ **27.** $\frac{2x - 1}{2x + 3}$ **29.** $-\frac{x + 7}{x + 6}$ **31.** $\frac{35ab^2}{24x^2y}$ **33.** $\frac{4x^3y^3}{3a^2}$ **35.** $\frac{3}{4}$ **37.** ab^2
39. $\frac{x^2(x - 1)}{y(x + 3)}$ **41.** $\frac{y(x - 1)}{x^2(x + 10)}$ **43.** $-ab^2$ **45.** $\frac{x + 5}{x + 4}$ **47.** 1 **49.** $-\frac{n - 10}{n - 7}$ **51.** $\frac{x(x + 2)}{2(x - 1)}$ **53.** $-\frac{x + 2}{x - 6}$
55. $\frac{x + 5}{x - 12}$ **59.** $\frac{7a^3y^2}{40bx}$ **61.** $\frac{4}{3}$ **63.** $\frac{3a}{2}$ **65.** $\frac{x^2(x + 4)}{y^2(x + 2)}$ **67.** $\frac{x(x - 2)}{y(x - 6)}$ **69.** $-\frac{3by}{ax}$ **71.** $\frac{(x + 6)(x - 3)}{(x + 7)(x - 6)}$
73. 1 **75.** $-\frac{x + 8}{x - 4}$ **77.** $\frac{2n + 1}{2n - 3}$ **81.** $\frac{x}{x + 8}$ **83.** $\frac{n - 2}{n + 3}$

SECTION 6.2

1. $24x^3y^2$ **3.** $30x^4y^2$ **5.** $8x^2(x + 2)$ **7.** $6x^2y(x + 4)$ **9.** $36x(x + 2)^2$ **11.** $6(x + 1)^2$
13. $(x - 1)(x + 2)(x + 3)$ **15.** $(2x + 3)^2(x - 5)$ **17.** $(x - 1)(x - 2)$ **19.** $(x - 3)(x + 2)(x + 4)$
21. $(x + 4)(x + 1)(x - 7)$ **23.** $(x - 6)(x + 6)(x + 4)$ **25.** $(x - 10)(x - 8)(x + 3)$ **27.** $(3x - 2)(x - 3)(x + 2)$

29. $(x+2)(x-3)$ **31.** $(x-5)(x+1)$ **33.** $(x-3)(x-2)(x-1)(x-6)$ **35.** $\frac{5}{ab^2}, \frac{6b}{ab^2}$ **37.** $\frac{15y^2}{18x^2y}, \frac{14x}{18x^2y}$

39. $\frac{ay+5a}{y^2(y+5)}, \frac{6y}{y^2(y+5)}$ **41.** $\frac{a^2y+7a^2}{y(y+7)^2}, \frac{ay}{y(y+7)^2}$ **43.** $\frac{b}{y(y-4)}, -\frac{b^2y}{y(y-4)}$ **45.** $-\frac{3y-21}{(y-7)^2}, \frac{2}{(y-7)^2}$

47. $\frac{2y^2}{y^2(y-3)}, \frac{3}{y^2(y-3)}$ **49.** $\frac{x^3+4x^2}{(2x-1)(x+4)}, \frac{2x^2+x-1}{(2x-1)(x+4)}$ **51.** $\frac{3x^2+15x}{(x-5)(x+5)}, \frac{4}{(x-5)(x+5)}$

53. $\frac{x^2-1}{(x+5)(x-3)(x+1)}, \frac{x^2-3x}{(x+5)(x-3)(x+1)}$ **55.** $\frac{800}{10^5}, \frac{9}{10^5}$ **57.** $\frac{x^3-x}{x^2-1}, \frac{x}{x^2-1}$

59. $\frac{3c^2-3cd}{3(6c+d)(c+d)(c-d)}, \frac{6cd+d^2}{3(6c+d)(c+d)(c-d)}$

SECTION 6.3

1. $\frac{11}{y^2}$ **3.** $-\frac{7}{x+4}$ **5.** $\frac{8x}{2x+3}$ **7.** $\frac{5x+7}{x-3}$ **9.** $\frac{2x-5}{x+9}$ **11.** $\frac{-3x-4}{2x+7}$ **13.** $\frac{1}{x+5}$ **15.** $\frac{1}{x-6}$ **17.** $\frac{3}{2y-1}$

19. $\frac{1}{x-5}$ **23.** $\frac{4y+5x}{xy}$ **25.** $\frac{19}{2x}$ **27.** $\frac{5}{12x}$ **29.** $\frac{19x-12}{6x^2}$ **31.** $\frac{52y-35x}{20xy}$ **33.** $\frac{13x+2}{15x}$ **35.** $\frac{7}{24}$

37. $\frac{x+90}{45x}$ **39.** $\frac{x^2+2x+2}{2x^2}$ **41.** $\frac{2x^2+3x-10}{4x^2}$ **43.** $\frac{-x^2-4x+4}{x+4}$ **45.** $\frac{4x+7}{x+1}$ **47.** $\frac{4x^2+9x+9}{24x^2}$

49. $\frac{3x-1-2xy-3y}{xy^2}$ **51.** $\frac{20x^2+28x-12xy+9y}{24x^2y^2}$ **53.** $\frac{9x^2-3x-2xy-10y}{18xy^2}$ **55.** $\frac{7x-23}{(x-3)(x-4)}$

57. $\frac{-y-33}{(y+6)(y-3)}$ **59.** $\frac{3x^2+20x-8}{(x-4)(x+6)}$ **61.** $\frac{3(4x^2+5x-5)}{(x+5)(2x+3)}$ **63.** $\frac{-4x+5}{x-6}$ **65.** $\frac{2(y+2)}{(y-4)(y+4)}$ **67.** $-\frac{4x}{(x+1)^2}$

69. $\frac{2x-1}{(1+x)(1-x)}$ **71.** $\frac{14}{(x-5)^2}$ **73.** $\frac{-2(x+7)}{(x-7)(x+6)}$ **75.** $\frac{x-4}{x-6}$ **77.** $\frac{2x+1}{x-1}$ **79.** $\frac{-3(x^2+8x+25)}{(x-3)(x+7)}$

81a. $\frac{19{,}200}{x}$ dollars **b.** $\frac{96{,}000}{x(x+5)}$ dollars **c.** \$128

SECTION 6.4

1. $\frac{x}{x-3}$ **3.** $\frac{2}{3}$ **5.** $\frac{y+3}{y-4}$ **7.** $\frac{2(2x+13)}{5x+36}$ **9.** $\frac{x+2}{x+3}$ **11.** $\frac{x-6}{x+5}$ **13.** $\frac{-x+2}{x+1}$ **15.** $x-1$ **17.** $\frac{1}{2x-1}$

19. $\frac{x-3}{x+5}$ **21.** $\frac{x-7}{x-8}$ **23.** $\frac{2y-1}{2y+1}$ **25.** $\frac{x-2}{2x-5}$ **27.** $\frac{-x-1}{4x-3}$ **29.** $\frac{x+1}{2(5x-2)}$ **31.** $\frac{5}{3}$ **33.** $-\frac{1}{x-1}$

35. $\frac{y+4}{2(y-2)}$ **37.** $\frac{x+1}{x-1}$ **39.** $\frac{y^2+x^2}{xy}$

SECTION 6.5

3. 3 **5.** 1 **7.** 9 **9.** 1 **11.** $\frac{1}{4}$ **13.** 1 **15.** −3 **17.** $\frac{1}{2}$ **19.** 8 **21.** 5 **23.** −1 **25.** 5

27. No solution **29.** 2 and 4 **31.** 4 and $-\frac{3}{2}$ **33.** 3 **35.** 4 **37.** 0 **39.** $-\frac{2}{5}$ **41.** 0 and $-\frac{2}{3}$

SECTION 6.6

3. 9 **5.** 12 **7.** 7 **9.** 6 **11.** 1 **13.** −6 **15.** 4 **17.** $-\frac{2}{3}$ **19.** 20,000 voters voted in favor.

21. The two cities are 175 mi apart. **23.** The sales tax will be \$97.50 higher. **25.** 67.5 in. **27.** There are an estimated 75 elk in the preserve. **29.** The length of side AC is 6.7 cm. **31.** The height of triangle ABC is 2.9 m. **33.** The perimeter of triangle DEF is 22.5 ft. **35.** The area of triangle ABC is 48 m². **37.** The length of BC is 6.25 cm. **39.** The length of DA is 6 in. **41.** The length of OP is 13 cm. **43.** The width of the river is 35 m. **45.** The first person's share of the winnings is \$1.25 million. **47.** The basketball player made 210 shots.

SECTION 6.7

1. $y=-3x+10$ **3.** $y=4x-3$ **5.** $y=-\frac{3}{2}x+3$ **7.** $y=\frac{2}{5}x-2$ **9.** $y=-\frac{2}{7}x+2$ **11.** $y=-\frac{1}{3}x+2$

13. $y = 3x + 8$ **15.** $y = -\frac{2}{3}x - 3$ **17.** $x = -6y + 10$ **19.** $x = \frac{1}{2}y + 3$ **21.** $x = -\frac{3}{4}y + 3$ **23.** $x = 4y + 3$

25. $t = \frac{d}{r}$ **27.** $T = \frac{PV}{nR}$ **29.** $l = \frac{P - 2w}{2}$ **31.** $b_1 = \frac{2A - hb_2}{h}$ **33.** $h = \frac{3V}{A}$ **35.** $S = C - Rt$ **37.** $P = \frac{A}{1 + rt}$

39. $w = \frac{A}{S + 1}$ **41a.** $S = \frac{F + BV}{B}$ **b.** The selling price per unit to break even is $180. **c.** The selling price per unit to break even is $75.

SECTION 6.8

3. It would take 2 h with both sprinklers working. **5.** It would take both skiploaders 3 h working together. **7.** It would take 30 h with both computers working. **9.** It would take 30 min with both air conditioners working. **11.** It would take the second pipeline 90 min to fill the tank. **13.** It would take the apprentice 15 h to construct the wall. **15.** It would take the second technician 3 h to complete the wiring. **17.** It would have taken one welder 40 h to complete the welds. **19.** It would take one machine 28 h working alone. **21.** In 2 h, the jogger ran 16 mi. **23.** The rate through the congested traffic is 20 mph. **25.** The rate of the jogger is 8 mph, and the rate of the cyclist is 20 mph. **27.** The rate of the jet is 360 mph. **29.** Her walking speed is 4 mph. **31.** The rate of the car is 48 mph. **33.** The rate of the wind is 20 mph. **35.** The rate of the gulf current is 6 mph. **37.** The rate of the trucker for the first 330 mi is 55 mph. **39.** The bus usually travels 60 mph.

CHAPTER REVIEW

1. $\frac{b^3y}{10ax}$ [6.1C] **2.** $\frac{7x + 22}{60x}$ [6.3B] **3.** $\frac{x - 2}{3x - 10}$ [6.4A] **4.** $-\frac{x + 6}{x + 3}$ [6.1A] **5.** $\frac{2x^4}{3y^7}$ [6.1A]

6. 62 [6.6A] **7.** $\frac{(3y - 2)^2}{(y - 1)(y - 2)}$ [6.1C] **8.** $\frac{by^3}{6ax^2}$ [6.1B] **9.** $\frac{x}{x - 7}$ [6.4A]

10. $\frac{3x^2 - x}{(2x + 3)(6x - 1)(3x - 1)}, \frac{24x^3 - 4x^2}{(2x + 3)(6x - 1)(3x - 1)}$ [6.2B] **11.** $a = \frac{T - 2bc}{2b + 2c}$ [6.7A] **12.** 2 [6.5A]

13. $c = \frac{100m}{i}$ [6.7A] **14.** The equation has no solution. [6.5A] **15.** $\frac{1}{x^2}$ [6.1C] **16.** $\frac{2y - 3}{5y - 7}$ [6.3B]

17. $\frac{1}{x + 3}$ [6.3A] **18.** $(5x - 3)(2x - 1)(4x - 1)$ [6.2A] **19.** $y = -\frac{4}{9}x + 2$ [6.7A] **20.** $\frac{2x + 1}{x + 2}$ [6.1B]

21. 5 [6.5A] **22.** $\frac{3x - 1}{x - 5}$ [6.3B] **23.** 10 [6.6A] **24.** 12 [6.6A] **25.** The length of QO is 15 cm. [6.6C] **26.** It would take 6 h to fill the pool using both hoses. [6.8A] **27.** The rate of the car is 45 mph. [6.8B] **28.** The rate of the wind is 20 mph. [6.8B] **29.** The pitcher's ERA is 1.35. [6.6B]

CHAPTER TEST

1. $\frac{x^2 - 4x + 5}{(x - 2)(x + 3)}$ [6.3B] **2.** -1 [6.6A] **3.** $\frac{(x - 5)(2x - 1)}{(x + 3)(2x + 5)}$ [6.1B] **4.** $\frac{2x^3}{3y^3}$ [6.1A] **5.** $t = \frac{d - s}{r}$ [6.7A]

6. 2 [6.5A] **7.** $-\frac{x + 5}{x + 1}$ [6.1A] **8.** $3(2x - 1)(x + 1)$ [6.2A] **9.** $\frac{5}{(2x - 1)(3x + 1)}$ [6.3B] **10.** $\frac{x + 5}{x + 4}$ [6.1C]

11. $\frac{x - 3}{x - 2}$ [6.4A] **12.** $\frac{3x + 6}{x(x - 2)(x + 2)}, \frac{x^2}{x(x - 2)(x + 2)}$ [6.2B] **13.** $\frac{2}{x + 5}$ [6.3A] **14.** $y = \frac{3}{8}x - 2$ [6.7A]

15. The equation has no solution. [6.5A] **16.** $\frac{x + 1}{x^3(x - 2)}$ [6.1B] **17.** The length of CE is 12.8 ft. [6.6C] **18.** Two additional pounds of salt are required. [6.6B] **19.** It would take 4 h to fill the pool with both pipes turned on. [6.8A] **20.** The rate of the wind is 20 mph. [6.8B] **21.** Fifty-four sprinklers are required. [6.6B]

CUMULATIVE REVIEW

1. $\frac{31}{30}$ [1.5B] **2.** 21 [2.1A] **3.** $5x - 2y$ [2.2A] **4.** $-8x + 26$ [2.2D] **5.** $-\frac{9}{2}$ [3.2A] **6.** -12 [3.3B]

7. 10 [3.1D] **8.** a^3b^7 [4.2A] **9.** $a^2 + ab - 12b^2$ [4.3C] **10.** $3b^3 - b + 2$ [4.5A] **11.** $x^2 + 2x + 4$ [4.5B]

12. $(4x + 1)(3x - 1)$ [5.3A/5.3B] **13.** $(y - 6)(y - 1)$ [5.2A] **14.** $a(2a - 3)(a + 5)$ [5.3B] **15.** $4(b - 5)(b + 5)$ [5.4A/5.4B] **16.** -3 and $\frac{5}{2}$ [5.5A] **17.** $\frac{2x^3}{3y^5}$ [6.1A] **18.** $-\frac{x - 2}{x + 5}$ [6.1A] **19.** 1 [6.1C] **20.** $\frac{3}{(2x - 1)(x + 1)}$ [6.3B] **21.** $\frac{x + 3}{x + 5}$ [6.4A] **22.** 4 [6.5A] **23.** 3 [6.6A] **24.** $t = \frac{f - v}{a}$ [6.7A] **25.** $5x - 13 = -8$; 1 [3.4A] **26.** The 120-gram alloy is 70% silver. [3.6B] **27.** The height is 6 in. and the base is 10 in. [5.5B] **28.** A policy of \$5000 would cost \$80. [6.6B] **29.** Working together, it would take the pipes 6 min to fill the tank. [6.8A] **30.** The rate of the current is 2 mph. [6.8B]

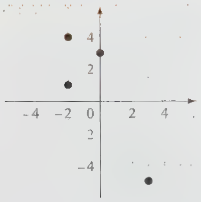
Answers to Chapter 7 Odd-Numbered Exercises

PREP TEST

1. 3 [1.5B] **2.** -1 [2.1A] **3.** $-3x + 12$ [2.2C] **4.** 2 [3.2A] **5.** 5 [3.2A] **6.** -2 [3.2A] **7.** $-4x + 5$ [4.5A] **8.** 4 [6.6A] **9.** $y = \frac{3}{5}x - 3$ [6.7A] **10.** $y = -\frac{1}{2}x - 5$ [6.7A]

SECTION 7.1

1.



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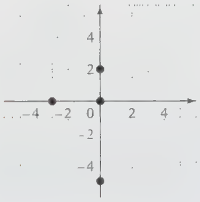

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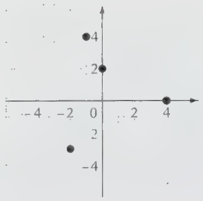
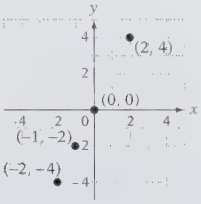


7. A is (2, 3), B is (4, 0), C is (−4, 1), and D is (−2, −2). **9.** A is (−2, 5), B is (3, 4), C is (0, 0), and D is (−3, −2). **11a.** The abscissa of point A is 2. The abscissa of point C is −4. **b.** The ordinate of point B is 1. The ordinate of point D is −3. **15.** Yes **17.** No **19.** No **21.** No **23.**



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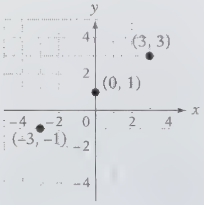

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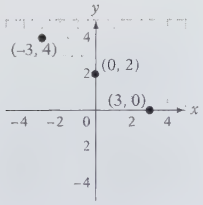
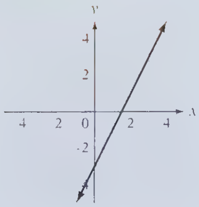


29. {(24, 600), (32, 750), (22, 430), (15, 300), (4.4, 68), (17, 370), (15, 310), (4.4, 55)}; No

31. {(15.7, 26), (15.3, 27), (9.0, 15), (7.9, 14), (7.6, 12), (7.1, 12), (7.1, 13)}; No **33.** Yes **35.** No **37.** Yes **39.** 8 **41.** 9 **43.** 2 **45.** −1 **47.** 22 **49.** $-\frac{3}{2}$ **51.** −7

SECTION 7.2

1.



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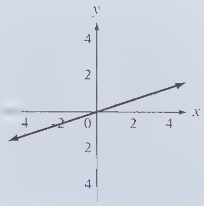

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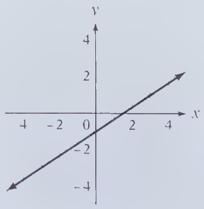

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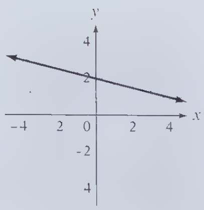

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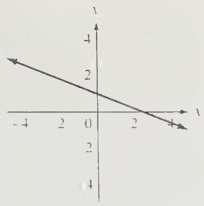

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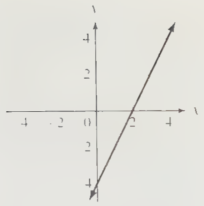

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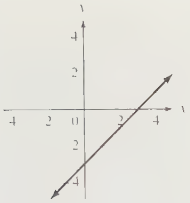

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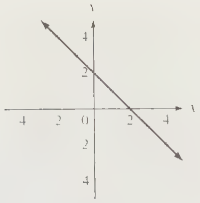

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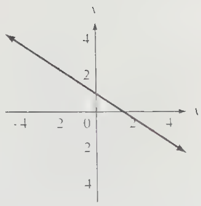

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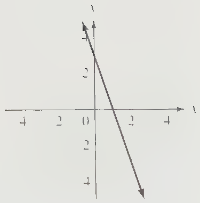
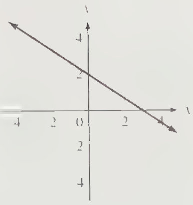


21.


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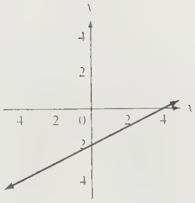

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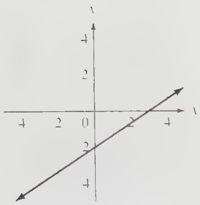

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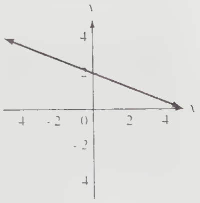

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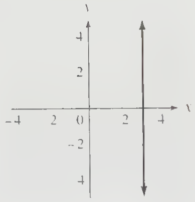

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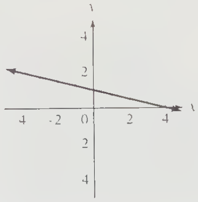

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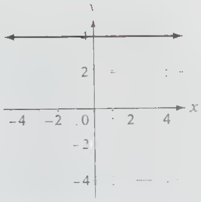

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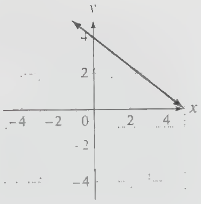
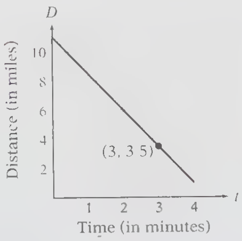
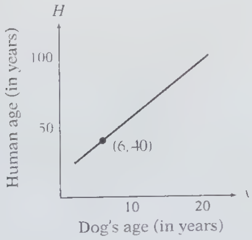


37. After flying for 3 min, the helicopter is 3.5 mi away from the victims.

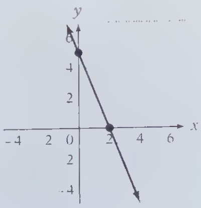


39. A dog 6 years old is equivalent in age to a human 40 years old.



SECTION 7.3

1. (3, 0), (0, −3) **3.** (2, 0), (0, −6) **5.** (10, 0), (0, −2) **7.** (−4, 0), (0, 12) **9.** (0, 0), (0, 0) **11.** (6, 0), (0, 3)

13.


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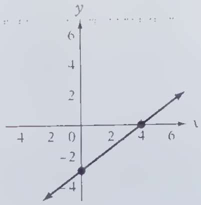

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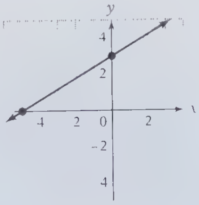
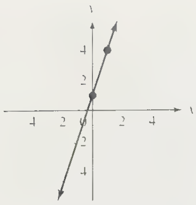


21. −2 **23.** $\frac{1}{3}$ **25.** $-\frac{5}{2}$

27. Undefined **29.** Zero **31.** $-\frac{1}{3}$ **33.** No **35.** Yes **37.** Yes **39.** $m = 620$. The sales of ebooks are increasing by $620 million per year. **41.** $m = -0.3$. The inflation rate declined 0.3% per year. **43.** $m = \frac{2}{3}$, (0, −2) **45.** $m = -\frac{2}{5}$, (0, 2) **47.** $m = \frac{1}{4}$, (0, 0)

49.



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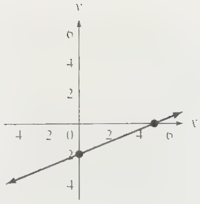

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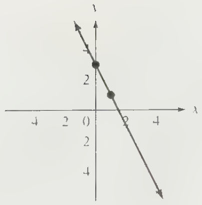

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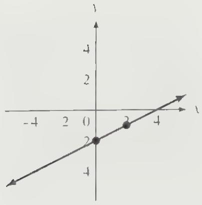

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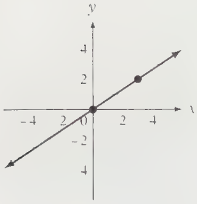

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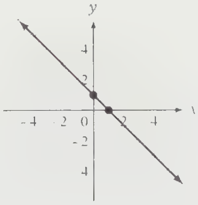

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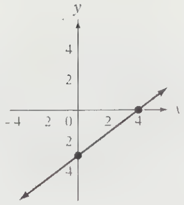

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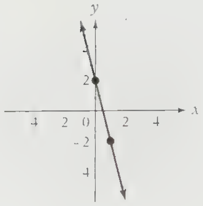


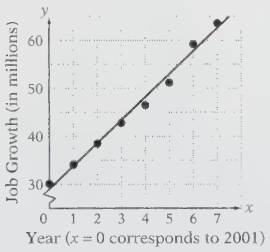
65. Yes

SECTION 7.4

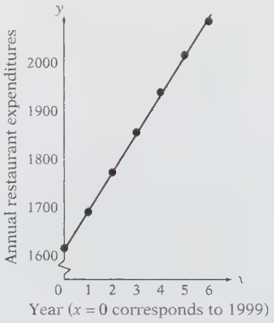
3. $y = 2x + 2$ **5.** $y = -3x - 1$ **7.** $y = \frac{1}{3}x$ **9.** $y = \frac{3}{4}x - 5$ **11.** $y = -\frac{3}{5}x$ **13.** $y = \frac{1}{4}x + \frac{5}{2}$

15. $y = 2x - 3$ **17.** $y = -2x - 3$ **19.** $y = \frac{2}{3}x$ **21.** $y = \frac{1}{2}x + 2$ **23.** $y = -\frac{3}{4}x - 2$ **25.** $y = \frac{3}{4}x + \frac{5}{2}$

27. The number of job openings for graduates with 4-year degrees is increasing by 4.8 million per year.



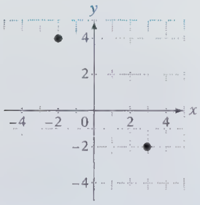
29. The amount spent by a family in restaurants is increasing by $80 per year.



31. No **33.** Yes **35.** $-\frac{3}{2}$ **37.** $n = -5$ **39.** $y = -\frac{2}{3}x + \frac{5}{3}$

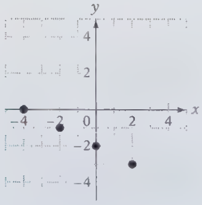
CHAPTER REVIEW

1. a.



b. The abscissa of point A is -2.
c. The ordinate of point B is -4. [7.1A]

2.

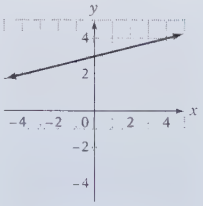


[7.1B]

3. $y = -\frac{8}{3}x + \frac{1}{3}$ [7.4B]

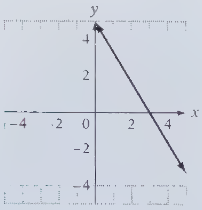
4. $y = -\frac{5}{2}x + 16$ [7.4A]

5.



[7.2A]

6.



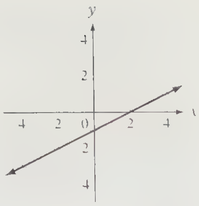
[7.2B]

7. No [7.3B]

8. -1 [7.1D] **9.** $y = -\frac{2}{3}x + \frac{11}{3}$ [7.4B] **10.** Yes [7.1C] **11.** $\frac{7}{11}$ [7.3B] **12.** $(8, 0)$, $(0, -12)$ [7.3A]

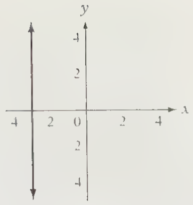
13. 0 [7.3B]

14.



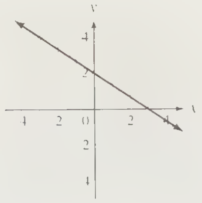
[7.3C]

15.



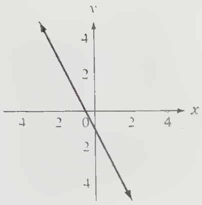
[7.2B]

16.



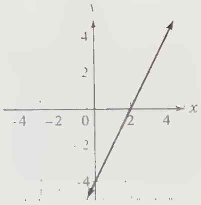
[7.3C]

17.



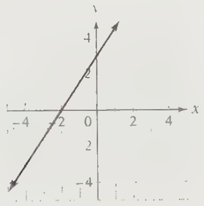
[7.2A]

18.



[7.3C]

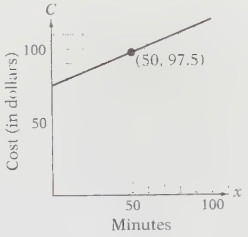
19.



[7.2B]

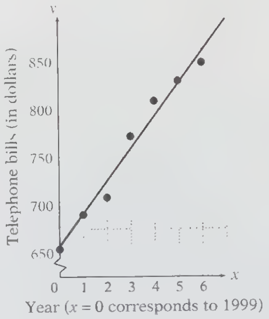
20. {(55, 95), (57,101), (53, 94), (57, 98), (60, 100), (61, 105), (58, 97), (54, 95)}; No [7.1C]

21. The cost of 50 min of access time for 1 month is $97.50.



[7.2C]

22. The average annual cost of telephone bills for a family is increasing by $34 per year.

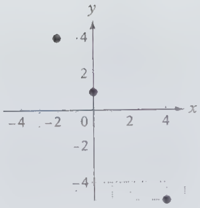


[7.4C]

CHAPTER TEST

1. (3, −3) [7.1B]

2.



[7.1B]

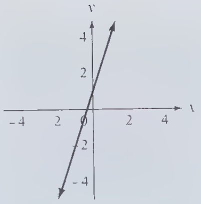
3. Yes [7.1C]

4. $f(2) = 6$ [7.1D]

5. $f(-1) = 3$ [7.1D]

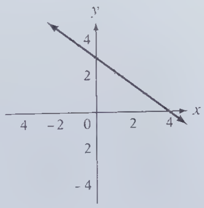
6. {(3.5, 25), (4.0, 30), (5.2, 45), (5.0, 38), (4.0, 42), (6.3, 12), (5.4, 34)}; No [7.1C]

7.



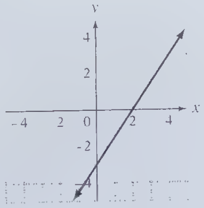
[7.2A]

8.



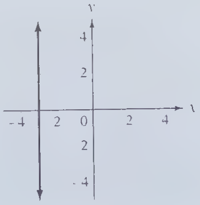
[7.2A]

9.



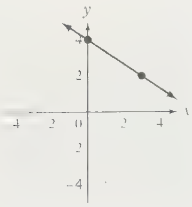
[7.2B]

10.



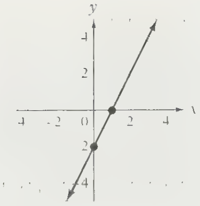
[7.2B]

11.



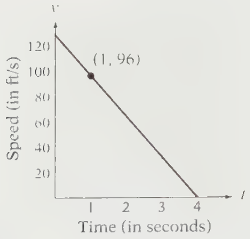
[7.3C]

12.



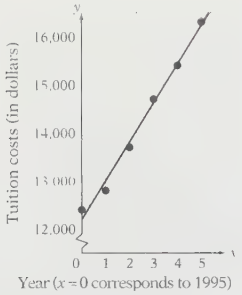
[7.3C]

13. After 1 s, the speed of the ball is 96 ft/s. [7.2C]



14. The slope of the line is 0.46. The average hourly wage is increasing by $.46 per year. [7.3B]

15.



The average annual tuition for a private 4-year college is increasing $809 per year. [7.4C]

16. (2, 0), (0, −3) [7.3A] **17.** (−2, 0), (0, 1) [7.3A] **18.** 2 [7.3B] **19.** Yes [7.3B] **20.** Undefined [7.3B]

21. $-\frac{2}{3}$ [7.3B] **22.** $y = 3x - 1$ [7.4A] **23.** $y = \frac{2}{3}x + 3$ [7.4A] **24.** $y = -\frac{5}{8}x - \frac{7}{8}$ [7.4B]

25. $y = -\frac{2}{7}x - \frac{4}{7}$ [7.4B]

CUMULATIVE REVIEW

1. −12 [1.5B] **2.** $-\frac{5}{8}$ [2.1A] **3.** $f(-2) = -\frac{2}{3}$ [7.1D] **4.** $\frac{3}{2}$ [3.2A] **5.** $\frac{19}{18}$ [3.3B] **6.** $\frac{1}{15}$ [1.4B]

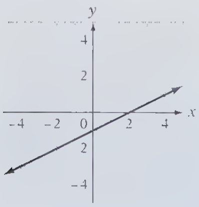
7. $-32x^8y^7$ [4.2B] **8.** $-3x^2$ [4.4A] **9.** $x + 3$ [4.5B] **10.** $5(x + 2)(x + 1)$ [5.2B] **11.** $(a + 2)(x + y)$ [5.1A]

12. 4 and −2 [5.5A] **13.** $\frac{x^3(x + 3)}{y(x + 2)}$ [6.1B] **14.** $\frac{3}{x + 8}$ [6.3A] **15.** 2 [6.5A] **16.** $y = \frac{4}{5}x - 3$ [6.7A]

17. (−2, −5) [7.1B] **18.** Zero [7.3B] **19.** $y = \frac{1}{2}x - 2$ [6.7A] **20.** $y = -3x + 2$ [7.4A]

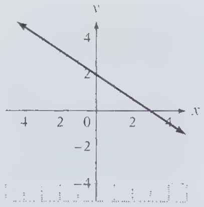
21. $y = 2x + 2$ [7.4A] **22.** $y = \frac{2}{3}x - 3$ [7.4A] **23.** The sale price is $62.30. [3.2B] **24.** The measure of the first angle is 46°, the measure of the second angle is 43°, and the measure of the third angle is 91°. [3.5C] **25.** The value of the home is $110,000. [6.6B] **26.** It would take $3\frac{3}{4}$ h with both the electrician and the apprentice working. [6.8A]

27.



[7.2A]

28.

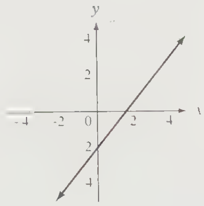


[7.3C]

Answers to Chapter 8 Odd-Numbered Exercises

PREP TEST

1. $y = \frac{3}{4}x - 6$ [6.7A] **2.** 0 [3.3B] **3.** $33y$ [2.2D] **4.** $10x$ 10 [2.2D] **5.** Yes [7.1B] **6.** (4, 0), (0, −3) [7.3A] **7.** Yes [7.3B] **8.**

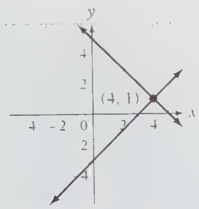


[7.3C] **9.** 5 ml [3.6B] **10.** 1.5 h [3.6C]

SECTION 8.1

1. I. c, II. a, III. b **3.** (2, −1) **5.** Dependent. The solutions are the ordered pairs that satisfy $y = -\frac{3}{2}x + 1$. **7.** The system of equations is inconsistent and has no solution **9.** (−2, 4) **11.** Yes **13.** No **15.** Yes **17.** Yes

19.



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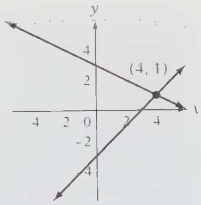

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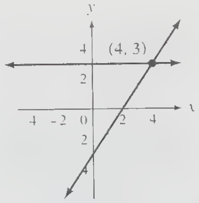

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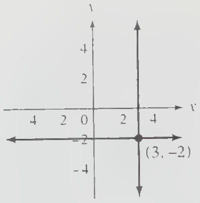

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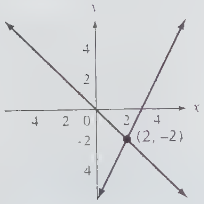

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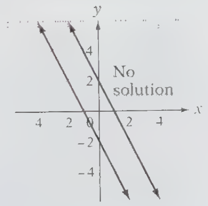

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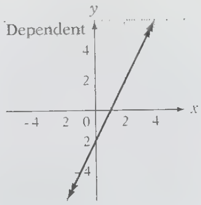
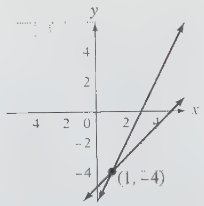


33.



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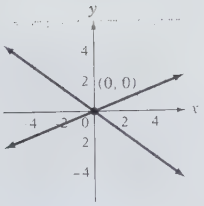

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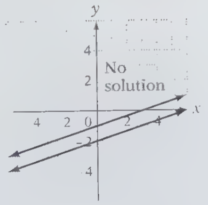


39a. Sometimes true **b.** Always true **c.** Never true **d.** Always true **43.** Answers will vary.

SECTION 8.2

3. (2, 1) **5.** (4, 1) **7.** (−1, 1) **9.** The system of equations is inconsistent and has no solution. **11.** The system of equations is inconsistent and has no solution. **13.** $\left(-\frac{3}{4}, -\frac{3}{4}\right)$ **15.** (1, 1) **17.** (2, 0) **19.** (1, −2) **21.** (0, 0) **23.** The system of equations is dependent. The solutions are the ordered pairs that satisfy the equation $2x - y = 2$. **25.** (−4, −2) **27.** (10, 31) **29.** (3, −10) **31.** (−22, −5) **33.** $1900 should be invested at 5% and $1600 should be invested at 7.5%. **35.** $3600 must be invested at 6% and $2400 must be invested at 9%. **37.** $4400 should be invested at 8% and $1600 should be invested at 11%. **39.** There was $21,000 invested at 6.5%. **41.** There was $12,000 invested at 8% and $8000 invested at 7%. **43.** There was $3750 invested in the second trust deed. **45.** You must dry 185 loads of clothes. **47.** 1 **49.** The assertion is not correct. The system of equations is independent. The solution is (0, 2). **51.** The research consultant invested $45,000. **53.** The simple interest earned was $400. The interest earned when compounded monthly was $415. The interest earned when compounded daily was $416.39.

SECTION 8.3

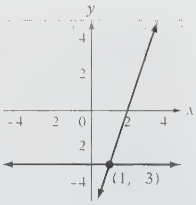
1. $(5, -1)$ **3.** $(1, 3)$ **5.** $(1, 1)$ **7.** $(3, -2)$ **9.** The system is dependent. The solutions are the ordered pairs that satisfy the equation $2x - y = 1$. **11.** $(3, 1)$ **13.** The system is dependent. The solutions are the ordered pairs that satisfy the equation $2x - 3y = 1$. **15.** $\left(\frac{2}{3}, \frac{1}{2}\right)$ **17.** $(2, 0)$ **19.** $(0, 0)$ **21.** $(5, -2)$ **23.** $\left(\frac{32}{19}, -\frac{9}{19}\right)$ **25.** $\left(\frac{7}{4}, -\frac{5}{16}\right)$ **27.** $(1, -1)$ **29.** No solution **31.** $(3, 1)$ **33.** $(-1, 2)$ **35.** $(1, 1)$ **39.** $A = 3; B = -1$ **41a.** $k = 1$ **b.** $k = \frac{3}{2}$ **c.** $k = 4$

SECTION 8.4

1. The rate of the whale in calm water is 35 mph. The rate of the current is 5 mph. **3.** The rowing rate in calm water is 14 km/h. The rate of the current is 6 km/h. **5.** The rate of the jet in calm air is 525 mph. The rate of the wind is 35 mph. **7.** The rate of the helicopter in calm air is 225 mph. The rate of the wind is 45 mph. **9.** The rate of the canoeist in calm water is 6 mph. The rate of the current is 1 mph. **11.** The wheat flour cost \$.65 per pound, and the rye flour cost \$.70 per pound. **13.** Reagent I is 25% hydrochloric acid; reagent II is 35% hydrochloric acid. **15.** There were 12 gal of 87-octane and 6 gal of 93-octane. **17.** There are 1 nickel and 2 dimes or 3 nickels and 1 dime. **19.** There were 12.5 acres of good land and 87.5 acres of bad land.

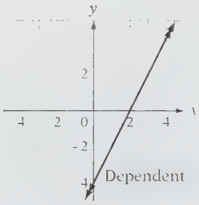
CHAPTER REVIEW

1. Yes [8.1A] **2.** No [8.1A] **3.**



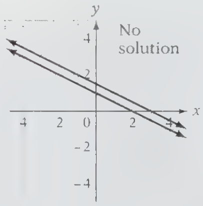
[8.1A]

4.



[8.1A]

5.

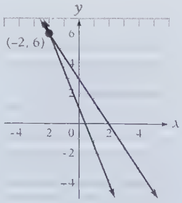


[8.1A]

6. $(-1, 1)$ [8.2A] **7.** $(1, 6)$ [8.2A] **8.** $(-3, 1)$ [8.3A] **9.** $\left(-\frac{5}{6}, \frac{1}{2}\right)$ [8.3A] **10.** The system is inconsistent and has no solution. [8.2A] **11.** $(1, 6)$ [8.2A] **12.** $(1, -5)$ [8.3A] **13.** The system is inconsistent and has no solution. [8.3A] **14.** The system is dependent. The solutions are the ordered pairs that satisfy the equation $y = -\frac{4}{3}x + 4$. [8.2A] **15.** $(-1, -3)$ [8.2A] **16.** The system is dependent. The solutions are the ordered pairs that satisfy the equation $3x + y = -2$. [8.3A] **17.** $\left(\frac{2}{3}, -\frac{1}{6}\right)$ [8.3A] **18.** The rate of the sculling team in calm water was 9 mph; the rate of the current was 3 mph. [8.4A] **19.** There were 1300 shares at \$6 per share and 200 shares at \$25 per share. [8.4B] **20.** The rate of the flight crew in calm air was 125 km/h; the rate of the wind was 15 km/h. [8.4A] **21.** The rate of the plane in calm air was 105 mph; the rate of the wind was 15 mph. [8.4A] **22.** The number of ads requiring \$.25 was 130; the number of ads requiring \$.45 was 60. [8.4B] **23.** There was \$7000 invested at 7% and \$5000 invested at 8.5%. [8.2B] **24.** There were 350 bushels of lentils and 200 bushels of corn. [8.4B] **25.** There was \$165,000 invested at 5.4% and \$135,000 invested at 6.6%. [8.2B]

CHAPTER TEST

1. Yes [8.1A] **2.** Yes [8.1A] **3.**



[8.1A] **4.** $(3, 1)$ [8.2A] **5.** $(1, -1)$ [8.2A]

6. (2, −1) [8.2A] **7.** $\left(\frac{22}{7}, -\frac{5}{7}\right)$ [8.2A] **8.** The system is inconsistent and has no solution. [8.2A]

9. (2, 1) [8.3A] **10.** $\left(\frac{1}{2}, -1\right)$ [8.3A] **11.** The system of equations is dependent. The solutions are the ordered pairs that satisfy the equation $x + 2y = 8$. [8.3A] **12.** (2, −1) [8.3A] **13.** (1, −2) [8.3A] **14.** The rate of the plane in calm air is 100 mph. The rate of the wind is 20 mph. [8.4A] **15.** The price of a reserved-seat ticket is $10. The price of a general-admission ticket is $6. [8.4B] **16.** There was $15,200 invested at 6.4% and $12,800 invested at 7.6%. [8.2B]

CUMULATIVE REVIEW

1. $\frac{3}{2}$ [2.1A] **2.** $-\frac{3}{2}$ [3.1C] **3.** $f(2) = 7$ [7.1D] **4.** $-6a^3 + 13a^2 - 9a + 2$ [4.3B] **5.** $-2x^5y^2$ [4.4A]

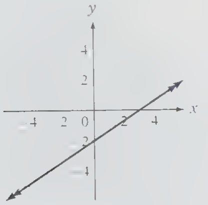
6. $2b - 1 + \frac{1}{2b - 3}$ [4.5B] **7.** $-\frac{4y}{x^3}$ [4.4A] **8.** $4y^2(xy - 4)(xy + 4)$ [5.4B] **9.** 4 and −1 [5.5A]

10. $x - 2$ [6.1C] **11.** $\frac{x^2 + 2}{(x - 1)(x + 2)}$ [6.3B] **12.** $\frac{x - 3}{x + 1}$ [6.4A] **13.** $-\frac{1}{5}$ [6.5A] **14.** $r = \frac{A - P}{Pt}$ [6.7A]

15. The x-intercept is (6, 0), and the y-intercept is (0, −4). [7.3A] **16.** $-\frac{7}{5}$ [7.3B] **17.** $y = -\frac{3}{2}x$ [7.4A]

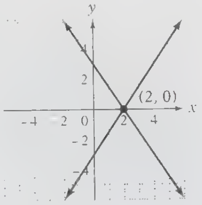
18. Yes [8.1A] **19.** (−6, 1) [8.2A] **20.** (4, −3) [8.3A] **21.** The amount invested at 9.6% is $3750. The amount invested at 7.2% is $5000. [8.2B] **22.** The rate of the freight train is 48 mph. The rate of the passenger train is 56 mph. [3.6C] **23.** A side of the original square measures 8 in. [5.5B] **24.** The rate of the wind is 30 mph. [8.4A]

25.



[7.2B]

26.



[8.1A]

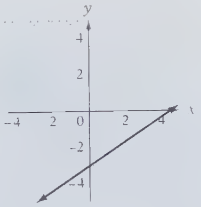
27. The rate of the boat in calm water is 14 mph. [8.4A] **28.** 35.3% of the mixture is sugar. [3.6A]

Answers to Chapter 9 Odd-Numbered Exercises

PREP TEST

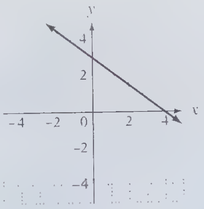
1. < [1.1A] **2.** $-7x + 15$ [2.2D] **3.** The same number can be added to each side of an equation without changing the solution of the equation. [3.1B] **4.** Each side of an equation can be multiplied by the same nonzero number without changing the solution of the equation. [3.1C] **5.** 0.45 lb [3.1D] **6.** $-\frac{1}{2}$ [3.2A] **7.** $-\frac{8}{3}$ [3.2A]

8. 2 [3.3B] **9.**



[7.2A]

10.

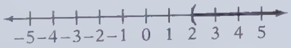


[7.2B]

SECTION 9.1

3. $A = \{16, 17, 18, 19, 20, 21\}$ **5.** $A = \{9, 11, 13, 15, 17\}$ **7.** $A = \{b, c\}$ **9.** $A \cup B = \{3, 4, 5, 6\}$ **11.** $A \cup B = \{-10, -9, -8, 8, 9, 10\}$ **13.** $A \cup B = \{a, b, c, d, e, f\}$ **15.** $A \cup B = \{1, 3, 7, 9, 11, 13\}$ **17.** $A \cap B = \{4, 5\}$ **19.** $A \cap B = \varnothing$ **21.** $A \cap B = \{c, d, e\}$ **23.** $\{x \mid x > -5, x \in \text{negative integers}\}$ **25.** $\{x \mid x > 30, x \in \text{integers}\}$ **27.** $\{x \mid x > 5, x \in \text{even integers}\}$ **29.** $\{x \mid x > 8, x \in \text{real numbers}\}$

31.



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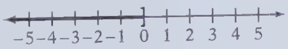

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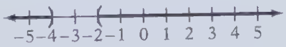
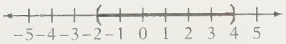
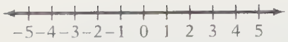
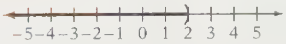


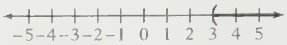
37. 

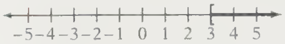
39. 

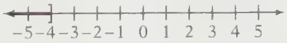
41a. Never true **b.** Always true **c.** Always true **43a.** Yes **b.** Yes

SECTION 9.2

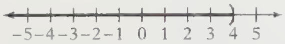
1. $x < 2$ 

3. $x > 3$ 

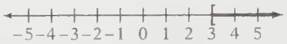
5. $n \geq 3$ 

7. $x \leq -4$ 

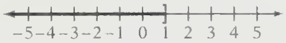
9. $x < 1$ **11.** $x \leq -3$ **13.** $y \geq -9$ **15.** $x < 12$ **17.** $x \geq 5$ **19.** $x < -11$ **21.** $x \leq 10$ **23.** $x \geq -6$ **25.** $x > 2$ **27.** $d < -\frac{1}{6}$ **29.** $x \geq -\frac{31}{24}$ **31.** $x < \frac{5}{8}$ **33.** $x < \frac{5}{4}$ **35.** $x > \frac{5}{24}$ **37.** $x < -3.8$ **39.** $x \leq -1.2$ **41.** $x < 5.6$

43. 

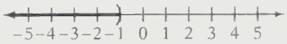
$x < 4$

45. 

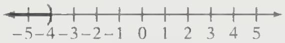
$y \geq 3$

47. 

$x \leq 1$

49. 

$x < -1$

51. 

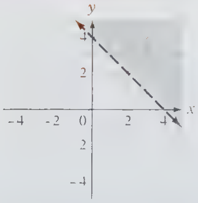
$b < -4$

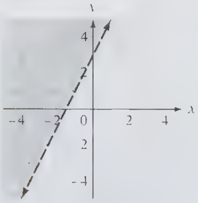
53. $y \leq 0$ **55.** $x > \frac{2}{7}$ **57.** $x \leq -\frac{5}{2}$ **59.** $x < 16$ **61.** $x \geq 16$ **63.** $x \geq -14$ **65.** $x \leq 21$ **67.** $x > 0$ **69.** $x \leq -\frac{12}{7}$ **71.** $x > \frac{2}{3}$ **73.** $x \leq \frac{2}{3}$ **75.** $x \leq 2.3$ **77.** $x < -3.2$ **79.** $x \leq 5$ **81.** The team must win at least 11 more games to qualify for the tournament. **83.** The organization must collect more than 440 lb of aluminum cans to collect the bonus. **85.** The person needs at least 50 mg of vitamin C. **87.** The student cannot earn an A grade for the course. **89.** $\{c \mid c > 0\}$ **91.** $\{c \mid c \in \text{real numbers}\}$ **93.** $\{c \mid c > 0\}$

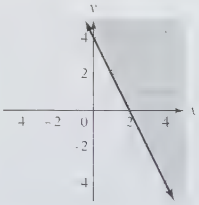
SECTION 9.3

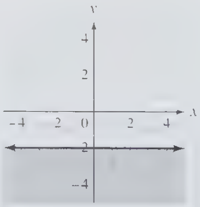
1. $x < 4$ **3.** $x < -4$ **5.** $x \geq 1$ **7.** $x < 5$ **9.** $x < 0$ **11.** $x < 20$ **13.** $y \leq \frac{5}{2}$ **15.** $x < \frac{25}{11}$ **17.** $n \leq \frac{11}{18}$ **19.** $x \geq 6$ **21.** The agent expects to sell at most \$20,000. **23.** To exceed the \$10 fee, a person must use the service more than 60 min. **25.** The maximum amount that can be added is 8 oz. **27.** The distance to the ski area is greater than 38 mi. **29.** $\{1, 2\}$ **31.** $\{3, 4, 5\}$ **33.** $\{x \mid x \in \text{real numbers}\}$

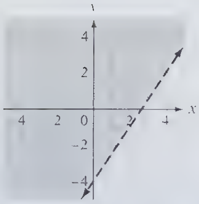
SECTION 9.4

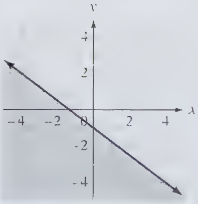
1. 

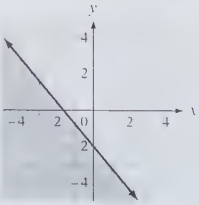
3. 

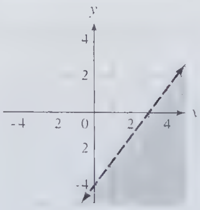
5. 

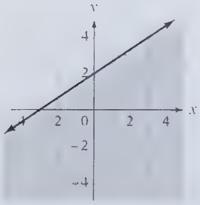
7. 

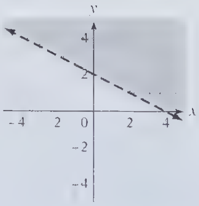
9. 

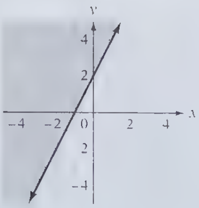
11. 

13. 

15. 

17. 

19. 

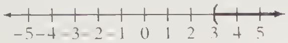
21. 

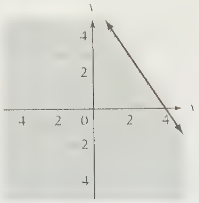
23. $x \leq 3$

CHAPTER REVIEW

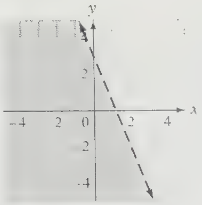
1. $x > 18$ [9.2A] **2.** $A \cap B = \varnothing$ [9.1A] **3.** $\{x \mid x > -8, x \in \text{odd integers}\}$ [9.1B]

4. $A \cup B = \{2, 4, 6, 8, 10\}$ [9.1A] **5.** $A = \{1, 3, 5, 7\}$ [9.1A] **6.** $x \geq 4$ [9.3A]

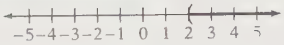
7. 

[9.1C] **8.** $x \geq -4$ [9.3A] **9.** 

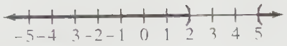
[9.4A]

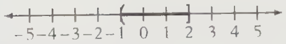
10. 

[9.4A]

11. $\{x \mid x > 3\}$ [9.1B] **12.** $x > 2$ 

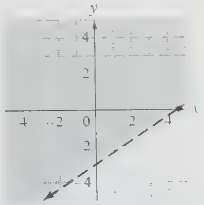
[9.2A]

13. $A \cap B = \{1, 5, 9\}$ [9.1A] **14.** 

[9.1C] **15.** 

[9.1C]

16. $x \geq -3$ [9.2B] **17.** $x > -18$ [9.3A] **18.** $x < \frac{1}{2}$ [9.3A] **19.** $x < -\frac{8}{9}$ [9.2B]

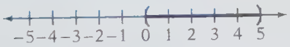
20. 

[9.4A]

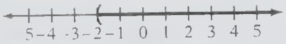
21. $x \geq 4$ [9.3A]

22. Five or more residents make florist B the more economical choice. [9.3B] **23.** The minimum length is 24 ft. [9.3B] **24.** The smallest integer that will satisfy the inequality is 32. [9.2C] **25.** The lowest score the student can receive is 72. [9.2C]

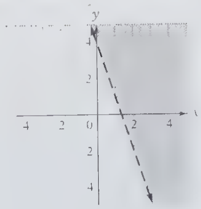
CHAPTER TEST

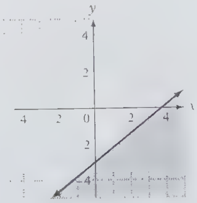
1. 

[9.1C] **2.** $\{x \mid x < 50, x \in \text{positive integers}\}$ [9.1B] **3.** $A = \{4, 6, 8\}$ [9.1A]

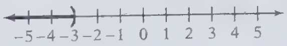
4. $x \leq -3$ [9.3A] **5.** $x > \frac{1}{8}$ [9.2A] **6.** 

[9.1C] **7.** $x < -1$ [9.3A]

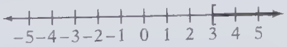
8. $\{x \mid x > -23, x \in \text{real numbers}\}$ [9.1B] **9.** 

[9.4A] **10.** 

[9.4A]

11. $A \cap B = \{12\}$ [9.1A] **12.** $x < -3$ 

[9.2A] **13.** $x \geq -\frac{40}{3}$ [9.2B]

14. $x < -\frac{22}{7}$ [9.3A] **15.** $x \geq 3$ 

[9.2B] **16.** $x \geq -4$ [9.3A] **17.** The child must grow at least 5 in. [9.2C] **18.** The maximum width is 11 ft. [9.3B] **19.** The allowable diameters are between 0.0389 in. and 0.0394 in. [9.2C] **20.** The broker processed less than or equal to \$75,000. [9.3B]

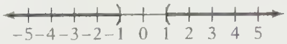
CUMULATIVE REVIEW

1. $40a - 28$ [2.2D] **2.** $\frac{1}{8}$ [3.2A] **3.** 4 [3.3B] **4.** $-12a^7b^4$ [4.2B] **5.** $-\frac{1}{b^4}$ [4.4A]

6. $4x - 2 - \frac{4}{4x - 1}$ [4.5B] **7.** $f(-1) = 0$ [7.1D] **8.** $3a^2(3x - 1)(3x + 1)$ [5.4B] **9.** $\frac{1}{x + 2}$ [6.1C]

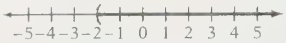
10. $\frac{18a}{(2a - 3)(a + 3)}$ [6.3B] **11.** $-\frac{5}{9}$ [6.5A] **12.** $C = S + Rt$ [6.7A] **13.** $-\frac{7}{3}$ [7.3B]

14. $y = \frac{3}{2}x - \frac{3}{2}$ [7.4A] **15.** (4, 1) [8.2A] **16.** (1, −4) [8.3A] **17.** $A \cup B = \{-10, -2, 0, 1, 2\}$ [9.1A]
18. $\{x \mid x < 48, x \in \text{real numbers}\}$ [9.1B] **19.**

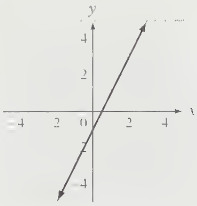


[9.1C]

20. $x > -2$

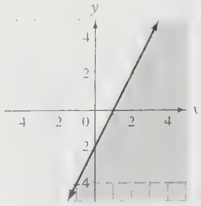


[9.2B] **21.** $x < -15$ [9.2B] **22.** $x > 2$ [9.3A]
23. $\{x \mid x \le -26, x \in \text{integers}\}$ [9.2C] **24.** The maximum number of miles is 359. [9.3B] **25.** There are 5000 fish in the lake. [6.6B] **26.** The measure of the first angle is 65°, the measure of the second angle is 35°, and the measure of the third angle is 80°. [3.5C] **27.**



[7.2A]

28.



[9.4A]

Answers to Chapter 10 Odd-Numbered Exercises

PREP TEST

1. −14 [1.1B] **2.** $-2x^2y - 4xy^2$ [2.2A] **3.** 14 [3.1C] **4.** $\frac{7}{5}$ [3.3A] **5.** x^6 [4.2A]
6. $x^2 + 2xy + y^2$ [4.3D] **7.** $4x^2 - 12x + 9$ [4.3D] **8.** $4 - 9v^2$ [4.3D] **9.** $a^2 - 25$ [4.3D] **10.** $\frac{x^2y^2}{9}$ [4.4A]

SECTION 10.1

3. 4 **5.** 7 **7.** $4\sqrt{2}$ **9.** $2\sqrt{2}$ **11.** $18\sqrt{2}$ **13.** $10\sqrt{10}$ **15.** $\sqrt{15}$ **17.** $\sqrt{29}$ **19.** $-54\sqrt{2}$ **21.** $3\sqrt{5}$ **23.** 0 **25.** $48\sqrt{2}$ **27.** 15.492 **29.** 16.971 **31.** 16 **33.** x^3 **35.** $y^7\sqrt{y}$ **37.** a^{10} **39.** x^2y^2 **41.** $2x^2$ **43.** $2x\sqrt{6}$ **45.** $2x^2\sqrt{15x}$ **47.** $7a^2b^4$ **49.** $3x^2y^3\sqrt{2xy}$ **51.** $2x^5y^3\sqrt{10xy}$ **53.** $4a^4b^5\sqrt{5a}$ **55.** $8ab\sqrt{b}$ **57.** x^3y **59.** $8a^2b^3\sqrt{5b}$ **61.** $6x^2y^3\sqrt{3y}$ **63.** $4x^3y\sqrt{2y}$ **65.** $5(a + 4)$ or $5a + 20$ **67.** $2(x + 2)^2$ or $2x^2 + 8x + 8$ **69.** $x + 2$ **71.** $y + 1$ **73a.** $12\sqrt{5}$ mph **b.** 46 mph **75.** No. For example, $\sqrt{4 + 9} \ne \sqrt{4} + \sqrt{9}$. **77.** No. $\sqrt{18}$ still contains a perfect-square factor. $6\sqrt{2}$ **79a.** 1 **b.** 3 **c.** $3\sqrt{3}$

SECTION 10.2

1. 2, 20, 50 **5.** $3\sqrt{2}$ **7.** $-\sqrt{7}$ **9.** $-11\sqrt{11}$ **11.** $10\sqrt{x}$ **13.** $-2\sqrt{y}$ **15.** $-11\sqrt{3b}$ **17.** $2x\sqrt{2}$ **19.** $-3a\sqrt{3a}$ **21.** $-5\sqrt{xy}$ **23.** $8\sqrt{5}$ **25.** $8\sqrt{2}$ **27.** $15\sqrt{2} - 10\sqrt{3}$ **29.** $\sqrt{x}$ **31.** $-12x\sqrt{3}$ **33.** $2xy\sqrt{x} - 3xy\sqrt{y}$ **35.** $-9x\sqrt{3x}$ **37.** $-13y^2\sqrt{2y}$ **39.** $4a^2b^2\sqrt{ab}$ **41.** $7\sqrt{2}$ **43.** $6\sqrt{x}$ **45.** $-3\sqrt{y}$ **47.** $-45\sqrt{2}$ **49.** $13\sqrt{3} - 12\sqrt{5}$ **51.** $32\sqrt{3} - 3\sqrt{11}$ **53.** $6\sqrt{x}$ **55.** $-34\sqrt{3x}$ **57.** $10a\sqrt{3b} + 10a\sqrt{5b}$ **59.** $-2xy\sqrt{3}$ **61.** $5\sqrt{2}$

SECTION 10.3

3. 5 **5.** 6 **7.** x **9.** x^3y^2 **11.** $3ab^6\sqrt{2a}$ **13.** $12a^4b\sqrt{b}$ **15.** $2 - \sqrt{6}$ **17.** $x - \sqrt{xy}$ **19.** $5\sqrt{2} - \sqrt{5x}$ **21.** $4 - 2\sqrt{10}$ **23.** $x - 6\sqrt{x} + 9$ **25.** $3a - 3\sqrt{ab}$ **27.** $10abc$ **29.** $-2 + 2\sqrt{5}$ **31.** $16 + 10\sqrt{2}$ **33.** $6x + 10\sqrt{x} - 4$ **35.** $15x - 22y\sqrt{x} + 8y^2$ **37.** $x - y$ **41.** 3 **43.** 4 **45.** $6x^2$ **47.** $2x^2\sqrt{2y}$ **49.** $4x\sqrt{y}$ **51.** $\frac{2\sqrt{3x}}{3y}$ **53.** $\frac{-1 + \sqrt{2}}{2}$ **55.** $-\frac{5\sqrt{7} + 15}{2}$ **57.** $-\frac{5\sqrt{3} + 9}{2}$ **59.** $3 + \sqrt{6}$ **61.** $\frac{-12 + 3\sqrt{2}}{7}$ **63.** $\frac{14 - 9\sqrt{2}}{17}$ **65.** $\sqrt{15} + 2\sqrt{5}$ **67.** $\frac{x\sqrt{y} + y\sqrt{x}}{x - y}$ **69.** $4\sqrt{6} + 12$ **71.** The complete solution is available in the *Solutions Manual.*

SECTION 10.4

1. 25 **3.** 144 **5.** 5 **7.** No solution **9.** -1 **11.** 1 **13.** 15 **15.** $\frac{7}{3}$ **17.** 45 **19.** No solution

21. The pitcher's mound is less than halfway between home plate and second base. **23.** The periscope would have to be 16.67 ft above the surface of the water. **25.** The height of the TV screen is 21.6 in. **27.** The child is 18.75 ft from the center of the merry-go-round. **29.** The perimeter of the triangle is 30 units. **33.** The area of the fountain is 244.78 ft^2.

CHAPTER REVIEW

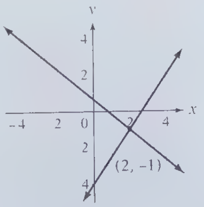
1. 3 [10.3A] **2.** $9a^2\sqrt{2ab}$ [10.1B] **3.** 12 [10.1A] **4.** $3a\sqrt{2} + 2a\sqrt{3}$ [10.3A] **5.** $-x\sqrt{3} - x\sqrt{5}$ [10.3B] **6.** $-6\sqrt{30}$ [10.1A] **7.** 20 [10.4A] **8.** $20\sqrt{3}$ [10.1A] **9.** $7x^2y^4$ [10.3B] **10.** No solution [10.4A] **11.** $18a\sqrt{5b} + 5a\sqrt{b}$ [10.2A] **12.** $20\sqrt{10}$ [10.1A] **13.** $7x^2y\sqrt{15xy}$ [10.2A] **14.** $8y + 10\sqrt{5y} - 15$ [10.3A] **15.** $26\sqrt{3x}$ [10.2A] **16.** No solution [10.4A] **17.** $\frac{8\sqrt{x} + 24}{x - 9}$ [10.3B] **18.** $36x^8y^5\sqrt{3xy}$ [10.1B] **19.** $2y^4\sqrt{6}$ [10.1B] **20.** $\frac{3}{5}$ [10.4A] **21.** $-6x^3y^2\sqrt{2y}$ [10.2A] **22.** $\frac{16\sqrt{a}}{a}$ [10.3B] **23.** The explorer would weigh 144 lb on the surface of Earth. [10.4B] **24.** The depth of the water is 100 ft. [10.4B] **25.** The radius of the sharpest corner is 25 ft. [10.4B]

CHAPTER TEST

1. $11x^4y$ [10.1B] **2.** $6x^2y\sqrt{y}$ [10.3A] **3.** $-5\sqrt{2}$ [10.2A] **4.** $3\sqrt{5}$ [10.1A] **5.** 9 [10.3B] **6.** 25 [10.4A] **7.** $4a^2b^5\sqrt{2ab}$ [10.1B] **8.** $7ab\sqrt{a}$ [10.3B] **9.** $\sqrt{3} + 1$ [10.3B] **10.** $4x^2y^2\sqrt{5y}$ [10.3A] **11.** No solution [10.4A] **12.** $21\sqrt{2y} - 12\sqrt{2x}$ [10.2A] **13.** $6x^3y\sqrt{2x}$ [10.1B] **14.** $y + 2\sqrt{y} - 15$ [10.3A] **15.** $-2xy\sqrt{3xy} - 3xy\sqrt{xy}$ [10.2A] **16.** $\frac{17 - 8\sqrt{5}}{31}$ [10.3B] **17.** $a - \sqrt{ab}$ [10.3A] **18.** $5\sqrt{3}$ [10.1A] **19.** The length of the pendulum is 7.30 ft. [10.4B] **20.** The larger integer is 51. [10.4B]

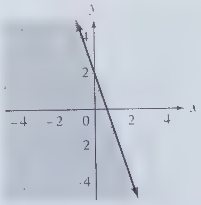
CUMULATIVE REVIEW

1. $-\frac{1}{12}$ [1.5B] **2.** $2x + 18$ [2.2D] **3.** $\frac{1}{13}$ [3.3B] **4.** $6x^5y^5$ [4.2A] **5.** $-2b^2 + 1 - \frac{1}{3b^2}$ [4.5A] **6.** $f(-3) = 1$ [7.1D] **7.** $2a(a - 5)(a - 3)$ [5.2B] **8.** $\frac{1}{4(x + 1)}$ [6.1B] **9.** $\frac{x + 3}{x - 3}$ [6.3B] **10.** $\frac{5}{3}$ [6.5A] **11.** $y = \frac{1}{2}x - 2$ [7.4A] **12.** $(1, 1)$ [8.2A] **13.** $(3, -2)$ [8.3A] **14.** $x \le -\frac{9}{2}$ [9.3A] **15.** $6\sqrt{3}$ [10.1A] **16.** $-4\sqrt{2}$ [10.2A] **17.** $4ab\sqrt{2ab} - 5ab\sqrt{ab}$ [10.2A] **18.** $14a^5b^2\sqrt{2a}$ [10.3A] **19.** $3\sqrt{2} - x\sqrt{3}$ [10.3A] **20.** 8 [10.3B] **21.** $-6 - 3\sqrt{5}$ [10.3B] **22.** 6 [10.4A] **23.** The book costs $24.50. [3.2B] **24.** 56 oz of pure water must be added. [3.6B] **25.** The numbers are 13 and 8. [5.5B] **26.** Working alone, it would take the small pipe 48 h to fill the tank. [6.8A] **27.** $(2, -1)$



[8.1A]

28.



[9.4A]

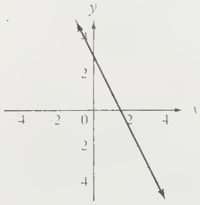
29. The smaller integer is 40. [10.4B] **30.** The building is 400 ft high. [10.4B]

Answers to Chapter 11 Odd-Numbered Exercises

PREP TEST

1. 41 [2.1A] **2.** $-\frac{1}{5}$ [3.2A] **3.** $(x + 4)(x - 3)$ [5.2A] **4.** $(2x - 3)^2$ [5.4A] **5.** Yes [5.4A] **6.** 3 [6.5A]

7.



[7.2A] **8.** $2\sqrt{7}$ [10.1A] **9.** $|a|$ [10.1B] **10.** 3.6 mi [3.6C]

SECTION 11.1

1. -3 and 5 **3.** $-\frac{5}{2}$ and $\frac{1}{3}$ **5.** -5 and 3 **7.** 1 and 3 **9.** -1 and -2 **11.** 3 **13.** 0 and $-\frac{2}{3}$ **15.** -2 and 5 **17.** $\frac{2}{3}$ and 1 **19.** -3 and $\frac{1}{3}$ **21.** $\frac{2}{3}$ **23.** $-\frac{1}{2}$ and $\frac{3}{2}$ **25.** $\frac{1}{2}$ **27.** -3 and 3 **29.** $-\frac{1}{2}$ and $\frac{1}{2}$ **31.** -3 and 5 **33.** 1 and 5 **35.** -6 and 6 **37.** -1 and 1 **39.** $-\frac{7}{2}$ and $\frac{7}{2}$ **41.** $-\frac{2}{3}$ and $\frac{2}{3}$ **43.** $-\frac{3}{4}$ and $\frac{3}{4}$ **45.** No real number solution **47.** $-2\sqrt{6}$ and $2\sqrt{6}$ **49.** -5 and 7 **51.** -7 and -3 **53.** $-\frac{1}{3}$ and $\frac{7}{3}$ **55.** $-\frac{2}{7}$ and $-\frac{12}{7}$ **57.** $4 + 2\sqrt{5}$ and $4 - 2\sqrt{5}$ **59.** No real number solution **61.** $-\frac{3}{4} + 2\sqrt{3}$ and $-\frac{3}{4} - 2\sqrt{3}$ **63.** $1, -1, -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}$ **65.** The annual percentage rate is 8%. **67.** Yes

SECTION 11.2

1. $x^2 - 8x + 16, (x - 4)^2$ **3.** $x^2 + 5x + \frac{25}{4}, \left(x + \frac{5}{2}\right)^2$ **5.** 1 and -3 **7.** 8 and -2 **9.** 2 **11.** No real number solution **13.** -1 and -4 **15.** -8 and 1 **17.** $-2 + \sqrt{3}$ and $-2 - \sqrt{3}$ **19.** $-3 + \sqrt{14}$ and $-3 - \sqrt{14}$ **21.** $1 + \sqrt{2}$ and $1 - \sqrt{2}$ **23.** $\frac{-3 + \sqrt{13}}{2}$ and $\frac{-3 - \sqrt{13}}{2}$ **25.** 2 and 1 **27.** $\frac{-1 + \sqrt{13}}{2}$ and $\frac{-1 - \sqrt{13}}{2}$ **29.** $-5 + 4\sqrt{2}$ and $-5 - 4\sqrt{2}$ **31.** $\frac{3 + \sqrt{29}}{2}$ and $\frac{3 - \sqrt{29}}{2}$ **33.** $\frac{1 + \sqrt{17}}{2}$ and $\frac{1 - \sqrt{17}}{2}$ **35.** No real number solution **37.** 1 and $\frac{1}{2}$ **39.** -3 and $\frac{1}{2}$ **41.** 2 and $\frac{3}{2}$ **43.** 1 and $-\frac{1}{2}$ **45.** -2 and $\frac{1}{3}$ **47.** -2 and $-\frac{2}{3}$ **49.** $\frac{1}{2}$ and $-\frac{3}{2}$ **51.** $\frac{1}{3}$ and $-\frac{3}{2}$ **53.** $-\frac{1}{2}$ and $\frac{4}{3}$ **55.** -5.372 and 0.372 **57.** 1.712 and -3.212 **59.** -1.151 and 0.651 **61.** $1 + \sqrt{7}$ and $1 - \sqrt{7}$ **63.** 0 and -1 **65.** $4 + \sqrt{3}$ and $4 - \sqrt{3}$ **67.** 0 and 8 **69.** $\frac{15 + 3\sqrt{17}}{2}$ and $\frac{15 - 3\sqrt{17}}{2}$ **71.** The ball will hit the ground in 4.81 s.

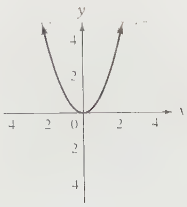
SECTION 11.3

1. -1 and 5 **3.** -3 and 5 **5.** -1 and 3 **7.** -5 and 1 **9.** $-\frac{1}{2}$ and 1 **11.** No real number solution **13.** 0 and 1 **15.** $-\frac{3}{2}$ and $\frac{3}{2}$ **17.** $-\frac{5}{2}$ and $\frac{3}{2}$ **19.** No real number solution **21.** $1 + \sqrt{6}$ and $1 - \sqrt{6}$ **23.** $-3 + \sqrt{10}$ and $-3 - \sqrt{10}$ **25.** $2 + \sqrt{13}$ and $2 - \sqrt{13}$ **27.** $\frac{-1 + \sqrt{2}}{3}$ and $\frac{-1 - \sqrt{2}}{3}$ **29.** $-\frac{1}{2}$ **31.** No real number solution **33.** $\frac{-4 + \sqrt{5}}{2}$ and $\frac{-4 - \sqrt{5}}{2}$ **35.** $\frac{1 + 2\sqrt{3}}{2}$ and $\frac{1 - 2\sqrt{3}}{2}$ **37.** $\frac{-5 + \sqrt{2}}{3}$ and $\frac{-5 - \sqrt{2}}{3}$

39. 5.690 and −3.690 **41.** 7.690 and −1.690 **43.** 4.589 and −1.089 **45.** −2.118 and 0.118 **47.** 1.105 and −0.905 **51a.** False **b.** False **c.** False **d.** True **53.** −3 and −4 **55.** No real number solution **57.** −1 and 11 **59.** After 2 h the distance between the planes is 1000 mi.

SECTION 11.4

1. Down **3.** Up **5.** 4 **7.** −5 **9.** −42 **11.**



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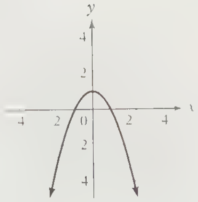

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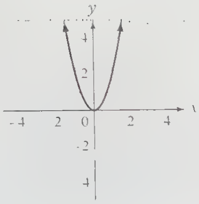

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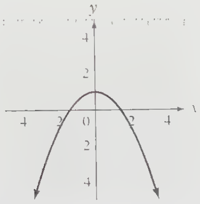

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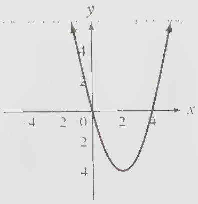

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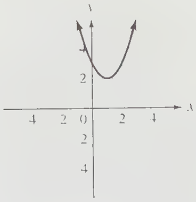
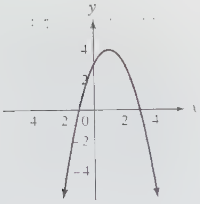


23.



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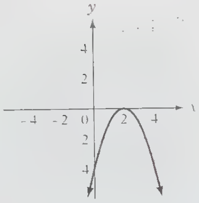


27. $(-6, 0), (1, 0); (0, -6)$ **29.** $(-6, 0); (0, 36)$

31. $(-2 - \sqrt{6}, 0), (-2 + \sqrt{6}, 0); (0, -2)$ **33.** No x-intercepts; $(0, 1)$ **35.** $\left(\frac{3}{2}, 0\right), (5, 0); (0, 15)$ **37.** $\left(\frac{-3 - \sqrt{33}}{6}, 0\right), \left(\frac{-3 + \sqrt{33}}{6}, 0\right); (0, 2)$ **39.** $y = 3x^2 + 6x + 5$ **41.** $y = 3x^2 - 6x$ **43.** $(-1, 0), (0, 0), (5, 0)$ **45.** $(-3, 0), (-1, 0), (1, 0)$

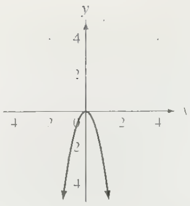
SECTION 11.5

1. The length is 4 m; the height is 10 m. **3.** The length of the batter's box is 6 ft; the width is 4 ft. **5.** The length of the pool is 100 ft; the width is 50 ft. **7.** The hang time is 5.5 s. **9.** The maximum velocity of the car is 78 ft/s. **11.** The radius of the large pizza is 7 in. **13.** The first computer alone would take 35 min. The second computer alone would take 14 min. **15.** The first engine alone would take 12 h. The second engine alone would take 6 h. **17.** The rate of the plane is 100 mph. **19.** The pizza would cost $24.

CHAPTER REVIEW

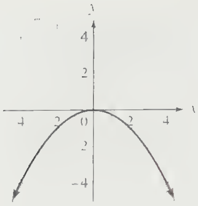
1. $\frac{4}{3}$ and $-\frac{7}{2}$ [11.1A] **2.** $-\frac{5}{7}$ and $\frac{5}{7}$ [11.1B] **3.** −6 and 4 [11.2A] **4.** −6 and 1 [11.3A] **5.** −4 and $\frac{3}{2}$ [11.2A] **6.** 2 and $\frac{5}{12}$ [11.1A] **7.** $-2 - 2\sqrt{6}$ and $-2 + 2\sqrt{6}$ [11.1B] **8.** 1 and $\frac{3}{2}$ [11.3A] **9.** $-\frac{1}{3}$ and $-\frac{1}{2}$ [11.1A] **10.** No real number solution [11.1B] **11.** $2 - \sqrt{3}$ and $2 + \sqrt{3}$ [11.2A] **12.** $\frac{3 - \sqrt{29}}{2}$ and $\frac{3 + \sqrt{29}}{2}$ [11.3A] **13.** No real number solution [11.2A] **14.** −7 and −10 [11.1A] **15.** −1 and 2 [11.1B] **16.** $\frac{-4 - \sqrt{23}}{2}$ and $\frac{-4 + \sqrt{23}}{2}$ [11.2A] **17.** No real number solution [11.3A]

18. -2 and $-\frac{1}{2}$ [11.3A] **19.**



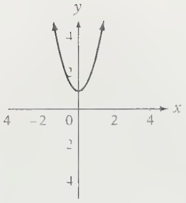
[11.4A]

20.



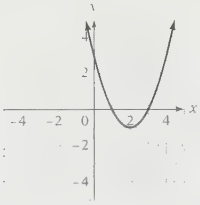
[11.4A]

21.



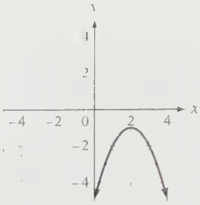
[11.4A]

22.



[11.4A]

23.



[11.4A]

24. x-intercepts: $(-3, 0)$, $(5, 0)$; y-intercept: $(0, -15)$ [11.4A] **25.** The rate of the hawk in calm air is 75 mph. [11.5A]

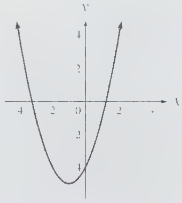
CHAPTER TEST

1. 6 and -1 [11.1A] **2.** -4 and $\frac{5}{3}$ [11.1A] **3.** 0 and 10 [11.1B] **4.** $-4 + 2\sqrt{5}$ and $-4 - 2\sqrt{5}$ [11.1B]

5. $-2 + 2\sqrt{5}$ and $-2 - 2\sqrt{5}$ [11.2A] **6.** $\frac{-3 + \sqrt{41}}{2}$ and $\frac{-3 - \sqrt{41}}{2}$ [11.2A] **7.** $\frac{3 + \sqrt{7}}{2}$ and $\frac{3 - \sqrt{7}}{2}$ [11.2A]

8. $\frac{-4 + \sqrt{22}}{2}$ and $\frac{-4 - \sqrt{22}}{2}$ [11.2A] **9.** $-2 + \sqrt{2}$ and $-2 - \sqrt{2}$ [11.3A] **10.** $\frac{3 + \sqrt{33}}{2}$ and $\frac{3 - \sqrt{33}}{2}$ [11.3A]

11. $-\frac{1}{2}$ and 3 [11.3A] **12.** $\frac{1 + \sqrt{13}}{6}$ and $\frac{1 - \sqrt{13}}{6}$ [11.3A] **13.**



[11.4A]

14. x-intercepts: $(-4, 0)$, $(3, 0)$; y-intercept: $(0, -12)$ [11.4A] **15.** The length is 8 ft. The width is 5 ft. [11.5A]
16. The rate of the boat in calm water is 11 mph. [11.5A]

CUMULATIVE REVIEW

1. $-28x + 27$ [2.2D] **2.** $\frac{3}{2}$ [3.1C] **3.** 3 [3.3B] **4.** $-12a^8b^4$ [4.2B] **5.** $x + 2 - \frac{4}{x - 2}$ [4.5B]

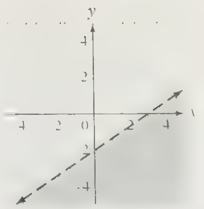
6. $x(3x - 4)(x + 2)$ [5.3A/5.3B] **7.** $\frac{9x^2(x - 2)^2}{(2x - 3)^2}$ [6.1C] **8.** $\frac{x + 2}{2(x + 1)}$ [6.3B] **9.** $\frac{x - 4}{2x + 5}$ [6.4A]

10. The x-intercept is $(3, 0)$, and the y-intercept is $(0, -4)$. [7.3A] **11.** $y = -\frac{4}{3}x - 2$ [7.4A] **12.** $(2, 1)$ [8.2A]

13. $(2, -2)$ [8.3A] **14.** $x > \frac{1}{9}$ [9.3A] **15.** $a - 2$ [10.3A] **16.** $6ab\sqrt{a}$ [10.3B] **17.** $\frac{-6 + 5\sqrt{3}}{13}$ [10.3B]

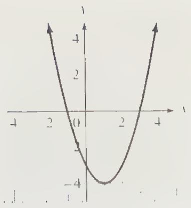
18. 5 [10.4A] **19.** $\frac{5}{2}$ and $\frac{1}{3}$ [11.1A] **20.** $5 + 3\sqrt{2}$ and $5 - 3\sqrt{2}$ [11.1B] **21.** $\frac{-7 + \sqrt{13}}{6}$ and $\frac{-7 - \sqrt{13}}{6}$ [11.2A]

22. 2 and $-\frac{1}{2}$ [11.3A] **23.** The cost of the mixture is \$2.25 per pound. [3.6A] **24.** 250 additional shares are required. [6.6B] **25.** The rate of the plane in still air is 200 mph. The rate of the wind is 40 mph. [8.4A] **26.** The student must receive a score of 77 or above. [9.2C] **27.** The integer is -5 or 5. [11.5A] **28.** The rate for the last 8 mi was 4 mph. [11.5A] **29.**



[9.4A]

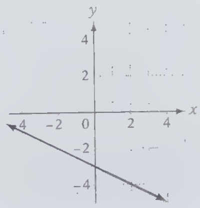
30.



[11.4A]

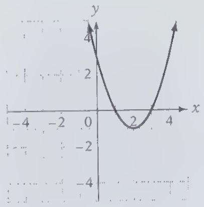
FINAL EXAM

1. -3 [1.1B] **2.** -6 [1.2B] **3.** -256 [1.5A] **4.** -11 [1.5B] **5.** $-\frac{15}{2}$ [2.1A] **6.** $9x + 6y$ [2.2A] **7.** $6z$ [2.2B] **8.** $16x - 52$ [2.2D] **9.** -50 [3.1C] **10.** -3 [3.3B] **11.** 12.5% [1.4B] **12.** 15.2 [3.1D] **13.** $-3x^2 - 3x + 8$ [4.1B] **14.** $81x^4y^{12}$ [4.2B] **15.** $6x^3 + 7x^2 - 7x - 6$ [4.3B] **16.** $-\frac{x^4y}{2}$ [4.4A] **17.** $\frac{3x}{y} - 4x^2 - \frac{5}{x}$ [4.5A] **18.** $5x - 12 + \frac{23}{x+2}$ [4.5B] **19.** $\frac{4y^6}{x^6}$ [4.4A] **20.** $\frac{3}{4}$ [7.1D] **21.** $(x - 6)(x + 1)$ [5.2A] **22.** $(3x + 2)(2x - 3)$ [5.3A/5.3B] **23.** $4x(2x - 1)(x - 3)$ [5.3A/5.3B] **24.** $(5x - 4)(5x + 4)$ [5.4A] **25.** $2(a + 3)(4 - x)$ [5.1B] **26.** $3y(5 - 2x)(5 + 2x)$ [5.4B] **27.** $\frac{1}{2}$ and 3 [5.5A] **28.** $\frac{2(x+1)}{x-1}$ [6.1B] **29.** $\frac{-3x^2 + x - 25}{(2x-5)(x+3)}$ [6.3B] **30.** $\frac{x^2 - 2x}{x - 1}$ [6.4A] **31.** 2 [6.5A] **32.** $a = b$ [6.7A] **33.** $\frac{2}{3}$ [7.3B] **34.** $y = -\frac{2}{3}x - 2$ [7.4A] **35.** $(6, 17)$ [8.2A] **36.** $(2, -1)$ [8.3A] **37.** $x \le -3$ [9.2B] **38.** $y \ge \frac{5}{2}$ [9.3A] **39.** $7x^3$ [10.1B] **40.** $38\sqrt{3a}$ [10.2A] **41.** $\sqrt{15} + 2\sqrt{3}$ [10.3B] **42.** 2 [10.4A] **43.** -1 and $\frac{4}{3}$ [11.1A] **44.** $\frac{1+\sqrt{5}}{4}$ and $\frac{1-\sqrt{5}}{4}$ [11.3A] **45.** $2x + 3(x - 2)$, $5x - 6$ [2.3B] **46.** The original value was $3000. [3.1D] **47.** The markup rate is 65%. [3.2B] **48.** An additional $6000 must be invested. [8.2B] **49.** The mixture costs $3 per pound. [3.6A] **50.** The percent concentration of the acid in the mixture is 36%. [3.6B] **51.** In the first hour, the plane flew 215 km. [3.6C] **52.** The measures of the angles are 50°, 60°, and 70°. [3.5C] **53.** The integer is 4 or −4. [11.5A] **54.** The length is 10 m. The width is 5 m. [5.5B] **55.** Sixteen ounces of dye are required for 120 oz of base paint. [6.6B] **56.** Working together, it would take them 0.6 h to prepare the dinner. [6.8A] **57.** The rate of the boat in calm water is 15 mph. The rate of the current is 5 mph. [8.4A] **58.** The rate of the wind is 25 mph. [11.5A] **59.**



[7.3C]

60.



[11.4A]

Glossary

abscissa The first number in an ordered pair. It measures a horizontal distance and is also called the first coordinate. (Sec. 7.1)

absolute value of a number The distance of the number from zero on the number line. (Sec. 1.1)

acute angle An angle whose measure is between 0° and 90°. (Sec. 3.5)

addend In addition, a number being added. (Sec. 1.2)

addition The process of finding the total of two numbers. (Sec. 1.2)

addition method An algebraic method of finding an exact solution of a system of linear equations. (Sec. 8.3)

additive inverses Numbers that are the same distance from zero on the number line, but on opposite sides; also called opposites. (Sec. 1.1, 2.2)

adjacent angles Two angles that share a common side. (Sec. 3.5)

alternate exterior angles Two angles that are on opposite sides of the transversal and outside the parallel lines. (Sec. 3.5)

alternate interior angles Two angles that are on opposite sides of the transversal and between the parallel lines. (Sec. 3.5)

analytic geometry Geometry in which a coordinate system is used to study the relationships between variables. (Sec. 7.1)

arithmetic mean of values Average determined by calculating the sum of the values and then dividing that result by the number of values. (Sec. 1.3)

axes The two number lines that form a rectangular coordinate system; also called coordinate axes. (Sec. 7.1)

base In exponential notation, the factor that is multiplied the number of times shown by the exponent. (Sec. 1.5)

basic percent equation Percent times base equals amount. (Sec. 3.1)

binomial A polynomial of two terms. (Sec. 4.1)

clearing denominators Removing denominators from an equation that contains fractions by multiplying each side of the equation by the LCM of the denominators. (Sec. 3.2, 6.5)

coefficient The number part of a variable term. (Sec. 2.1)

combining like terms Using the Distributive Property to add the coefficients of like variable terms; adding like terms of a variable expression. (Sec. 2.2)

complementary angles Two angles whose sum is 90°. (Sec. 3.5)

completing the square Adding to a binomial the constant term that makes it a perfect-square trinomial. (Sec. 11.2)

complex fraction A fraction whose numerator or denominator contains one or more fractions. (Sec. 6.4)

conjugates Binomial expressions that differ only in the sign of a term. The expressions $a + b$ and $a - b$ are conjugates. (Sec. 10.3)

consecutive even integers Even integers that follow one another in order. (Sec. 3.4)

consecutive integers Integers that follow one another in order. (Sec. 3.4)

consecutive odd integers Odd integers that follow one another in order. (Sec. 3.4)

constant term A term that includes no variable part; also called a constant. (Sec. 2.1)

coordinate axes The two number lines that form a rectangular coordinate system; also simply called axes. (Sec. 7.1)

coordinates of a point The numbers in an ordered pair that is associated with a point. (Sec. 7.1)

corresponding angles Two angles that are on the same side of the transversal and are both acute angles or are both obtuse angles. (Sec. 3.5)

cost The price that a business pays for a product. (Sec. 3.2)

decimal notation Notation in which a number consists of a whole-number part, a decimal point, and a decimal part. (Sec. 1.4)

degree A unit used to measure angles. (Sec. 3.5)

degree of a polynomial in one variable The largest exponent that appears on the variable. (Sec. 4.1)

dependent system of equations A system of equations that has an infinite number of solutions. (Sec. 8.1)

dependent variable In a function, the variable whose value depends on the value of another variable known as the independent variable. (Sec. 7.1)

descending order The terms of a polynomial in one variable arranged so that the exponents on the variable decrease from left to right. The polynomial $9x^5 - 2x^4 + 7x^3 + x^2 - 8x + 1$ is in descending order. (Sec. 4.1)

discount The amount by which a retailer reduces the regular price of a product for a promotional sale. (Sec. 3.2)

discount rate The percent of the regular price that the discount represents. (Sec. 3.2)

domain The set of first coordinates of the ordered pairs in a relation. (Sec. 7.1)

element of a set One of the objects in a set. (Sec. 1.1, 9.1)

empty set The set that contains no elements; also called the null set. (Sec. 9.1)

equation A statement of the equality of two mathematical expressions. (Sec. 3.1)

equilateral triangle A triangle in which all three sides are of equal length. (Sec. 3.5)

equivalent equations Equations that have the same solution. (Sec. 3.1)

evaluating a function Replacing x in $f(x)$ with some value and then simplifying the numerical expression that results. (Sec. 7.1)

evaluating a variable expression Replacing each variable by its value and then simplifying the resulting numerical expression. (Sec. 2.1)

even integer An integer that is divisible by 2. (Sec. 3.4)

exponent In exponential notation, the elevated number that indicates how many times the base occurs in the multiplication. (Sec. 1.5)

exponential form The expression 2^5 is in exponential form. Compare *factored form*. (Sec. 1.5)

factor In multiplication, a number being multiplied. (Sec. 1.3)

factor a polynomial To write the polynomial as a product of other polynomials. (Sec. 5.1)

factor a trinomial of the form $x^2 + bx + c$ To express the trinomial as the product of two binomials. (Sec. 5.1)

factored form The expression $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ is in factored form. Compare *exponential form*. (Sec. 1.5)

first coordinate The first number in an ordered pair. It measures a horizontal distance and is also called the abscissa. (Sec. 7.1)

FOIL A method of finding the product of two binomials; the letters stand for First, Outer, Inner, and Last. (Sec. 4.3)

formula A literal equation that states rules about measurements. (Sec. 6.7)

function A relation in which no two ordered pairs that have the same first coordinate have different second coordinates. (Sec. 7.1)

functional notation A function designated by $f(x)$, which is the value of the function at x. (Sec. 7.1)

graph of a relation The graph of the ordered pairs that belong to the relation. (Sec. 7.1)

graph of an equation in two variables A graph of the ordered-pair solutions of the equation. (Sec. 7.2)

graph of an integer A heavy dot directly above that number on the number line. (Sec. 1.1)

graph of an ordered pair The dot drawn at the coordinates of the point in the plane. (Sec. 7.1)

graphing a point in the plane Placing a dot at the location given by the ordered pair; also called plotting a point in the plane. (Sec. 7.1)

greater than A number a is greater than another number b, written $a > b$, if a is to the right of b on the number line. (Sec. 1.1)

greater than or equal to The symbol $\geq$ means "is greater than or equal to." (Sec. 1.1)

greatest common factor The greatest common factor (GCF) of two or more integers is the greatest integer that is a factor of all the integers. The greatest common factor of two or more monomials is the product of the GCF of the coefficients and the common variable factors. (Sec. 5.1)

half-plane The solution set of an inequality in two variables. (Sec. 9.4)

hypotenuse In a right triangle, the side opposite the 90° angle. (Sec. 10.4)

inconsistent system of equations A system of equations that has no solution. (Sec. 8.1)

independent system of equations A system of equations that has one solution. (Sec. 8.1)

independent variable In a function, the variable that varies independently and whose value determines the value of the dependent variable. (Sec. 7.1)

inequality An expression that contains the symbol $>$, $<$, $\geq$ (is greater than or equal to), or $\leq$ (is less than or equal to). (Sec. 9.1)

integers The numbers $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$. (Sec 1.1)

intersection of sets *A* and *B* The set that contains the elements that are common to both A and B. (Sec. 9.1)

irrational number The decimal representation of an irrational number never repeats or terminates and can only be approximated. (Sec. 1.4, 10.1)

isosceles triangle A triangle that has two equal angles and two equal sides. (Sec. 3.5)

least common denominator The smallest number that is a multiple of each denominator in question. (Sec. 1.4)

least common multiple (LCM) The LCM of two or more numbers is the smallest number that contains the prime factorization of each number. The LCM of two or more polynomials is the polynomial of least degree that contains the factors of each polynomial. (Sec. 1.4, 6.2)

less than A number a is less than another number b, written $a < b$, if a is to the left of b on the number line. (Sec. 1.1)

less than or equal to The symbol $\leq$ means "is less than or equal to". (Sec. 1.1)

like terms Terms of a variable expression that have the same variable part. (Sec. 2.2)

line of best fit A line drawn to approximate data that are graphed as points in a coordinate system. (Sec. 7.4)

linear equation in two variables An equation of the form $y = mx + b$, where m and b are constants; also called a linear function. (Sec. 7.2)

linear function An equation of the form $y = mx + b$, where m and b are constants; also called a linear equation in two variables. (Sec. 7.2)

linear model A first-degree equation that is used to describe a relationship between quantities. (Sec. 7.4)

literal equation An equation that contains more than one variable. (Sec. 6.7)

markdown The amount by which a retailer reduces the regular price of a product for a promotional sale. (Sec. 3.2)

markup The difference between selling price and cost. (Sec. 3.2)

markup rate The percent of retailer's cost that the markup represents. (Sec. 3.2)

monomial A number, a variable, or a product of numbers and variables; a polynomial of one term. (Sec. 4.1)

multiplicative inverse of a number The reciprocal of a number. (Sec. 2.2)

natural numbers The numbers 1, 2, 3, ... (Sec. 1.1)

negative integers The numbers ..., −4, −3, −2, −1. (Sec. 1.1)

negative slope A property of a line that slants downward to the right. (Sec. 7.3)

nonfactorable over the integers A polynomial that does not factor using only integers. (Sec. 5.2)

null set The set that contains no elements; also called the empty set. (Sec. 9.1)

numerical coefficient The number part of a variable term. When the numerical coefficient is 1 or −1, the 1 is usually not written. (Sec. 2.1)

obtuse angle An angle whose measure is between 90° and 180°. (Sec. 3.5)

odd integer An integer that is not divisible by 2. (Sec. 3.4)

opposite of a polynomial The polynomial created when the sign of each term of the original polynomial is changed. (Sec. 4.1)

opposites Numbers that are the same distance from zero on the number line, but on opposite sides; also called additive inverses. (Sec. 1.1)

ordered pair Pair of numbers, such as (a, b) that can be used to identify a point in the plane determined by the axes of a rectangular coordinate system. (Sec. 7.1)

Order of Operations Agreement A set of rules that tell us in what order to perform the operations that occur in a numerical expression. (Sec. 1.6)

ordinate The second number in an ordered pair. It measures a vertical distance and is also called the second coordinate. (Sec. 7.1)

origin The point of intersection of the two coordinate axes that form a rectangular coordinate system. (Sec. 7.1)

parabola The graph of a quadratic equation in two variables. (Sec. 11.4)

parallel lines Lines that never meet; the distance between them is always the same. Parallel lines have the same slope. (Sec. 3.5, 7.3)

percent Parts of 100. (Sec. 1.4)

perfect square The square of an integer (Sec. 10.1)

perfect-square trinomial A trinomial that is the square of a binomial. (Sec. 5.4)

perimeter The distance around a plane geometric figure. (Sec. 3.5)

perpendicular lines Intersecting lines that form right angles. (Sec. 3.5)

plane Flat surface determined by the intersection of two lines. (Sec. 7.1)

plotting a point in the plane Placing a dot at the location given by the ordered pair; also called graphing a point in the plane. (Sec. 7.1)

point-slope formula If (x_1, y_1) is a point on a line with slope m, then $y - y_1 = m(x - x_1)$. (Sec. 7.4)

polynomial A variable expression in which the terms are monomials. (Sec. 4.1)

positive integers The integers, 1, 2, 3, 4, ... (Sec. 1.1)

positive slope A property of a line that slants upward to the right. (Sec. 7.3)

prime polynomial A polynomial that is nonfactorable over the integers. (Sec. 5.2)

principal square root The positive square root of a number. (Sec. 10.1)

product In multiplication, the result of multiplying two numbers. (Sec. 1.3)

proportion An equation that states the equality of two ratios or rates. (Sec. 6.6)

Pythagorean Theorem The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs. (Sec. 10.4)

quadrant One of the four regions into which the two axes of a rectangular coordinate system divide the plane. (Sec. 7.1)

quadratic equation An equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants and a is not equal to zero; also called a second-degree equation. (Sec. 5.5, 11.1)

quadratic equation in two variables An equation of the form $y = ax^2 + bx + c$, where a is not equal to zero. (Sec. 11.4)

quadratic function A quadratic function is given by $f(x) = ax^2 + bx + c$, where a is not equal to zero. (Sec. 11.4)

radical The symbol $\sqrt{\ }$, which is used to indicate the positive, or principal, square root of a number. (Sec. 10.1)

radical equation An equation that contains a variable expression in a radicand. (Sec. 10.4)

radicand In a radical expression, the expression under the radical sign. (Sec. 10.1)

range The set of second coordinates of the ordered pairs in a relation. (Sec. 7.1)

rate The quotient of two quantities that have different units. (Sec. 6.6)

rate of work That part of a task that is completed in one unit of time. (Sec. 6.8)

ratio The quotient of two quantities that have the same unit. (Sec. 6.6)

rational expression A fraction in which the numerator or denominator is a polynomial. (Sec. 6.1)

rational number A number that can be written in the form a/b, where a and b are integers and b is not equal to zero. (Sec. 1.4)

rationalizing the denominator The procedure used to remove a radical from the denominator of a fraction. (Sec. 10.3)

real numbers The rational numbers and the irrational numbers. (Sec. 1.4)

reciprocal Interchanging the numerator and denominator of a rational number yields that number's reciprocal. (Sec. 2.2, 6.1)

rectangular coordinate system System formed by two number lines, one horizontal and one vertical, that intersect at the zero point of each line. (Sec. 7.1)

relation Any set of ordered pairs. (Sec. 7.1)

repeating decimal Decimal that is formed when dividing the numerator of its fractional counterpart by the denominator results in a decimal part wherein a block of digits repeats infinitely. (Sec. 1.4)

right angle An angle whose measure is 90 degrees. (Sec. 3.5)

roster method Method of writing a set by enclosing a list of the elements in braces. (Sec. 1.1, 9.1)

scatter diagram A graph of collected data as points in a coordinate system. (Sec. 7.4)

scientific notation Notation in which each number is expressed as the product of two factors, one a number between 1 and 10 and the other a power of ten. (Sec. 4.4)

second coordinate The second number in an ordered pair. It measures a vertical distance and is also called the ordinate. (Sec. 7.1)

second-degree equation An equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants and a is not equal to zero; also called a quadratic equation. (Sec. 11.1)

selling price The price for which a business sells a product to a customer. (Sec. 3.2)

set A collection of objects. (Sec. 1.1, 9.1)

set-builder notation A method of designating a set that makes use of a variable and a certain property that only elements of that set possess. (Sec. 9.1)

similar objects Similar objects have the same shape but not necessarily the same size. (Sec. 6.6)

simplest form of a fraction A fraction is in simplest form when there are no common factors in the numerator and the denominator. (Sec. 1.4)

simplest form of a rational expression A rational expression is in simplest form when the numerator and denominator have no common factors. (Sec. 6.1)

slope The measure of the slant of a line. The symbol for slope is m. (Sec. 7.3)

slope-intercept form The slope-intercept form of an equation of a straight line is $y = mx + b$. (Sec. 7.3)

solution of a system of equations in two variables An ordered pair that is a solution of each equation of the system. (Sec. 8.1)

solution of an equation A number that, when substituted for the variable, results in a true equation. (Sec. 3.1)

solution of an equation in two variables An ordered pair whose coordinates make the equation a true statement. (Sec. 7.1)

solution set of an inequality A set of numbers, each element of which, when substituted for the variable, results in a true inequality. (Sec. 9.2)

solving an equation Finding a solution of the equation. (Sec. 3.1)

square root A square root of a positive number x is a number a for which $a^2 = x$. (Sec. 10.1)

standard form A quadratic equation is in standard form when the polynomial is in descending order and equal to zero. $ax^2 + bx + c = 0$ is in standard form. (Sec. 5.5, 11.1)

straight angle An angle whose measure is 180 degrees. (Sec. 3.5)

substitution method An algebraic method of finding an exact solution of a system of equations. (Sec. 8.2)

sum In addition, the total of two or more numbers. (Sec. 1.2)

supplementary angles Two angles whose sum is 180°. (Sec. 3.5)

system of equations Equations that are considered together. (Sec. 8.1)

terminating decimal Decimal that is formed when dividing the numerator of its fractional counterpart by the denominator results in a remainder of zero. (Sec. 1.4)

terms of a variable expression The addends of the expression. (Sec. 2.1)

transversal A line intersecting two other lines at two different points. (Sec. 3.5)

triangle A three-sided closed figure. (Sec. 3.5)

trinomial A polynomial of three terms. (Sec. 4.1)

undefined slope A property of a vertical line. (Sec. 7.3)

uniform motion The motion of a moving object whose speed and direction do not change. (Sec. 3.6, 6.8)

union of sets *A* and *B* The set that contains all the elements of A and all the elements of B. (Sec. 9.1)

value of the variable The number assigned to the variable. (Sec. 2.1)

variable A letter of the alphabet used to stand for a number that is unknown or that can change. (Sec. 1.1)

variable expression An expression that contains one or more variables. (Sec. 2.1)

variable part In a variable term, the variable or variables and their exponents. (Sec. 2.1)

variable term A term composed of a numerical coefficient and a variable part. (Sec. 2.1)

vertical angles Two angles that are on opposite sides of the intersection of two lines. (Sec. 3.5)

***x*-coordinate** The abscissa in an xy-coordinate system. (Sec. 7.1)

***x*-intercept** The point at which a graph crosses the x-axis. (Sec. 7.3, 11.4)

***xy*-coordinate system** A rectangular coordinate system in which the horizontal axis is labeled x and the vertical axis is labeled y. (Sec. 7.1)

***y*-coordinate** The ordinate in an xy-coordinate system. (Sec. 7.1)

***y*-intercept** The point at which a graph crosses the y-axis. (Sec. 7.3)

zero slope A property of a horizontal line. (Sec. 7.3)

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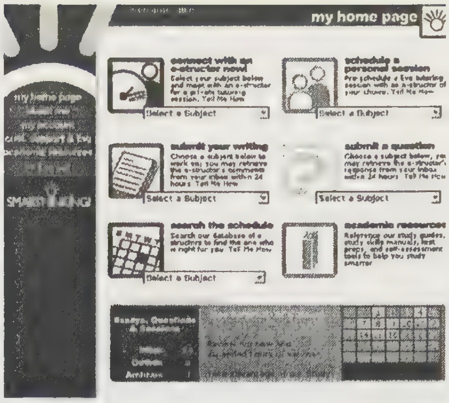
There's more than one way to *succeed* this semester...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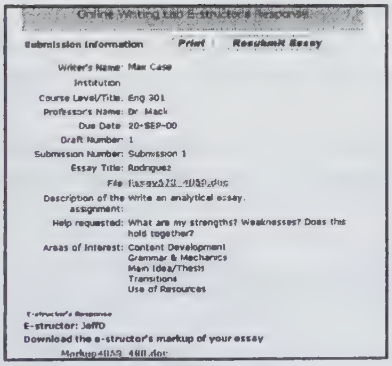

Online Writing Lab E-structor's Response

Submission Information

Writer's Name: Max Case
Institution
Course Level/Title: Eng 301
Professor's Name: Dr. Mack
Due Date: 20-SEP-00
Draft Number: 1
Submission Number: Submission 1
Essay Title: Rodriguez
File: Essay573_4059.doc
Description of the assignment: Write an analytical essay.
Help requested: What are my strengths? Weaknesses? Does this hold together?
Areas of Interest: Content Development
Grammar & Mechanics
Main Idea/Thesis
Transitions
Use of Resources

E-structor's Response
E-structor: JeffO
Download the e-structor's markup of your essay
Markup4059_488.doc

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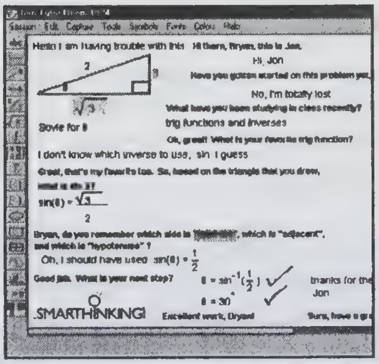

Hello I am having trouble with this
Hi there, Bryan, this is Jon.
Hi, Jon
Have you gotten started on this problem yet
No, I'm totally lost
What have you been studying in class recently?
trig functions and inverses
Oh, great! What is your favorite trig function?
Solve for θ
I don't know which inverse to use, sin I guess
Great, that's my favorite too. So, based on the triangle that you drew, what is sin θ?
$\sin(\theta) = \frac{\sqrt{3}}{2}$
Bryan, do you remember which side is [illegible], which is "adjacent", and which is "hypotenuse"?
Oh, I should have used $\sin(\theta) = \frac{1}{2}$
Good job. What is your next step?
$\theta = \sin^{-1}(\frac{1}{2})$ ✓
$\theta = 30°$ ✓
thanks for the
Jon
Excellent work, Bryan!
Sure, have a gr

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Table of Equations and Formulas

[illegible]

[illegible]

[illegible]

$y = mx + b$

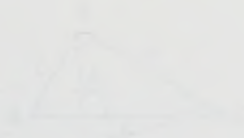
[illegible]

[illegible]

Quadratic Formula

[illegible]

Perimeter and Area of a Triangle, and Sum of the Measures of the Angles

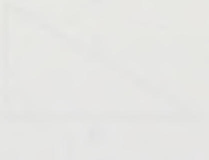


$P = a + b + c$

$A = \frac{1}{2}bh$

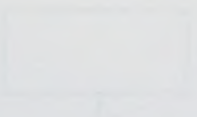
$A + B + C = 180°$

Pythagorean Theorem



$a^2 + b^2 = c^2$

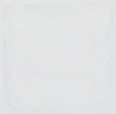
Perimeter and Area of a Rectangle



$P = 2l + 2w$

$A = lw$

Perimeter and Area of a Square



$P = 4s$

$A = s^2$

Circumference and Area of a Circle

$C = 2\pi r$

$A = \pi r^2$

Volume of a Rectangular Solid

$V = lwh$

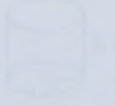
Volume of a Cube

$V = s^3$

Volume of a Sphere

$V = \frac{4}{3}\pi r^3$

Volume of a Right Circular Cylinder



$V = \pi r^2 h$

Table of Equations and Formulas

Slope of a straight line

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1},\ x_1 \neq x_2$$

Slope-intercept form of a straight line

$$y = mx + b$$

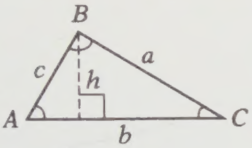
Point–Slope formula

$$y - y_1 = m(x - x_1)$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Perimeter and Area of a Triangle, and Sum of the Measures of the Angles

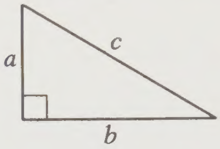


$$P = a + b + c$$

$$A = \frac{1}{2}bh$$

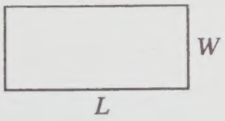
$$A + B + C = 180°$$

Pythagorean Theorem



$$a^2 + b^2 = c^2$$

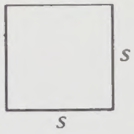
Perimeter and Area of a Rectangle



$$P = 2L + 2W$$

$$A = LW$$

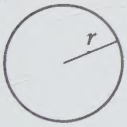
Perimeter and Area of a Square



$$P = 4s$$

$$A = s^2$$

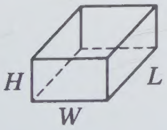
Circumference and Area of a Circle



$C = 2\pi r$ or $C = \pi d$

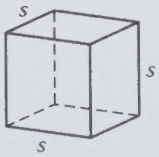
$$A = \pi r^2$$

Volume of a Rectangular Solid



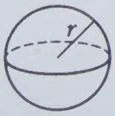
$$V = LWH$$

Volume of a Cube



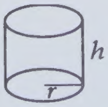
$$V = s^3$$

Volume of a Sphere



$$V = \frac{4}{3}\pi r^3$$

Volume of a Right Circular Cylinder



$$V = \pi r^2 h$$

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